The Role of Beliefs and Motivational Variables in Enhancing Word Problem Solving

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Abstract

A Word Problem Enrichment programme (WPE) has been found to increase student word problem solving performance when facing non-routine and application problems. However, it is unknown if the WPE has an impact on student beliefs about word problem solving, and how the WPE works for students with different motivation in learning mathematics. This study investigated the impact of the WPE on student beliefs about word problem solving by using advanced statistical methods (LPA and SEM) to analyse relations among the different cognitive, motivation, and belief factors. A total of 170 fourth- and sixth-grade students from elementary schools participated. Results showed that the effects of WPE are various depending on students' initial motivation level. The impacts of the WPE on student beliefs were found only in students with a low initial motivation level, while its impacts on student problem-solving performance were found only in students with a high initial motivation level.

Keywords: word problem (mathematics); word problem solving; motivation; beliefs

Word problem solving and mathematical modelling are widely seen as important aims of mathematics learning, which can prepare students to use mathematics in everyday situations (e.g., Verschaffel, Greer, & De Corte, 2000). However, teachers face difficulties in teaching mathematics that go beyond arithmetic (Greer, 1997). Several researchers in the field of mathematical problem solving research have shown that the differences in student performance cannot be explained purely as the result of differences in cognitive skills, but the role of beliefs and motivational variables must be taken into account in order to provide adequate explanation of the individual's differences in problem solving performance (Brose, Schmiedek, Lövdén, & Lindenberger, 2012; Maaß & Schlöglmann, 2009; Mason & Scrivani, 2004; McLeod & Adams, 1989; Pepin & Roesken-Winter, 2015; Pintrich & Schunk, 2002; Schoenfeld, 1992; Seegers & Boekaerts, 1993; Wigfield & Cambria, 2010). This study is about a Word Problem Enrichment programme (WPE) designed to encourage teachers to use innovative, self-created word problems to improve mathematical modelling and word problem solving performance in students. A positive impact of the WPE on student word problem solving performance was found in a previous study focusing on cognitive factors only (Pongsakdi, Laine, Veermans, Hannula-Sormunen, & Lehtinen, 2016). However, it is unknown whether the WPE would have an impact on student beliefs about mathematical word problem solving, and how the intervention works for students with different mathematics motivation. The aim of this study was to investigate the impact of the WPE on student beliefs about word problem solving by using Latent Profile Analysis (LPA) and Structural Equation Modelling (SEM) to analyse relations among the different cognitive, motivation, and belief factors.

Word problem solving and mathematical modelling

Word problems have been assigned an important role in elementary school mathematics curriculum (e.g., National Council of Teachers of Mathematics, 2010) as a means of teaching and learning mathematical modelling and problem solving which can prepare students to apply mathematics in everyday situations (Verschaffel et al., 2000). A word problem is text which describes a situation with question(s) to be answered by performing mathematical operation(s) based on a given set of descriptions (Verschaffel et al., 2000). Word problem solving, therefore, refers to an entire process of dealing with a word problem in order to solve it (Pongsakdi et al., 2016). One of the main rational for using word problems in mathematics education is that they can prepare students for mathematical modeling. Some researchers, for example Niss (2015), distinguished two kinds of mathematical modelling namely descriptive and prescriptive modelling which serve different purposes. Descriptive modelling attempts to capture with some extra mathematical domain in the real world and answer practical questions with the help of mathematical tools, while the aim of prescriptive modelling is to design, organize, or structure certain aspects of extra mathematical domains. Descriptive modelling is described with a circle which consists of interpretation of the real word situation, mathematisation of the questions, mathematical solutions and interpretation the mathematical solution in the real life context (e.g. Niss, 2015). Similar cyclic models have been used in defining processes of mathematical word problems. Verschaffel and colleagues (Depaepe, De Corte, & Verschaffel, 2015; Verschaffel, 2000) reviewed different descriptions of processes of mathematical modelling and word problem solving (e.g., Blum and Niss, 1991; Burkhardt, 1994; Mason, 2001), and concluded that, fundamentally, they consist of six major components which do not necessarily follow a linear order: 1) understanding and defining problem situation leading to a situation model; 2) developing a mathematical model base on proper situation model; 3) working through the mathematical model to acquire some mathematical results; 4) interpreting the result in respect to the original problem situation; 5) checking whether the interpreted mathematical result is suitable and reasonable for its purpose; and 6) communicating the acquired solution of the original word problem. Figure 1 presents these phases, but also possibilities for more superficial processes students may use when solving word problems in school context.

Figure 1. A process of mathematical word problem solving (adapted from Verschaffel et al., 2000, p.168)

Figure 1

Over the past few decades, a series of studies have been conducted in different countries to find evidence about a phenomenon called "suspension of sense-making" (see Verschaffel et al., 2000, for a review of the earlier replications). These studies revealed that many students have a habit of applying superficial comprehension strategies and do not develop a sufficient understanding of the situations (situation model) described in the given problems (see Figure 1). They see the choice of the mathematical operations with the given numbers to be computed as important without basing the mathematical model on a proper situation model (Reusser & Stebler, 1997). Even though students use deeper comprehension strategies, they have difficulties in making proper use of realistic thinking when solving word problems requiring the use of realistic considerations (Greer, 1993; Verschaffel, De Corte, & Lasure, 1994). For example, when students are asked to find out how many planks of 1 m can Steve get if he has bought 4 planks of 2.5 m each, only 13% of students gave the realistic answer, 8 planks (of 1 m), while 73% answered 10 (without using realistic considerations that these planks are not connected, and, therefore, Steve can get only 2 planks of 1 meter and a half-meter plank left over from each 2.5-meter plank.) (Verschaffel et al., 1994). The students showed a strong tendency to exclude realistic considerations in solving the word problems. Several researches suggested that the reason for students' superficial interpretations and unrealistic responses is not a cognitive deficit. It is assumed to be due to student beliefs, which were gradually developed in the context of schooling (Schoenfeld, 1991; Verschaffel et al., 2000).

Beliefs in mathematics learning

Research interest in the role of beliefs in learning mathematics increased during the 1980s (Hart, 1989), when researchers, who had initially included only cognitive components, failed to explain mathematical problem solving behaviour, and later found that other variables such as beliefs play an important role as constituting elements of problem solving processes (e.g., Depaepe et al., 2015; Garofalo, 1989; Garofalo & Lester, 1985; Schoenfeld, 1985a; Schoenfeld, 1991). Researchers have explained the term "belief" in various ways. Some have seen belief as one of the main variables in an affective construct (e.g., McLeod, 1992), while others have used the term "belief" overlapped and synonymously with terms such as attitude, perception, and value (see Leder & Forgasz, 2002). Op 't Eynde and colleagues (2002) reviewed and developed a framework of student mathematics-related beliefs by integrating the major components of different models presented in previous studies (e.g., Kloosterman, 1996; McLeod, 1992; Pehkonen, 1995). Op 't Eynde et al. (2002) defined beliefs as subjective conceptions that students regard as true, and they classified student beliefs into three categories: (1) beliefs about mathematics, mathematical learning and problem solving, and mathematics teaching; (2) beliefs about the self in relation to mathematics; and (3) beliefs about the social norms in class.

Currently, it is commonly accepted that student beliefs have a significant influence on mathematical learning and problem solving (De Corte, Op 't Eynde, & Verschaffel, 2002; Depaepe et al., 2015; Schommer-Aikins, Duell & Hutter, 2005), and there is a general assumption that the impact of student beliefs on their learning and problem solving behaviour is mediated through three processes: cognitive, conative (motivational and volitional), and affective (Op 't Eynde et al., 2002). First, several studies have shown that student beliefs about mathematics has an influence on the ways they engage in mathematical activities and how they approach problems (e.g., Garofalo, 1989; Schoenfeld, 1983, 1985a, 1989). For example, students who believe that mathematics involves mostly memorizing facts and formulas tend to handle mathematical problems in a very mechanical fashion, such as attempting to recall the most suitable methods to solve problems (Garofalo, 1989; Schoenfeld, 1989). Moreover, student beliefs about mathematics have an impact on what cognitive strategies, or techniques will be used when solving problems. For instance, in Schoenfeld's study (1985a), students failed to use learned mathematical knowledge because they perceived that the knowledge was not meaningful.

Second, there is substantial number of evidence supporting the notion that student beliefs about mathematics and mathematics learning are related to their motivation in learning mathematics (e.g., Kloosterman, 2002; Kloosterman, Raymond, & Emenaker, 1996). It is acknowledged that students will not be highly motivated in learning unless they see the importance of what they learn (Eccles et al., 1983; Schunk, 1991).

Lastly, student beliefs about mathematics contribute an important part to the development of their emotional responses to mathematical situations (McLeod, 1991, 1992). For instance, in regular classroom mathematics, students are often asked to solve routine problems. When solving these routine problems, student actions are based on previously learned procedures.

Students expect that most mathematical problems can be solved in a short period of time, without any obstacles or delays. Therefore, if there are any obstacles that interrupt problem solving activities, the emotional responses of students can become quite intense (Mandler, 1984; McLeod, 1989).

Beliefs about word problem solving

An important study on student beliefs in the field of mathematical word problem solving is the in-depth and systematic work by Verschaffel et al. (2000). Their studies pointed out that the students showed a strong tendency to apply superficial strategies and exclude realistic considerations in modelling processes. This tendency towards responding mechanistically is likely to be due to student beliefs which constructed through the accumulated experience of traditional classroom mathematics (Depaepe et al., 2015; Schoenfeld, 1991; Verschaffel et al., 2000). Reusser and Stebler (1997) presented empirical evidence to support this assumption based on interviews with students who explained their reasons for superficial interpretations and unrealistic responses. Reusser and Stebler (1997) identified assumptions that students typically developed in the culture of traditional school mathematics. For instance, students assumed that every word problem used in the classroom makes sense, and there was only one correct answer to every problem. Moreover, they believed that to look at keywords or at previously solved word problems would help them to determine mathematical operation(s) when they do not understand the problem. By solving word problems in this manner, students merely practice computation skills by recalling facts and imitating a solution procedure illustrated in the examples (Jonsson, Norqvist, Liljekvist, & Lithner, 2014; Lithner, 2008).

Several researchers have conducted intervention studies on mathematics education in realistic and powerful learning environments (e.g., Cognition and Technology Group at

Vanderbilt [CTGV], 1992; Kajamies, Vauras, & Kinnunen, 2010; Verschaffel & De Corte, 1997). These studies, however, did not specifically investigate the change in student mathematical beliefs. One of the few studies that addressed the possibility of developing appropriate beliefs in the new classroom culture was the study by Higgins (1997), who examined the impact of one year of heuristic problem solving instruction on middle school student beliefs about mathematics and problem solving. The results revealed that students in the experimental classroom had more sophisticated beliefs about mathematics than the students exposed only to traditional classroom teaching.

Verschaffel and his colleagues (1999) set up a design experiment in which a learning environment for solving application word problems was developed and implemented in fifth-grade classes. Student beliefs about the role of real-world knowledge in mathematical modelling and problem solving were examined. The results indicated that students in the experimental group had more positive beliefs towards learning and teaching of mathematical word problem solving. However, the effect of the programme on student beliefs was quite small. Unlike previous studies, Mason and Scrivani (2004) conducted a small-scale intervention study aimed specifically at ascertaining student beliefs about mathematics and mathematical learning. Over three months, forty-six fifth graders received instruction that focused on the development of student beliefs by changing the traditional learning environment. The results showed a positive impact of the intervention on student mathematical beliefs and performance in solving word problems.

The studies mentioned above have shown that the programmes do have positive outcomes, and have indicated that it is possible to foster appropriate beliefs about mathematics and problem solving, as well as to improve mathematical modelling and word problem solving performance in students. However, other important factors that may have an important influence on student beliefs and word problem solving performance, such as motivational variables, were not investigated. Researchers in the field of mathematical problem solving have shown that the differences in student performance involve several factors, and that also includes motivational variables (Brose et al., 2012; Pintrich & Schunk, 2002; Wigfield & Cambria, 2010). In order to develop an adequate explanation for student word problem solving, the present study examined not only the role of student beliefs and word problem solving performance, but also motivational variables, by investigating how students' initial motivation in learning mathematics influences their beliefs and word problem solving performance.

Motivation in learning mathematics

Motivation is another variable that is often used to explain individual differences in problem solving performance. It is evident that motivation not only has a role in predicting mathematical achievement (Chiu & Xihua, 2008; Shores & Shannon, 2007; Singh, Granville, & Dika, 2002), but also a crucial role in predicting advancement during mathematics education studies (Hannula, Kaasila, Pehkonen, & Laine, 2007). Even though there are several theories of motivation that are relevant to student learning, in this study, student motivation in learning mathematics was investigated through the lens of expectancy-value theory, since it has been widely utilised by several studies on mathematics learning (e.g., Berger & Karabenick, 2011; Greene, DeBacker, Ravindran, & Krows, 1999).

Expectancy-value theory

Eccles and her colleagues (1983; Wigfield & Eccles, 2002) have a developed modern expectancy-value model that emphasises the crucial influence of an individual's judgement on his or her ability to succeed at a task, as well as the incentive value of an outcome, as proximal determinants of achievement performance, choice, and persistence. The model consists of two main constructs: a) expectancies for success, and b) task values.

Expectancies for success are represented by self-efficacy, which is defined as one's perception about his or her own capability to accomplish a specific task (Bandura, 1997). It is evident that students who perceive themselves capable of doing well on the task are much more likely to be motivated with respect to effort, persistence, and behaviour than those who have a lower sense of efficacy (Bandura, 1997; Eccles, Wigfield, & Schiefele, 1998; Pintrich & Schunk, 2002). For example, students who view themselves competent to do maths are more willing to confront challenging, non-routine problems. In contrast, students who are not confident or who view themselves as incompetent tend to avoid solving tasks that seem complex or difficult.

Task values comprise four major components: interest, attainment value, utility, and cost. Intrinsic or interest value is the enjoyment an individual experiences in doing the task, for example, students choose to learn mathematics because maths is exciting to them, while attainment value involves a sense of personal importance in doing well on the given task. Students who hold this value believe that it is important to be good at maths. Utility value, or usefulness of the task, refers to how useful the task is for one's future plans, for instance, students choose to learn maths because it will help them in the future. Finally, cost is defined as opportunities lost due to engagement in the task (e.g., I have to give up a lot to do well in math), as well as the effort that one needs to make in order to complete the task (see Eccles et al., 1983, for discussion of these components).

Word Problem Enrichment programme (WPE)

This present study is about a Word Problem Enrichment (WPE) programme designed to encourage teachers to use innovative, self-created problems to improve mathematical modelling and word problem solving performance in students. The WPE included only the professional development of teachers, without systematic instruction on how the experimental-group teachers should implement the new method in the classroom. This approach follows the general idea of teacher autonomy in Finnish comprehensive schools, which holds that the teachers have the freedom to design their own teaching. Therefore, the effects of the programme were dependent on how teachers in the experimental group applied the new ideas and skills (e.g., skills to produce on their own pedagogically meaningful word problems) provided by the WPE in their own teaching. In the previous study, the effectiveness of the WPE on student word problem solving performance was found by including cognitive factors only (Pongsakdi, Laine, Veermans, Hannula-Sormunen, & Lehtinen, 2016). The present study aims to extend the focus also to the role of motivation and beliefs in improving word problem solving performance.

Research questions of the study

(1) Does the WPE have an impact on student beliefs about word problem solving?

(2) Do students' initial beliefs about word problem solving have an influence on their word problem solving performance, and are the effects of the WPE on the word problem solving performance mediated through changes in beliefs?

(3) Is the impact of the WPE on student word problem solving performance and beliefs about word problem solving depending on students' initial mathematics motivation?

Method

Participants and overall design

The study adopted a quasi-experimental pre-test–post-test design. Teachers in the experimental condition of this study participated in a WPE professional learning course. Participants in the study consisted of ten classroom teachers and 170 students, 75 boys and 95 girls, from the fourth and sixth grades. Although the students were drawn from different elementary schools located in socioeconomically varied areas in southwest Finland, the households were predominantly middle-class. A majority of students are Finnish (95.3%), and none of them were reported with learning disabilities.

Table 1. Number of participants per grade and experimental condition

Table 1

Based on open call, five classroom teachers volunteered to participate in the professional development programme. This group of teachers (n = 5) and their 98 students (the number of students in each class: 19, 21, 22, 19, and 17 students, respectively) were assigned as an experimental group, while other volunteer classroom teachers (n = 5) and their 72 students (the number of students in each class: 12, 18, 19, 12 and 11 students, respectively) served as a control group (see Table 1). The control-group teachers did not know about this open call (for participating in the professional development programme), but after introducing general ideas of this study, they volunteered to participate in the study as a control group because of their own interest in the use of word problem solving in mathematics education.

Professional development programme

A researcher from the University of Turku worked collaboratively with an expert teacher, who worked for the Centre of Teacher Training in Mathematics (Turun Matikkamaa), to develop the programme and act as the professional development facilitators. The experimental teachers took part in the programme, which was organised over three afternoon seminars, each lasting around three hours. Seminars were arranged once a month between January and March 2013.

In the first seminar, several important issues related to typical use of word problems in traditional mathematics lessons, such as beliefs about word problems (e.g., there is only one correct way to solve any word problem, word problems have only one right answer) and the stereotypical nature of school word problems, were highlighted and actively discussed. Teachers were shown the empirical evidence concerning the impact of traditional textbooks and pedagogical practice on student word problem solving and realistic reasoning skills (Verschaffel et al., 2000), for example, how students often applied straightforward strategy (e.g., keyword approaches) to solve word problems without adequate understanding of the context of the problem (solving word problems without a proper situation model) and excluded realistic considerations in modelling process.

In the second seminar, the main purpose was to emphasise that regular textbooks mainly include routine word problems, and that there is a lack of non-routine and application word problems that are important to the development of students' realistic mathematical modelling and word problem solving performance. Three types of word problems were discussed: routine, non-routine, and non-routine word problems requiring the use of realistic considerations (also called application word problems). First, routine word problems are word problems that can be solved straightforwardly by a routine application such as the keyword approach (a strategy to solve word problems with the help of individual words, e.g., "in total" = addition). Students could solve this type of word problem by applying simple arithmetical operations using the numbers provided in the word problem. In contrast, non-routine word problems are word problems that cannot be solved by straightforward strategies. They require students to have an adequate understanding of the context of the word problems in order to solve them correctly. Lastly, non-routine word problems requiring the use of realistic considerations (application word problems) are similar to non-routine word problems. One additional requirement is the use of non-direct translation of the word problem texts on the basis of real-world knowledge and assumptions into the mathematical model.

The teachers were encouraged to use different types of word problems, especially non-routine and application word problems with real-world situations, to improve students' realistic mathematical modelling and word problem solving performance. Several examples of non-routine and application word problems (e.g., plank problem) that resembled those word problems presented in Verschaffel and De Corte (1997) and CTGV (1992) were introduced to the teachers, with additional guidelines on how they could create innovative word problems themselves or together with students. When solving demanding, non-routine and application word problems, the teachers were advised to instruct their students to apply two steps of heuristic strategies based on Verschaffel et al.'s (1999) study: a) build a mental representation (e.g., draw a picture, make a list, distinguish relevant from irrelevant data, use real-world knowledge), and b) decide how to solve the problem (e.g., make a flow chart, guess and check, look for a pattern, simplify the numbers). Then the teachers were guided on how to assess student word problem solving performance. Several word problem solving assessment criteria (e.g., Charles, Lester, & O'Daffer, 1987; Kallick, & Brewer, 1997; Stenmark, 1991) were discussed and provided to teachers.

In the last seminar, teachers were guided on how to create non-routine and application word problems that are interesting for their students and also related to real-world situations. Real-world situations refer to situations that occur in the world that one experiences both directly and indirectly (e.g., through the media, other people's experience) in everyday life. However, the situations that various people perceive as "real" might differ, because of their previous experiences and cultural backgrounds (Pongsakdi, Brezovszky, Hannula-Sormunen, & Lehtinen, 2013). For instance, for elementary school students, the fantasy world of fairy tales could also be considered to be realistic, since they are real in students' minds (Depaepe, De Corte, & Verschaffel, 2009). Therefore, the real-world situations that were emphasised in the seminar are not limited to their possible occurrence in the real world, but rather were based on situations that students might possibly conceive. The teachers were suggested to use various sources available on the Internet (e.g., pictures and videos) to create stories for the word problems. Ideally, the word problems would be similar to those word problems presented in Verschaffel and De Corte (1997) and CTGV (1992), which provided opportunities for students to use their imagination and real-life applications to think of possible solutions, and to discuss their thoughts in small groups to develop solutions. At the end of the last seminar, the teachers were asked to provide open-ended writings on their experiences during the professional development programme and on how the programme has affected their strategies to teach word problem solving. All teachers were very satisfied with the course and reported that it has been useful and inspiring for their teaching of word problems. In addition teachers reported several individual elements of the course, such as drawings and everyday experiences, which they had started applying in their teaching.

Measures

Data were collected by classroom teachers using three test instruments: Word Problem Solving Test, Motivation Questionnaire, and Word Problem Solving Beliefs Questionnaire. The pre-test was administered to students by their teachers at the beginning of the professional development programme (the beginning of the spring semester of 2013). Two months after the professional development programme (the end of the spring semester of 2013), the post-test given to students included only two test instruments: the parallel test of the Word Problem Solving Test and the Word Problem Solving Beliefs Questionnaire.

For the Word Problem Solving Test, students were instructed to describe how they solved each word problem, as well as explain how they understood the word problem either by writing short descriptions or using visual representations (e.g., drawing a picture or chart). For the Motivation and Word Problem Solving Beliefs Questionnaires, students responded to all items on a five-point Likert scale ranging from 1 (Completely disagree) to 5 (Completely agree). On both pre- and post-test, students had around 45 minutes to complete the measurements.

Word Problem Solving Test

Student word problem solving performance was assessed with a word problem solving test containing five word problems: 1 routine, 3 non-routine, and 1 application (see Appendix 1). A *routine word problem* was adapted from a typical routine word problem presented in textbooks. *Non-routine word problems* were constructed in such a way that they could not be solved by straightforward strategies. For example, the word problems avoided using keywords and provided meaningful data in the written form instead of numbers. An application word problem requiring realistic considerations was adapted from an original word problem in Depaepe and colleagues' (2009) study. A parallel version of the word problem pre-test was developed for the post-test. The problems were structurally identical, but the problem contexts differed. The number of word problems included in the test was quite small. However, in this study, it is important for us to understand how students solved the problems when there was no time pressure or an overwhelming number of word problems.

All word problems were analysed by using the same scoring system: one point was given for each correct answer and zero points for an incorrect answer, or no response. The routine word problem appeared to be too easy (a ceiling effect occurred with the pre-test and post-test). To examine the WPE's effectiveness on student word problem solving performance, the sum score of non-routine and application items was used in SEM. The reliability of these items was sufficient. Cronbach's alphas for the pre-test and the post-test were .62 and .66 respectively, which are considered sufficient (Hair, Black, Babin, Anderson, & Tatham, 2006).

Motivation scale

The 14-item questionnaire was used to measure student motivation. The items were adapted from the original scale used by Berger and Karabenick (2011). The items were framed specifically for mathematics, and modified to be suitable for primary school students. Factor analysis was conducted. One item was discarded from the analysis since the item did not correlate with the other items of the subscale. Five factors (Interest, Utility, Attainment value, Cost, and Self-efficacy) were established, supporting the expectancy-value model developed by Eccles and colleagues (1983; Wigfield & Eccles, 2002). The factors explained 78.71% of the variance.

Table 2. Five factors of the motivation scale and their Cronbach's alphas

Table 2

Beliefs about word problem solving

The 13-item questionnaire was used to measure student beliefs about word problem solving. Seven items related to typical beliefs about mathematics and beliefs about oneself as a problem solver were adapted from the original scale developed by Schoenfeld (1985b). These items were abbreviated to make them easier for primary school students to comprehend, and framed specifically for word problem solving. The other six items were developed based on important aspects of word problem solving (e.g., keyword approach, importance of situation model) discussed in previous studies (Verschaffel et al., 2000). Factor analysis was conducted. One item was excluded from the analyses because it loaded to different factors in the pre- and post-tests.

Four factors were constructed on the beliefs related to Situation model, Conventional school math, Liking, and Keyword approach. The factors explained 64.98% and 63.41% of the variance for the pre- and post-test respectively. However, due to a small sample size of this data, it was not possible to include all belief factors in the structural model. Therefore, only the theoretically most important factor that was stressed in this study, the situation model, was included in SEM.

Table 3. Four factors of beliefs about word problem solving scale and their Cronbach's alphas

Table 3

Analysis

Modelling

All analyses (LPA and SEM) were conducted using M*plus* version 7.0 (Muthén & Muthén, 1998-2012), and employed Maximum Likelihood Robust Estimator (MLR), which is a full-information approach to handle missing completely at random data and deviations from multivariate normality.

Latent Profile Analysis (LPA)

LPA was used to explore patterns of students' initial motivation in learning mathematics. LPA is a model-based classification technique that classifies students into homogeneous groups or latent person profiles, based on their similarities in observed variables. It differs from other traditional person-oriented methods, such as cluster analysis, since it is model based and has more strict criteria for identifying the number of profiles or clusters (Muthén, 2001; Lubke & Muthén, 2005). The LPA was conducted with 300 and 30 random start values. The most representative model was selected based on these six main criteria: 1) low values for AIC (Akaike Information Criterion), 2) low values for BIC (Bayesian Information Criterion), 3) high values for entropy, 4) a significant result of the BLRT (Bootstrapped Likelihood Ratio Test), 5) a significant result of the LMR (Lo-Mendell-Rubin test), and 6) the class solution has a meaningful theoretical interpretation (Muthén & Muthén, 2000; Nagin, 2005).

Structural Equation Modelling (SEM)

SEM was employed to test a theoretical model to explain the relationships among the different cognitive, motivation and belief factors. It is a statistical methodology that applies a hypothesis-testing method to the analysis of a structural theory supporting on some phenomenon (Byrne, 2012). SEM methodology provides several important features that are improvements over the older generation of multivariate procedures. For example, SEM offers explicit estimates of error variance parameters, while traditional multivariate procedures are unable to either assess or correct for measurement error. This may lead to serious inaccuracies, especially when the errors are sizeable. Moreover, SEM procedures allow us to incorporate both unobserved (i.e., latent) and observed variables, whereas former methods are based on observed measurements only (Byrne, 2012).

Figure 2. A structure of the hypothesised model

Figure 2

Based on the literature review, the hypothesised model with the expected cross-lag paths (represented by dotted lines) and auto-correlated residuals (covariances between residuals of each item in pre- and post-test) was proposed, as shown in Figure 2. The model included sum scores of non-routine and application word problems for the pre- and post-test (Math pre-test and Math post-test), a latent variable of student beliefs about the situation model for the pre- and post-test (Beliefs about situation model pre-test and Beliefs about situation model post-test), and

WPE intervention effect (1 = experimental group, 0 = control group). From this hypothesised model, math performance at pre-test might have an effect on beliefs at post-test, whereas beliefs at the pre-test phase could have an effect on math performance at post-test. Moreover, beliefs might have an effect on math performance at post-test.

First, the fit of the hypothesised model was investigated. Then it was followed by metric and scalar invariance tests to determine the extent of measurement invariance between different levels of the initial motivation groups (based on LPA results). A metric invariance test was conducted in order to confirm that we are measuring the same constructs across groups, while a scalar invariance test was carried out to investigate whether the intercepts of items would be the same across groups (Byrne, 2012). In these invariance analyses, the Satorra-Bentler scaled chi-square difference test was used to test for differences in nested models that use the MLR estimator (Bryant & Satorra, 2012). The fit of each model was assessed using the Comparative Fit Index (CFI), the Root Mean Square Error of Approximation (RMSEA), the Standardized Root Mean Square Residual (SRMR), Chi-square (χ^2), and the ratio of χ^2/df (when the p value associated to χ^2 was significant) (cf. Schermelleh-Engel, Moosbrugger, & Müller, 2003).

Results

Latent Profile Analysis (LPA) of student initial motivation

To investigate the patterns of student initial motivation in learning mathematics, LPA was conducted. One student was excluded from the analysis, because the student did not answer the motivation questionnaire (n = 169). Table 4 shows the fit indices for five different class solutions. It indicated that two- and three-class models had potential to be the most representative models for this data, because both models have small BIC values and high values for entropy. However, a significant result of the LMR suggested that the two-class model is superior to others. Moreover, the two-class model is able to clearly distinguish students into two classes: high (n = 89) and low level (n = 80) of initial motivation in learning mathematics.

Table 4. Fit indices of one- to five-class latent profile models

Table 4

Figure 3: Latent profiles for the two-class model

Figure 3

Structural Equation Modelling (SEM) of cognitive, motivation, and belief factors

To investigate the relationships between the different cognitive, motivation, and belief factors, first the fit of the hypothesised model was investigated. The results are shown in Figure 4. Only significant paths at the level of alpha = .05 were retained in the model. The testing of the hypothesised model revealed that the fit indices did not perfectly reach the model fit criteria, χ^2 (19, N = 170) = 42.48, p < .001, χ^2/df = 2.24, CFI = .94, RMSEA = .09. However, based on suggested modification indices, residual-covariance between item no.1 and item no. 3 at post-test was added to the model. The addition of this residual covariance significantly improved the fit of the model, the Satorra-Bentler scaled $\Delta\ \chi^2$ (1) = 11.80, p < .001. The testing of the modified model showed a fit qualified as acceptable following Schermelleh-Engel et al.'s (2003) guidelines, χ^2 (18, N = 170) = 32.82, p < .05, χ^2/df = 1.82, CFI = .96, RMSEA = .07.

Figure 4: The modified structural model with the standardised path coefficients

Figure 4

Note: *p < .05, **p < .01, ***p < .001.

As shown in Figure 4, the path coefficients indicated that the WPE has a direct effect not only on word problem solving performance (β = .14), but also on beliefs about the situation model (β = .22) at post-test. There is a significant effect (β = .46) of beliefs about the situation model on word problem solving performance in post-test. However, there is no indirect effect from the WPE, which is mediated through beliefs about the situation model on word problem solving performance at post-test. Moreover, there were no significant paths from beliefs about the situation model at pre-test to word problem solving performance at post-test, and from word problem solving performance at pre-test to beliefs about the situation model at post-test.

The role of students' initial motivation on the impact of the WPE

To investigate whether the impact of the WPE on student word problem solving performance and beliefs about situation model depend on students' initial mathematics motivation, we first included a two-class latent profile model (which classified students into two groups: high and low level of initial motivation) in the modified structural model as a moderator variable. Then we conducted the measurement invariance test (metric and scalar invariance tests) to examine the extent of measurement invariance between the groups with high and low levels of initial motivation.

To test metric invariance, the same structural model (Figure 5) was specified for both waves, and loadings were constrained to be equal. The Satorra-Bentler scaled $\Delta\chi^2$ (6) = 2.43, p > .05, from the fit of the constrained model (loadings were freely estimated). Thus it can be concluded that there was invariance in the factor construct between the groups with high and low levels of initial motivation. For the scalar invariance test, the intercepts of items were constrained to be equal across the two groups. The results revealed that the intercept of items are not equal across the two groups, the Satorra-Bentler scaled $\Delta\chi^2$ (7) = 16.80, p < .05. Therefore, partial scalar invariance was tested by releasing the intercept of one item in the scalar model. The results indicated that intercepts of items, except the intercept of one item in post-test, are equal across the two groups, the Satorra-Bentler scaled $\Delta\ \chi^2$ (6) = 8.33, $p < 0.22$. After the invariance measurement test, the fit of this integrated model (Figure 5) was investigated. The results indicated acceptable model fit, χ^2 (44, N = 169) = 62.91, $p < .05$, χ^2/df = 1.43, CFI = .95, RMSEA = .07.

Figure 5: The integrated structural model for high and low level of initial motivation group with the standardised path coefficients

Figure 5

Note: ns = non-significant, *$p < .05$, **$p < .01$, ***$p < .001$.

Figure 5 shows the standardised path coefficients of the integrated structural model for groups with high and low levels of initial motivation. The results indicated that the impact of the WPE on student word problem solving performance and beliefs about the situation model depends on student initial mathematics motivation. For students who have a high initial motivation level, there is no effect of the WPE on their beliefs about the situation model. The effect of WPE on beliefs about the situation model was found only in students who have a low initial motivation level (β = .44). In contrast, there is an effect of the WPE on word problem solving performance only in students with a high initial motivation level (β = .20), but for students with a low initial motivation level, no effect was found. Differences in initial beliefs about the situation model between groups of students with either a low or high initial motivation were compared. An independent-sample t-test revealed that there was a significant difference regarding initial beliefs about the situation model between the high (M = 3.74, SD = 0.90) and low initial motivation groups (M = 2.96, SD = 0.92); $t(163) = 5.55$, $p < .001$. Students who belong to the high initial motivation group have higher initial beliefs about the situation model than those who belong to the low initial motivation group.

Discussion

Several researchers in the field of mathematical problem solving research have shown that individual differences in problem solving performance are complex, and cannot be explained purely as the result of differences in cognitive skills. The role of beliefs and motivational variables must be considered in order to provide adequate explanation for the individuals' differences in problem solving performance (Maaß & Schlöglmann, 2009; Mason & Scrivani, 2004; McLeod & Adams, 1989; Pintrich & Schunk, 2002). The aim of this study was to investigate the impact of the WPE on student beliefs by using LPA and SEM to analyse interactions among the different cognitive, motivation, and belief factors. The study differs from previous studies (e.g., Mason & Scrivani, 2004; Verschaffel et al., 1999) aimed at enhancing word problem solving in two major respects. First, the WPE included only the professional development programme for teachers, without any strict instructions on how the experimental-group teachers should implement new methods in the classroom. Concerning the ecological validity, this approach follows the nature of Finland's education system that provides teachers autonomy with respect to how they design their own instruction. In addition, the focus on teachers' pedagogical thinking without any standardised classroom practice makes it feasible (in terms of resources) to apply this kind of programme in a large-scale training in the future (e.g., pre-service and in-service teacher curriculum) (see Pongsakdi et al., 2016). Second, this study examined not only the role of student beliefs and word problem solving, but also how motivational variables are related to individual differences in word problem solving performance.

Overall, the results revealed that WPE, aimed at facilitating teachers to enrich word problems used in mathematics teaching, has a positive impact not only on student word problem solving performance, but also on their beliefs about the nature of word problem solving. As proposed in earlier studies, a successful change in the classroom culture of word problems requires a major change in the types of word problems used in mathematics lessons (CTGV, 1992; Verschaffel & De Corte, 1997), instructional approach towards word problem solving (Depaepe, De Corte, & Verschaffel, 2010; Higgins, 1997; Mason & Scrivani, 2004; Verschaffel et al., 1999;), assessment approaches (Charles et al., 1987; Kallick, & Brewer, 1997; Stenmark, 1991), and teacher beliefs (see Depaepe et al., 2015 for overview). In the WPE, we not only promoted the use of more variety of non-routine and application word problems in the classroom, but we also emphasized to teachers how the traditional practice of word problems in classroom mathematics impacts students, and why the current practice needs to be changed. Practically, we tried to convince teachers to change their beliefs about the educational relevance about word problems. Based on teachers' open-ended writings on how the programme has affected the way they use word problems in their teaching, all teachers gave positive remarks on the programme and reported its usefulness and inspirational effects on their own teaching of word problem solving. Although evidence from this study suggested positive outcomes of the WPE on student word problem solving performance and beliefs about situation model, direct investigations on the effects of the WPE on the teachers' content knowledge, pedagogical content knowledge, beliefs about word problems as well as their classroom practices (the use of word problems, instructional and assessment approach), should also be conducted in future studies on the effects of the WPE.

Based on previous studies (e.g. Mason & Scrivani, 2004), it can be hypothesised that advancement in beliefs about the role of situation model results in improved skills to solve non-routine word problems. However, there was no indirect effect of the WPE on word problem solving performance as mediated through the changes in beliefs about the situation model. This might be due to the fact that after the professional development programme, teachers had relatively short time to apply the new ideas and skills that they learned from the programme in their teaching. In addition student beliefs and word problem solving performance were measured during the same post-test. In order to see the indirect effect of WPE on student word problem solving performance as mediated through student beliefs about situation model, it might require a longitudinal study in which teachers implement the new teaching method in the classroom for a longer period of time and in which several measurement points are used.

The results showed that the effects of WPE are various depending on students' initial motivation in learning mathematics. The impacts of the WPE on beliefs about situation model were found only in students with a low level of initial motivation in learning mathematics. This might be because students who have a high initial motivation level had a high level of beliefs about situation model already during the pre-test, while students who had a low initial motivation level also had a low level of initial beliefs about situation model. In contrast, the effects of WPE on word problem solving performance were found only in students with a high level of initial motivation in learning mathematics. These results could be explained by expectancy-value theory (Wigfield & Eccles, 2002), suggesting that students who see themselves competent to do maths are more willing to confront challenging word problems. In contrast, students who are not confident to do maths may avoid engagement in the new pedagogical practices, where more demanding word problems are used, because they experience them too complex or difficult. For these students, the programme can be effective only if it manages to strengthen student confidence. In their theoretical analysis about the role of motivational and epistemic beliefs in self-regulation in mathematics learning, De Corte and his colleagues (2011) present the hypothesis that beliefs about mathematics and motivational beliefs can develop reciprocally. However, it was not possible to deal with the reciprocal development of these constructs in this present study because general mathematics motivation was only measured as background variable. For the future longitudinal study, it would be important to highlight more motivational aspects in developing WPE and to investigate whether the modified programme has any impact on student motivation in learning mathematics and to analyse the developmental interaction between mathematics beliefs and motivation.

Even though the results suggested WPE's positive impact on student word problem solving performance and beliefs about situation model, the limitation of the quasi-experimental design used in this study must be taken into account. The experimental-group teachers were those who volunteered to participate in the professional development programme. This may imply that the level of teacher interest might be different between the two groups. However, even though the control-group teachers were not asked to participate the WPE training, they volunteered to participate in this study because of their own interest in the use of word problem solving in mathematics education. For future studies, it could be important to use randomized experimental design and to examine how teachers implement the new approaches in their teaching and how their interest in developing word problems mediates the effect of the professional development programme. An additional limitation is that the results of student word problem solving performance are based on four items. The small number of word problem tasks was due to restricted time available for measures of this study. The tasks were carefully planned, but more convincing results about the effects of the training program WPE require larger and better balanced set of word problem. Moreover, due to a small sample size, it was not possible to include all belief factors in the structural model. Only the theoretically most important factor that was stressed in this study, the situation model, was included in SEM. To clarify these issues, the future studies should use a larger set of word problems and include all belief factors with larger samples.

Educational implications

Several studies pointed out that student beliefs are influenced by teachers through their practice in the classroom (Depaepe et al., 2015; Pehkonen, 1998; Pehkonen & Törner, 1996), and it is important to improve student mathematical performance by changing their beliefs about the domain, as well as beliefs of their teachers (Mason & Scrivani, 2004). However, the results of this study showed that is not necessarily the case, especially for students with a low initial motivation level in learning mathematics. Even though their beliefs about situation model were improved, they did not have an impact on student word problem solving performance. These findings suggested that it may not be enough to focus merely on changing student beliefs. Student mathematics motivation needs to be considered as well. Students, particularly those with a low initial motivation, could feel overwhelmed when dealing with challenging non-routine and application word problems. Therefore, in classroom practice, it is important that teachers will provide adequate support for students to be more confident and feel less overwhelmed when facing non-routine and application word problems.

Furthermore, this study was conducted within the context of Finnish education system in which teachers have the freedom to design their own teaching, and are used to develop own pedagogical practices on the basis of general principles. The programme included only the professional development of teachers, without any strict instructions about classroom practices and teachers were assumed to be able to apply the new ideas in their own teaching. However it is an open question if this approach would work in other contexts.

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