Two New Classes of Locating-Dominating Codes

Ville Junnila, **Tero Laihonen** and **Tuomo Lehtilä***
Department of Mathematics and Statistics
University of Turku, FI-20014 Turku, Finland
viljun@utu.fi, terolai@utu.fi and tualeh@utu.fi

1 Introduction

Sensor networks are systems designed for environmental monitoring. Various location detection systems such as fire alarm and surveillance systems can be viewed as examples of sensor networks. For location detection, a sensor can be placed in any location of the network. The sensor monitors its neighbourhood (including the location of the sensor itself) and reports possible irregularities such as a fire or an intruder in the neighbouring locations. Based on the reports of the sensors, a central controller attempts to determine the location of a possible irregularity in the network. Usually, the aim is to minimize the number of sensors in the network. More explanation regarding location detection in sensor networks can be found in [4, 12, 15].

A sensor network can be modeled as a simple, undirected and connected graph $G = (V, E)$ as follows: the set of vertices V of the graph represents the locations of the network and the edge set E of the graph represents the connections between the locations. In other words, a sensor can be placed in each vertex of the graph and the sensor placed in the vertex u monitors u itself and the vertices neighbouring u. In what follows, we present some basic terminology and notation regarding graphs. The *open neighbourhood* of $u \in V$ consists of the vertices adjacent to u and it is denoted by $N(u)$. The *closed neighbourhood* of u is defined as $N[u] = \{u\} \cup N(u)$. A nonempty subset C of V is called a *code* and the elements of the code are called *codewords*. In this paper, the code C (usually) represents the set of locations where the sensors have been placed on. For the set of sensors monitoring a vertex $u \in V$, we use the following notation:

$$I(G, C; u) = I(C; u) = I(u) = N[u] \cap C.$$

We call $I(u)$ the *identifying set* (or the *I-set*) of u. The notation of identifying set can also be generalized for a subset U of V as follows:

$$I(G, C; U) = I(C; U) = I(U) = \bigcup_{u \in U} I(C; u).$$

As stated above, a sensor $u \in V$ reports that an irregularity has been detected if there is (at least) one in the closed neighbourhood $N[u]$. In what follows, we divide into two different situations depending on the capability of a sensor to distinguish whether the irregularity has been spotted in the location of the sensor itself or in its (open) neighbourhood. More precisely, we have the following two cases:

(i) In the first case, we assume that a sensor $u \in V$ reports 1 if the there is an irregularity in $N[u]$, and otherwise it reports 0.

(ii) In the second case, we assume that a sensor $u \in V$ reports 2 if there is an irregularity in u, it reports 1 if there is one in $N(u)$ and no irregularities in u, and otherwise it reports 0.

*Research supported by the University of Turku Graduate School (UTUGS).

Assume first that the sensors work as in (i). Notice then that if the sensors in the code C are located in such places that $I(C;u)$ is nonempty and unique for all $u \in V$, then an irregularity in the network can be located by comparing $I(C;u)$ to identifying sets of other vertices. This leads to the following definition of *identifying codes*, which were first introduced by Karpovsky *et al.* in [11]. For various papers regarding identification and related problems, we refer to the online bibliography [13].

Definition 1. A code $C \subseteq V$ is *identifying* in G if for all distinct $u, v \in V$ we have $I(C;u) \neq \emptyset$ and

$$I(C;u) \neq I(C;v).$$

An identifying code C in a finite graph G with the smallest cardinality is called *optimal* and the number of codewords in an optimal identifying code is denoted by $\gamma^{ID}(G)$.

Let C be an identifying code in G. By the definition, the identifying code C works correctly if there is simultaneously at most one irregularity in the network. However, using the identifying code C, we cannot locate or even detect more than one irregularity in the network. Indeed, there might exist vertices $u, v_1, v_2 \in V$ such that $I(C;u) = I(C;\{v_1, v_2\})$. If now the sensors in $I(C;u)$ output 1 and all the other sensors output 0, then it is deduced that the irregularity is in u. However, as the irregularities could also be in v_1 and v_2, we might determine a false location and more disturbingly not even notice that something is wrong. To overcome this problem, in [7], so called self-identifying codes, which are able to locate one irregularity and detect multiple ones, were introduced. (Notice that in the original paper self-identifying codes are called 1^+-identifying.) The formal definition of self-identifying codes is given as follows.

Definition 2. A code $C \subseteq V$ is called *self-identifying* in G if the code C is identifying in G and for all $u \in V$ and $U \subseteq V$ such that $|U| \geq 2$ we have

$$I(C;u) \neq I(C;U).$$

A self-identifying code C in a finite graph G with the smallest cardinality is called *optimal* and the number of codewords in an optimal self-identifying code is denoted by $\gamma^{SID}(G)$.

In addition to [7], self-identifying codes have also been previously discussed in [9, 10]. In these papers, two useful characterizations have been presented for self-identifying codes. These characterizations are presented in the following theorem.

Theorem 3 ([7, 9, 10])**.** *Let C be a code in G. Then the following statements are equivalent:*

(i) The code C is self-identifying in G.

(ii) For all distinct $u, v \in V$, we have $I(C;u) \setminus I(C;v) \neq \emptyset$.

(iii) For all $u \in V$, we have $I(C;u) \neq \emptyset$ and

$$\bigcap_{c \in I(C;u)} N[c] = \{u\}.$$

As stated earlier, self-identifying codes can locate one irregularity and detect multiple ones. Besides that, the characterization (iii) of the previous theorem also gives another useful property for self-identifying codes. Namely, the location of an irregularity can be determined without comparison to other identifying sets, since for all $u \in V$ the neighbourhoods of the codewords in $I(u)$ intersect uniquely in u.

So far, we have discussed the case where it is assumed that each sensor outputs 1 or 0 depending on whether there is an irregularity in the neighbourhood or not. In what follows, we now focus on the case (ii) where a sensor can also distinguish if the irregularity is on the location of the sensor itself. Then notice that if the sensors in the code C are located in such places that $I(C;u)$

is nonempty and unique for all $u \in V \setminus C$, then an irregularity in the network can be located by comparing $I(C; u)$ to identifying sets of other non-codewords. Indeed, we do not have to worry about vertices in C as an irregularity in such locations is immediately determined by a sensor outputting 2. This leads to the following definition of *locating-dominating codes (or sets)*, which were first introduced by Slater in [14, 16, 17].

Definition 4. A code $C \subseteq V$ is *locating-dominating* in G if for all distinct $u, v \in V \setminus C$ we have $I(C; u) \neq \emptyset$ and

$$I(C; u) \neq I(C; v).$$

A locating-dominating code C in a finite graph G with the smallest cardinality is called *optimal* and the number of codewords in an optimal locating-dominating code is denoted by $\gamma^{LD}(G)$.

Comparing the definitions of identifying and locating-dominating codes, we immediately notice their apparent similarities; in the case of identification we require that the identifying sets $I(u)$ are unique for all vertices and in the case of location-domination the same is required for non-codewords. Therefore, as self-identifying codes are a natural specialization of regular identifying codes, it is obvious to consider if something similar could be done for locating-dominating codes. Indeed, the characterizations of Theorem 3 gives two natural ways to define new types of locating-dominating codes with similar kind of beneficial properties as self-identifying codes have over regular identifying codes. The definitions of these codes are given as follows.

Definition 5. A code $C \subseteq V$ is *self-locating-dominating* in G if for all $u \in V \setminus C$ we have $I(C; u) \neq \emptyset$ and

$$\bigcap_{c \in I(C;u)} N[c] = \{u\}.$$

A self-locating-dominating code C in a finite graph G with the smallest cardinality is called *optimal* and the number of codewords in an optimal self-locating-dominating code is denoted by $\gamma^{SLD}(G)$.

Definition 6. A code $C \subseteq V$ is *solid-locating-dominating* in G if for all distinct $u, v \in V \setminus C$ we have

$$I(C; u) \setminus I(C; v) \neq \emptyset.$$

A solid-locating-dominating code C in a finite graph G with the smallest cardinality is called *optimal* and the number of codewords in an optimal solid-locating-dominating code is denoted by $\gamma^{DLD}(G)$.

In the following theorem, we present characterizations for self-locating-dominating and solid-locating-dominating codes. Comparing these characterizations to the original definitions of the codes, the differences of the codes become apparent. In particular, it immediate that each self-locating-dominating code is also a solid-locating-dominating code.

Theorem 7. *We have the following characterizations:*

(i) A code $C \subseteq V$ is self-locating-dominating if and only if for all distinct $u \in V \setminus C$ and $v \in V$ we have

$$I(C; u) \setminus I(C; v) \neq \emptyset.$$

(ii) A code $C \subseteq V$ is solid-locating-dominating if and only if for all $u \in V \setminus C$ we have

$$\left(\bigcap_{c \in I(C;u)} N[c] \right) \setminus C = \{u\}.$$

As discussed earlier, self-identifying codes have benefits over regular identifying codes; they detect more than one irregularity and locate one irregularity without comparison to other identifying sets. In what follows, we study the same properties concerning self-locating-dominating and solid-locating-dominating codes:

- Let us begin by considering the ability to locate an irregularity without comparison to other identifying sets. For self-locating-dominating codes, this property immediately follows from the definition. Analogously, the property is obtained for solid-locating-dominating codes by Theorem 7(ii).
- It can also be shown that if C is a self-locating-dominating code in G, then it can detect if there are multiple irregularities. On the other hand, solid-locating-dominating codes may have problems detecting multiple irregularities if they can happen in locations with sensors.

In the paper, our main focus is on the new types of locating-dominating codes. However, we also present some results for regular locating-dominating codes. In Section 2, we consider the different types of locating-dominating codes in the Cartesian product of two complete graphs, which is sometimes also called the rook's graph. Furthermore, in the rook's graphs, we obtain optimal codes for regular location-domination, self-location-domination and solid-location-domination. In Section 3, we consider similar problems in the binary Hamming space (or hypercube) $\mathbb{F}^n$, where n is a positive integer. In particular, we present an infinite family of optimal self-locating-dominating codes and construct regular locating-dominating codes with the smallest known cardinalities; especially proving that $309 \leq \gamma^{LD}(\mathbb{F}^{11}) \leq 320$.

2 Location-domination in the rook's graphs

In this section, we consider the different locating-dominating codes in the Cartesian product of two complete graphs. The Cartesian product of graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is $G_1 \square G_2 = (V_1 \times V_2, E)$ where $(x, y)(x', y') \in E$ if and only if $x = x'$ and $yy' \in E_2$ or $y = y'$ and $xx' \in E_1$. If K_n and K_m are two complete graphs of order n and m, respectively, then $K_n \square K_m$ is known as rook's graph and can be viewed as a chess board with m rows and n columns. The closed neighbourhood of a vertex is determined by the movement of a rook in chess. We denote $V(K_n) = \{x_1, \dots, x_n\}$, $V(K_m) = \{y_1, \dots, y_m\}$ and kth *row* with $R_k = \{(x_i, y_k) \mid i = 1, \dots, n, 1 \leq k \leq m\}$ (resp. hth *column* $P_h = \{(x_h, y_i) \mid i = 1, \dots, m, 1 \leq k \leq n\}$).

Previously identification in rook's graphs has been studied in [6] and [5] and self-identification in [9] and [10]. In what follows, we are going to find optimal locating-dominating, self-locating-dominating and solid-locating-dominating codes in the rook's graphs. In the following theorem, we first begin by considering self-locating-dominating codes.

Theorem 8. *Let $G = K_n \square K_m$ be a rook's graph with $m \geq n \geq 1$. We have*

$$\gamma^{SLD}(G) = \begin{cases} m, & m \geq 2n, \text{ or } n = 1, \\ 2n, & 2n > m > n \geq 2, \\ 2n - 1, & m = n > 2, \\ 4, & n = m = 2. \end{cases}$$

In the following theorem, we will see that optimal solid-locating-dominating codes are mostly of the same size as optimal self-locating-dominating codes. However, this is only a superficial similarity. Actually, the structures of solid-locating-dominating codes vary more and there are more of them. For example, code $R_1 \cup P_1$ is an optimal solid-locating-dominating code when $n = m$ but it is not a self-locating-dominating code.

Theorem 9. *Let $G = K_n \square K_m$ be a rook's graph with $m \geq n \geq 1$. We have*

$$\gamma^{DLD}(G) = \begin{cases} m, & m \geq 2n \geq 4 \text{ or } n = 2, \\ 2n, & 2n > m > n > 2, \\ 2n - 1, & m = n > 2, \\ m - 1, & n = 1. \end{cases}$$

Finally, in the following theorem, we present optimal bounds for locating-dominating codes in the rook's graphs.

Theorem 10. *Let $G = K_n \square K_m$ be a rook's graph with $m \geq n \geq 1$. We have*

$$\gamma^{LD}(G) = \begin{cases} m-1, & m \geq 2n, \\ \lceil \frac{2n+2m}{3} \rceil - 1, & n \leq m \leq 2n-1. \end{cases}$$

Remark 11. In a forthcoming paper, we will discuss codes for location in graphs which are Cartesian products of several cliques. In particular, we will show that the conjecture of Goddard and Wash [5, Conjecture 4.3] concerning the cardinality of identifying codes in $K_n \square K_n \square K_n$ does not hold.

3 Location-domination in the binary Hamming spaces

In this section, we consider self-locating-dominating and solid locating-dominating codes in binary Hamming spaces of length n. A binary Hamming space of length n is a graph with the vertex set $\mathbb{F}^n = \{0,1\}^n$, and two vertices have an edge between them if they differ in exactly one coordinate. Vertices of $\mathbb{F}^n$ are called *words*. The distance $d(x,y)$ is the number of coordinates where words x and y differ. The *minimum distance* of code C is $\min\{d(c_1,c_2) \mid c_1, c_2 \in C\}$. The sizes of optimal self-locating-dominating and solid-locating-dominating codes in $\mathbb{F}^n$ are denoted by $\gamma^{SLD}(\mathbb{F}^n) = \gamma^{SLD}(n)$ and $\gamma^{DLD}(\mathbb{F}^n) = \gamma^{DLD}(n)$, respectively.

In what follows, we first concentrate on the case of self-locating-dominating codes. In particular, we present an infinite family of optimal self-locating-dominating codes in binary Hamming spaces. This result is based on the following theorem, where we give a characterization for self-locating-dominating codes. The characterization is based on the fact that if a non-codeword has at least three codewords in its I-set, then no other word can have those same codewords in its I-set.

Theorem 12. *A code C is a self-locating-dominating code in $\mathbb{F}^n$ if and only if for each non-codeword w we have $|I(w)| \geq 3$.*

Compare this characterization to an analogous result for self-identifying codes presented in [7]: a code C is self-identifying in $\mathbb{F}^n$ if and only if for each $x \in \mathbb{F}^n$ we have $|I(C;x)| \geq 3$. With the characterization of Theorem 12 we form a lower bound for self-locating-dominating codes.

Theorem 13. *Let $n \geq 3$. We have*

$$\gamma^{SLD}(n) \geq \left\lceil \frac{3 \cdot 2^n}{n+3} \right\rceil .$$

The following cardinalities of optimal self-locating-dominating codes in $\mathbb{F}^n$ are gained with constructions based on linear code similar to the well-known Hamming codes.

Theorem 14. *Let n and k be positive integers such that $n = 3(2^k - 1)$. Then we have*

$$\gamma^{SLD}(n) = 2^{3(2^k-1)-k}.$$

Let $C \subseteq \mathbb{F}^n$ and $D \subseteq \mathbb{F}^m$ be codes. Then the *direct sum* of C and D is defined as $C \oplus D = \{(x,y) \mid x \in C, y \in D\}$. In the following theorem, it is shown that new self-locating-dominating codes can be formed from known ones using a direct sum.

Theorem 15. *Let $C \subseteq \mathbb{F}^n$ be a self-locating-dominating code. Then $D = C \oplus \mathbb{F}$ is also a self-locating-dominating code.*

Let us then concentrate on solid-locating-dominating codes. We will first give a lower bound such that its ratio to $2\frac{2^n}{n+1}$ approaches 1 as n tends to infinity. After that we will give an infinite sequence of solid-locating-dominating codes with the same limit. When we compare the sizes of optimal self-locating-dominating and solid-locating-dominating codes we see from Theorems 13 and 14 that optimal solid-locating-dominating codes are essentially smaller. In the following theorem, we first give a lower bound for solid-locating-dominating codes.

Theorem 16. *Let n be an integer such that $n \geq 5$. Then we have*

$$\gamma^{DLD}(n) \geq \left\lceil \left(1 + \frac{n-1}{n^2+n+2}\right) \frac{2^{n+1}}{n+1} \right\rceil.$$

In the following remark, we briefly compare the previously obtained lower bound to one for locating-dominating codes locating multiple irregularities.

Remark 17. In this paper, we have mainly studied locating-dominating codes which can locate one and detect multiple irregularities. Previously, in [8], so called $(1, \leq l)$-locating-dominating codes of type B ($(1, \leq l)$-LDB codes for short), which can locate multiple irregularities, have been studied. In [8, Theorem 5], the lower bound $\left\lceil \frac{2^{n+1}}{n+1} \right\rceil$ for $(1, \leq 2)$-LDB codes has been achieved. Since it can be shown that every $(1, \leq 2)$-LDB code is also a solid-locating-dominating code, our lower bound in Theorem 16 improves the lower bound for $(1, \leq 2)$-LDB codes in Hamming spaces.

Let us define the *covering radius* of C as $R(C) = \max_{x \in \mathbb{F}^n} \min_{c \in C}\{d(x,c) \mid x \in \mathbb{F}^n, c \in C\}$. When $n \geq 5$, the lower bound in Theorem 16 is attained by choosing as codewords all codewords and their neighbours of a code with covering radius two and minimum distance five. Unfortunately, codes like this are only known when $n = 5$ [2, Theorem 11.2.2]. Using this code, the following theorem is obtained.

Theorem 18. *We have $\gamma^{DLD}(5) = 12$.*

In general, solid-locating-dominating codes can be constructed from codes with covering radius two.

Theorem 19. *Let $D \subseteq \mathbb{F}^n$ be a code with $R(D) = 2$. Then*

$$C = \{c \in \mathbb{F}^n \mid c \in N[d], d \in D\}$$

is a solid-locating-dominating code.

In [2, Theorem 4.5.8], Struik has constructed an infinite sequence of codes with covering radius two such that we can build on top of it such a sequence of solid-locating-dominating codes that they converge to our lower bound.

Theorem 20. *There exists such a sequence of solid-locating-dominating codes $(C_n)_{n=1}^{\infty}$ that*

$$\lim_{n\to\infty} \frac{|C_n|}{2\frac{2^n}{n+1}} = 1.$$

Using direct sum, we can construct new solid-locating-dominating codes from existing ones in a similar fashion as with self-locating-dominating codes.

Theorem 21. *Let $C \subseteq \mathbb{F}^n$ be a solid-locating-dominating code. Then $D = C \oplus \mathbb{F}$ is also a solid-locating-dominating code.*

Above, we have discussed self-locating-dominating and solid-locating-dominating codes in binary Hamming spaces. In what follows, we briefly consider regular locating-dominating codes. In particular, for certain lengths, we provide locating-dominating codes with the smallest known cardinalities. Previously, locating-dominating codes in $\mathbb{F}^n$ have been considered, for example, in [3, 8]. For future considerations, we first define the mapping $\pi : \mathbb{F}^n \to \mathbb{F}$ as follows:

$$\pi(u) = \begin{cases} 0, & \text{if } u \text{ has an even number of ones;} \\ 1, & \text{otherwise.} \end{cases}$$

Then, in the following theorem, we introduce a novel approach for constructing new locating-dominating codes based on known (suitable) identifying codes.

Theorem 22. *Let C be an identifying code in $\mathbb{F}^n$ such that $|I(C;u)| \geq 2$ for all $u \in \mathbb{F}^n \setminus C$. Then*

$$D = \{(\pi(u), u, u + c) \mid u \in \mathbb{F}^n, c \in C\}$$

is a locating-dominating code in $\mathbb{F}^{2n+1}$.

The best known upper bounds on $\gamma^{LD}(\mathbb{F}^n)$ for $1 \leq n \leq 10$ have been presented in [3, Table 3]. For lengths $n > 10$, the smallest known locating-dominating codes are actually identifying codes. (Recall that by the definitions any identifying code is also locating-dominating.) The currently best known upper bounds on $\gamma^{ID}(\mathbb{F}^n)$ can be found in [1]. In the following corollary, we present locating-dominating codes in $\mathbb{F}^n$ with the smallest known cardinalities for the lengths $n = 11$ and $n = 17$. These constructions significantly improve on the known upper bounds $\gamma^{LD}(\mathbb{F}^{11}) \leq \gamma^{ID}(\mathbb{F}^{11}) \leq 352$ and $\gamma^{LD}(\mathbb{F}^{17}) \leq \gamma^{ID}(\mathbb{F}^{17}) \leq 18558$.

Corollary 23. *We have $\gamma^{LD}(\mathbb{F}^{11}) \leq 320$ and $\gamma^{LD}(\mathbb{F}^{17}) \leq 16384$.*

With the help of the following theorem, which has been shown in [8, Theorem 7], we can construct new improved locating-dominating codes from codes attained in Corollary 23.

Theorem 24 ([8])**.** *If $C \subseteq \mathbb{F}^n$ is a locating-dominating code, then $C \oplus \mathbb{F}$ is also a locating-dominating code.*

The smallest currently known upper bounds for locating-dominating codes of lengths $n = 12$ and $n = 18$ are 684 and 35604 respectively [1].

Corollary 25. *We have $\gamma^{LD}(\mathbb{F}^{12}) \leq 640$ and $\gamma^{LD}(\mathbb{F}^{18}) \leq 32768$.*

In [8, Theorem 15], a lower bound, which is currently the best known, for $\gamma^{LD}(\mathbb{F}^n)$ has been presented. Applying the lower bound on the lengths $n = 11$, $n = 12$, $n = 17$ and $n = 18$, we obtain that $\gamma^{LD}(\mathbb{F}^{11}) \geq 309$, $\gamma^{LD}(\mathbb{F}^{12}) \geq 576$, $\gamma^{LD}(\mathbb{F}^{17}) \geq 13676$ and $\gamma^{LD}(\mathbb{F}^{18}) \geq 26006$. Thus, comparing the lower bounds to the constructions of the previous corollaries, we can state the obtained codes are rather small.

References

[1] I. Charon, G. Cohen, O. Hudry, and A. Lobstein. New identifying codes in the binary Hamming space. *European J. Combin.*, 31(2):491–501, 2010.

[2] G. Cohen, I. Honkala, S. Litsyn, and A. Lobstein. *Covering codes*, volume 54 of *North-Holland Mathematical Library*. North-Holland Publishing Co., Amsterdam, 1997.

[3] G. Exoo, V. Junnila, T. Laihonen, and S. Ranto. Locating vertices using codes. In *Proceedings of the Thirty-Ninth Southeastern International Conference on Combinatorics, Graph Theory and Computing*, volume 191, pages 143–159, 2008.

[4] N. Fazlollahi, D. Starobinski, and A. Trachtenberg. Connected identifying codes. *IEEE Trans. Inform. Theory*, 58(7):4814–4824, 2012.

[5] W. Goddard and K. Wash. ID codes in Cartesian products of cliques. *J. Combin. Math. Combin. Comput.*, 85:97–106, 2013.

[6] S. Gravier, J. Moncel, and A. Semri. Identifying codes of Cartesian product of two cliques of the same size. *Electron. J. Combin.*, 15(1):Note 4, 7, 2008.

[7] I. Honkala and T. Laihonen. On a new class of identifying codes in graphs. *Inform. Process. Lett.*, 102(2-3):92–98, 2007.

[8] I. Honkala, T. Laihonen, and S. Ranto. On locating-dominating codes in binary Hamming spaces. *Discrete Math. Theor. Comput. Sci.*, 6(2):265–281, 2004.

[9] V. Junnila and T. Laihonen. Collection of codes for tolerant location. In *Proceedings of the Bordeaux Graph Workshop*, pages 176–179, 2016.

[10] V. Junnila and T. Laihonen. Tolerant location detection in sensor networks. Submitted, 2016.

[11] M. G. Karpovsky, K. Chakrabarty, and L. B. Levitin. On a new class of codes for identifying vertices in graphs. *IEEE Trans. Inform. Theory*, 44(2):599–611, 1998.

[12] M. Laifenfeld and A. Trachtenberg. Disjoint identifying-codes for arbitrary graphs. In *Proceedings of International Symposium on Information Theory, 2005. ISIT 2005*, pages 244–248, 2005.

[13] A. Lobstein. Watching systems, identifying, locating-dominating and discrminating codes in graphs, a bibliography. Published electronically at `http://perso.enst.fr/~lobstein/debutBIBidetlocdom.pdf`.

[14] D. F. Rall and P. J. Slater. On location-domination numbers for certain classes of graphs. *Congr. Numer.*, 45:97–106, 1984.

[15] S. Ray, D. Starobinski, A. Trachtenberg, and R. Ungrangsi. Robust location detection with sensor networks. *IEEE Journal on Selected Areas in Communications*, 22(6):1016–1025, August 2004.

[16] P. J. Slater. Domination and location in acyclic graphs. *Networks*, 17(1):55–64, 1987.

[17] P. J. Slater. Dominating and reference sets in a graph. *J. Math. Phys. Sci.*, 22:445–455, 1988.