

P. Busch, P. Lahti, J.-P. Pellonpää, K. Ylinen

Quantum Measurement

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Preface

Quantum Measurement is a book on the mathematical and conceptual foundations of quantum mechanics, with a focus on its measurement theory. It has been written primarily for students of physics and mathematics with a taste for mathematical rigour and conceptual clarity in their quest to understand quantum mechanics. We hope it will also serve as a useful reference text for researchers working in a broad range of subfields of quantum physics and its foundations.

The exposition is divided into four Parts entitled *Mathematics* (Chapters 2-8), *Elements* (Chapters 9-13), *Realisations* (Chapters 14-19), and *Foundations* (Chapters 20-23). An overview of each Part is given in Chapter 1, and each Chapter begins with a brief non-technical outline of its contents.

A glance through the table of contents shows that different chapters require somewhat different backgrounds and levels of prerequisite knowledge on the part of the reader. The material is arranged in a logical (linear) order, so it should be possible to read the book from beginning to end and gain the relevant skills along the way, either from the text itself or occasionally from other sources cited. However, the reader should also be able to start with any part or chapter of her or his interest and turn to earlier parts where needed.

Part I is designed to be accessible to a reader possessing an undergraduate level of familiarity with linear algebra and elementary metric space theory. Chapters 2 and 3 can be read as an introduction to the part of Hilbert space theory which does not need measure and integration theory. The latter becomes an essential tool from Chapter 4 onwards, so we give a summary of the key concepts and some relevant results. Starting with Section 4.10, and more essentially from Chapter 6 on, we occasionally need the basic notions of general topology and topological vector spaces. Elements of the theory of C^* -algebras and von Neumann algebras are briefly summarised in Chapter 6, but their role is very limited in the sequel.

While prior study of quantum mechanics might be found useful, it is not a prerequisite for a successful study of the book. The essence of the work is the development of tools for a rigorous approach to central questions of quantum mechanics, which are often considered in a more intuitive and heuristic style in the literature. In this way the authors hope to contribute to the clarification of some key issues in the discussions concerning the foundations and interpretation of quantum mechanics.

The bibliography is fairly extensive, but it is not intended to be comprehensive in any sense. It contains many works on general background and key papers in the development of the field of quantum measurement. Naturally, most of especially the more recent references relate to the topics central to this book, in which the authors and their collaborators have also had their share.

The reader will notice that the word *measure* is used in a variety of meanings, which should, however, be clear from the context. A measure as a math-

emational concept is a set function which can be specified by giving the value space: we talk about (positive) measures, probability measures, complex measures, operator measures etc. We also speak about measures of quantifiable features such as accuracy, disturbance, or unsharpness. The etymologically related word *measurement* may be taken to refer to a process, but it is also given a precise mathematical content that can be viewed as an abstraction of this process.

Much of the material in this book has been extracted and developed from various series of lecture notes for graduate and postgraduate courses in mathematics and theoretical physics held over many years at the universities of Helsinki, Turku and York. In its totality, however, the work is considerably more comprehensive than the union of these courses. It reflects the development of its subject from the early days of quantum mechanics while the selection of topics is inevitably influenced by the authors' research interests. In fact, the book emerged in its present shape from a decade-long collective effort alongside our investigations into quantum measurement theory and its applications. At this point we wish to express our deep gratitude and appreciation to the many colleagues, scientific friends and, not least, our students with whom we have been fortunate to collaborate and discuss fundamental problems of quantum physics.

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