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All Quantum Resources Provide an Advantage in Exclusion Tasks

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A key ingredient in quantum resource theories is a notion of measure. Such as a measure should have a number of fundamental properties, and desirably also a clear operational meaning. Here we show that a natural measure known as the convex weight, which quantifies the resource cost of a quantum device, has all the desired properties. In particular, the convex weight of any quantum resource corresponds exactly to the relative advantage it offers in an exclusion (or antidistinguishability) task. After presenting the general result, we show how the construction works for state assemblages, sets of measurements, and sets of transformations. Moreover, in order to bound the convex weight analytically, we give a complete characterization of the convex components and corresponding weights of such devices.

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Introduction.—Quantum theory allows for concepts that have no analogue in classical physics. Most prominent examples include entangled states, incompatible measurements, and quantum memories. An important question is to characterize these genuinely quantum resources, in particular to quantify their nonclassicality. A natural approach to this problem is to view these genuine quantum properties as a resource for some task, and ask to what extent a given quantum device deviates from the classical scenario. Recently, a general framework of quantum resource theories has been developed to address these questions (see Ref. [1] for a recent review). These ideas have already been formally applied to a broad range of quantum properties, such as entanglement [2], joint measurability [3,4], steering [5,6], thermal operations [7], asymmetry [8], and coherence [9].

In general, a resource theory is defined via a set of free resources (for instance, associated to classical resources), and a set of free operations. Applying a free operation to a free resource should always give back a free resource, and more generally free operations cannot boost the available resource. Hence, classical pre- and postprocessings are usually part of free operations, which implies that the set of free resources must be convex. This motivates the use of convexity-based measures in order to quantify quantum resources, i.e., to measure their nonclassicality.

Recently, a large body of work has been devoted to one of these measures, namely, the generalized robustness [5,10–19]. The latter quantifies the resource of a given device, by asking by how much it can be mixed with another (arbitrary) device before the resource is lost (i.e., the mixture belongs to the free set). Loosely speaking, this captures the distance between a given device and the set of free devices. Since its introduction, the generalized robustness has been found to possess three very attractive and fundamental properties: (i) faithfulness, i.e., it is zero if and only if a device is free, (ii) convexity, (iii) monotonic under free operations, (iv) it quantifies the outperformance of a quantum device with respect to all classical ones in an explicit task, namely, a discrimination game, (v) it can be calculated efficiently when the free set can be expressed through semidefinite constraints (thereby forming a certificate).

In this Letter, we prove that another, also well-motivated, quantifier has the five fundamental properties mentioned above. This quantifier is known as the convex weight. It has a natural interpretation in the context of resource theories. Namely, it characterizes how a large fraction of a given resource device can be generated with free (or classical) resources. In this sense, the convex weight provides a direct quantifier of the resource cost, and is thus complementary to the generalized robustness. Consider, for instance, a resource that is extremal, but very close to the free set. While the generalized robustness is very small for this resource, the convex weight will nevertheless be equal to 1.

To prove property (iv), we construct explicitly a task for which the convex weight quantifies exactly the relative advantage provided by the resource over any free device. This task corresponds to an exclusion (or antidistinguishability) task. That is, given a randomly chosen element x_k from a known list of elements $\{x_i\}$, one should provide as the answer any $x_i \neq x_k$. After discussing the general framework, we discuss the cases where the quantum devices correspond to sets states, sets of measurements, and quantum channels. For instance, any set of incompatible quantum measurements provides an advantage in a task of state exclusion, and the convex weight represents the relative advantage over any set of compatible (i.e., jointly measurable) measurements. Finally, we show that the convex weight can be easily bounded (and in simple cases even decided) by fully analytical methods, a fact that we illustrate by characterizing all devices and corresponding weights that can appear in a convex decomposition of a given device.

Convex weight of quantum devices.—We concentrate on three categories of quantum devices: quantum states, measurements, and transformations. We may also extend the notion to include sets of such devices, e.g., a collection of states or measurements. Formally, states correspond to positive unit-trace operators denoted by ϱ, measurements to collections of positive operators denoted by $\{M_j\}$ with the normalization $\sum_j M_j = \mathbb{1}$, and transformations are given as completely positive trace nonincreasing maps denoted by $\mathcal{I}$ (or Λ if trace preserving). For a convex and compact subset F of (a given class of) quantum devices one defines the convex weight $\mathcal{W}_F(D)$ of a device D as the maximum relative number of times a device from the set F can be used to produce D. Formally,

$$\begin{aligned} &\mathcal{W}_F(D) = \min \lambda \\ &\text{s.t.:}\ D = (1-\lambda)D_F + \lambda\tilde{D}, \end{aligned} \tag{1}$$

where the optimization runs over devices D_F in the subset F and general devices $\tilde{D}$ outside of F.

The weight has the following appealing properties of a resource quantifier: (i) faithfulness, i.e., it is zero if and only if a device is free, (ii) convexity, (iii) monotonic under free operations, (iv) task-oriented interpretation, (v) simple to bound analytically. The properties (i) and (iii) follow directly from the definition, the properties (ii) and (v) are proven in the Supplemental Material [20] and the property (iv) is the main message of this Letter. We note that as a by-product of proving property (v) we provide a complete characterization of the convex components of a given device.

Exclusion input-output games.—An input-output game $\mathcal{G}$ is defined as a triplet $\mathcal{G} = (\mathcal{E}, \mathcal{M}, \Omega)$, where $\mathcal{E} = \{p(i)\varrho_i\}$ is a state ensemble, $\mathcal{M} = \{M_j\}$ forms a POVM, and $\Omega = \{\omega_{ij}\}$ is a real-valued reward function. The task is to find a transformation $\mathcal{I}$ that minimizes the payoff defined as $P(\mathcal{I}, \mathcal{G}) \coloneqq \sum_{ij} \omega_{ij} p(i) \mathrm{tr} \mathcal{I}(\varrho_i) M_j$. Note that in the case $\mathcal{I} = id$ and $\omega_{ij} = \delta_{ij}$ the payoff corresponds to the exclusion probability in a minimum error state discrimination task. Note also that in contrast to discrimination input-output games, where the task is to maximize the payoff, in exclusion input-output games we are interested in minimization. One could argue that there is not much difference between input-output games and their exclusion variants, as one can flip the signs in the reward functions and look at the absolute value of the payoff. The difference between the games becomes evident, however, when looking at canonical input-output games (see below) that remove all covariance between the payoff and the reward function. As it turns out, such elimination of covariance is necessary for the connection between resource measures and quantum games [19,24]. This duality between the games also highlights the duality between the concepts of generalized robustness and the convex weight.

In order to define input-output games for sets of devices, we define the games for each device separately and as payoff we take the sum of the individual payoffs. Formally, in the definition of a game we replace the state ensemble $\mathcal{E} = \{p(i)\varrho_i\}$ with a state assemblage $\mathcal{A} = \{p(i,x)\varrho_{i|x}\}$, the single POVM $\mathcal{M} = \{M_j\}$ with a measurement assemblage $\mathcal{M}_A = \{M_{j|x}\}$, and the reward function $\Omega = \{\omega_{ij}\}$ with a fine-tuned reward function $\Omega_f = \{\omega_{ijx}\}$. We refer to the triplet $(\mathcal{A}, \mathcal{M}_A, \Omega_f)$ with the same symbol $\mathcal{G}$ as used above when there is no risk of confusion. Now the payoff for a set of transformations $\{\mathcal{I}_x\}$ reads $P(\{\mathcal{I}_x\},\mathcal{G}) \coloneqq \sum_{ijx} \omega_{ijx} p(i,x) \mathrm{tr} \mathcal{I}_x(\varrho_{i|x}) M_{j|x}$. Again, the case $\{\mathcal{I}_x\} = \{id\}$ and $\omega_{ijx} = \delta_{ij}$ corresponds to a minimum error discrimination task.

Input-output games that do not relate to minimum error discrimination have some redundancy: a game can be transformed into another one by scaling the reward function or by adding a constant to it. This results in a scaled or shifted payoff. In order to treat such games on an equal footing, we eliminate the scaling and shifting covariance by defining canonical versions of the games. A canonical version of a game is obtained by first shifting the lowest payoff to zero when optimized over (sets of) transformations and then scaling the highest payoff to one.

Main idea.—The convex weight $\mathcal{W}_F(D)$ of a device D with respect to a free set F is defined in Eq. (1). Solving this equation for $\tilde{D}$ and defining $\hat{D}_F \coloneqq (1-\lambda)D_F$ results in

$$\begin{aligned} &\mathcal{W}_F(D) = \min \lambda \\ &\text{s.t.:}\ \frac{1}{\lambda}(D - \hat{D}_F) \in \mathrm{Dev}, \qquad \hat{D}_F \in C_F, \end{aligned} \tag{2}$$

where Dev is the set of all devices and $C_F = \{\alpha D_F | \alpha \geq 0, D_F \in F\}$ is a cone based on the subset F. Note that the two optimization constraints are tied to each other in that $\lambda + \alpha = 1$.

Of the two optimization constraints the conic one is linear and it reads the same for all three categories of devices (or sets thereof). The other constraint, however, is nonlinear and as such we check it in more detail. For states and measurements the constraint reduces to positive semidefiniteness of $D - \hat{D}_F$. This is due to the fact that the normalization, i.e., having unit trace or a sum equal to identity, of $(1/\lambda)(D - \hat{D}_F)$ is automatic. A transformation can be seen as an element of a quantum instrument, i.e., a collection of completely positive trace nonincreasing maps summing to a completely positive trace preserving map. We consider instruments in place of transformations from here on. This way the nonlinear optimization constraint becomes linear. Namely, as the normalization is now automatic, the operator $(1/\lambda)(D - \hat{D}_F)$ being an instrument corresponds to positive semidefiniteness of the Choi picture of $D - \hat{D}_F$.

With the above modification we have brought the problem of calculating the weight of a quantum device D with respect to the set F into a linear problem with conic constraints. Such optimization problems are called conic programs [25,26]. Below we spell out these programs explicitly in their dual form for states, measurements and channels as a special case of instruments. The treatment of general instruments is presented in the Supplemental Material [20]. The connection between convex weight and the performance in input-output games follows directly from the dual.

For a state assemblage $\mathcal{A} = \{p(i,x)\varrho_{i|x}\}$ the primal of the cone program (2) reads

$$1 - \mathcal{W}_F(\mathcal{A}) = \max \sum_{i,x} \mathrm{tr}[\sigma_{i|x}] \tag{3}$$

$$\text{s.t.: } p(i,x)\varrho_{i|x} \geq \sigma_{i|x} \quad \forall\ i,x. \qquad \{\sigma_{i|x}\}_{i,x} \in C_F. \tag{4}$$

This optimization problem is an example of a cone program, the dual of which is given by [25,26]

$$1 - \mathcal{W}_F(\mathcal{A}) = \min_{Y \geq 0} \sum_{i,x} p(i,x)\mathrm{tr}\varrho_{i|x} Y_{i|x} \qquad \text{s.t.: } \sum_{i,x} \mathrm{tr} T_{i|x} Y_{i|x} \geq 1 \quad \forall\ \{T_{i|x}\} \in F, \tag{5}$$

where $Y = \bigoplus_{i,x} Y_{i|x}$ is a witness. The solution of the dual problem equals that of the primal given that the so-called Slater condition holds, which in this case can be verified by choosing $Y = \alpha\mathbb{1}$ for large enough $\alpha > 0$. This removes the redundant parts of the free cone as well.

To see the objective function of the dual problem as an instance of an input-output game, we define another witness as $\tilde{Y}_{i|x} \coloneqq Y_{i|x}/N$, where $N \coloneqq \|\sum_{i,x} Y_{i|x}\|$. For each x we can add an extra term to $\{\tilde{Y}_{i|x}\}_i$, namely, $\mathbb{1} - \sum_i \tilde{Y}_{i|x}$, which ensures that we get a witness corresponding to a set of POVMs. Note that in the process we embed the state assemblage back into the larger space if needed and complete the witness into a POVM on the larger space by adding the missing parts to the last outcome. The new witness results in an objective function that is a scaled version of the success probability in a specific minimum error discrimination task. More precisely, $p_{\mathrm{succ}}(\mathcal{A}, \mathcal{M}_A) \coloneqq \sum_{i,x} p(i,x)\mathrm{tr}\varrho_{i|x} M_{i|x}$. Clearly p_{succ} is linear in the first argument and so from Eq. (1) we get $p_{\mathrm{succ}}(\mathcal{A}, \mathcal{M}_A) \geq [1 - \mathcal{W}_F(\mathcal{A})] \min_{\mathcal{A}_F \in F} p_{\mathrm{succ}}(\mathcal{A}_F, \mathcal{M}_A)$. This inequality can be saturated by taking an optimal witness in Eq. (5), writing the corresponding object function as a minimum error state discrimination task, and noting that the scaling N does not affect the quotient in the following expression:

$$\inf_{\mathcal{M}_A} \frac{p_{\mathrm{succ}}(\mathcal{A}, \mathcal{M}_A)}{\min_{\mathcal{A}_F \in F} p_{\mathrm{succ}}(\mathcal{A}_F, \mathcal{M}_A)} = 1 - \mathcal{W}_F(\mathcal{A}). \tag{6}$$

More precisely, the use of an optimal witness shows that the left-hand side (l.h.s.) of the above expression is a lower bound for the right-hand side and, hence, we get the equality in the expression. Here we have used the standard convention that the optimization is performed over those measurement assemblages $\mathcal{M}_A$ for which the l.h.s. is finite. We are ready to state our first observation.

Observation 1.—Let F be a convex subset of state assemblages. For any state assemblage $\mathcal{A} \notin F$ there exists a set of measurements that antidistinguishes the assemblage better than any assemblage in F. Moreover, the relative advantage is exactly quantified by the convex weight of $\mathcal{A}$ with respect to F.

As possible examples of the set F we mention unsteerable assemblages and their generalization to assemblages that can be prepared by states with an upper-bounded Schmidt number. In this case, the antidistinguishing POVM can be alternatively interpreted as an instance of the task of subchannel exclusion supported by one-way local operations and classical communication (one-way LOCC), see Ref. [5] for the details of such interpretation. In the case of state ensembles (i.e., assemblages with only one input $x = 1$), the antidistinguishing POVM relates to the task of subchannel exclusion on a single system; see Refs. [13,18] for details. In the case of single states, one can relate the POVM to the task as a phase exclusion, see Ref. [18]. In these cases, the possible examples include trivial ensembles, separable states, and states with a positive partial transpose.

When the free set F consists of trivial ensembles, i.e., ensembles that code no information about which state was sent $\mathcal{E} \coloneqq \{(1/n)\varrho\}_{x=1}^n$, the weight corresponds to a measure of antidistinguishability. The corresponding optimization constraints read $Y \geq 0$ and $\sum_i Y_i \geq n\mathbb{1}$. One can rewrite the second constraint as $\sum_i Y_i \geq \mathbb{1}$ by multiplying the object function with $1/n$. Assuming that an optimal witness $\{\tilde{Y}_i\}$ does not satisfy the equality $\sum_i \tilde{Y}_i = \mathbb{1}$, equality follows when dividing the witness by $\sum_i \|Y_i\|$ and adding an extra outcome. Hence, the relative advantage in exclusion tasks can include inconclusive events, whereas the other common measure of generalized robustness measures the relative advantage in discrimination tasks without inconclusive events. Interestingly, this shows that instead of searching for a POVM that gives the optimal relative advantage in (anti-)distinguishing tasks, one can search for a single state defining the trivial ensemble $\mathcal{E}$ that optimizes the corresponding convex distance, see Supplemental Material [20] for more details.

As another interesting free set, consider separable states (or any other set with pure free states). For such sets, the convex weight shows a peculiar feature: when moving from the free set to a resourceful pure state, the weight jumps from zero to one. However, continuity is not essential for the quantum advantage. Namely, the quantum advantage remains regardless of small disturbances. This is due to the fact that the convex weight is in general lower semi-continuous similarly to the generalized robustness as shown in the Supplemental Material [20], see also Ref. [27].

For a measurement assemblage $\mathcal{M}_A = \{M_{i|x}\}$ the primal problem reads

$$1 - \mathcal{W}_F(\mathcal{M}_A) = \max \frac{1}{|X|d} \sum_{i,x} \mathrm{tr}[O_{i|x}] \tag{7}$$

$$\text{s.t.:}\ M_{i|x} \geq O_{i|x} \quad \forall\ i, x, \qquad \{O_{i|x}\}_{i,x} \in C_F, \tag{8}$$

where $|X|$ is the number of measurements and d is the dimension of the Hilbert space. The dual of this reads

$$1 - \mathcal{W}_F(\mathcal{M}_A) = \min_{Y \geq 0} \sum_{i,x} \mathrm{tr} M_{i|x} Y_{i|x}$$
$$\text{s.t.:}\ \sum_{i,x} \mathrm{tr} T_{i|x} Y_{i|x} \geq 1 \quad \forall\ \{T_{i|x}\} \in F, \tag{9}$$

where $Y = \bigoplus_{i,x} Y_{i|x}$ is again a witness. With a similar argument as in the case of state assemblages, one checks that the Slater condition is valid. To get an expression similar to Eq. (6), we decompose the witness as $Y_{i|x} = \tilde{N} p(i,x) \varrho_{i|x}$, where $\varrho_{i|x} \coloneqq Y_{i|x}/\mathrm{tr} Y_{i|x}$, $p(i,x) \coloneqq \mathrm{tr} Y_{i|x}/\tilde{N}$, and $\tilde{N} \coloneqq \sum_{i,x} \mathrm{tr} Y_{i|x}$. Noting again that scaling does not affect the desired expression, we write

$$\inf_{\mathcal{A}} \frac{p_{\mathrm{succ}}(\mathcal{A}, \mathcal{M}_A)}{\min_{\mathcal{O}_A \in F} p_{\mathrm{succ}}(\mathcal{A}, \mathcal{O}_A)} = 1 - \mathcal{W}_F(\mathcal{M}_A), \tag{10}$$

where the optimization is performed over those state assemblages $\mathcal{A}$ for which the l.h.s. is finite. We arrive at our second observation.

Observation 2.—Let F be a convex subset of sets of POVMs. For any set of POVMs $\mathcal{M}_A \notin F$ there exists a state exclusion task where $\mathcal{M}_A$ outperforms any set of POVMs in F. Moreover, the relative advantage is exactly quantified by the convex weight of $\mathcal{A}$ with respect to F.

As examples of convex sets of measurements we mention joint measurability, informativity of a POVM (see Supplemental Material [20] for the explicit form of the weight), and simulability with projective (or any fixed subset of) POVMs. Note that in the case of sets of POVMs, the task includes the classical communication of the label x from the preparing party to the measuring party. This allows the measuring party to choose the measurement setting after receiving the label. In the case of discrimination tasks, such sacenario is referred to as state discrimination with premeasurement information [16].

In the case of quantum channels, i.e., completely positive trace-preserving maps, we start by writing the cone program in the Choi picture

$$1 - \mathcal{W}_F(\Lambda) = \max\ \mathrm{tr} J_{\hat{\Gamma}}$$
$$\text{s.t.:}\ J_\Lambda - J_{\hat{\Gamma}} \geq 0, \qquad J_{\hat{\Gamma}} \in C_{J_F}, \tag{11}$$

where J_Λ is the Choi state of Λ, and similarly for $\hat{\Gamma}$. The above optimization problem is an instance of a cone program. Such a program comes with a dual formulation given by

$$1 - \mathcal{W}_F(\Lambda) = \min_Y\ \mathrm{tr} Y J_\Lambda$$
$$\text{s.t.:}\ Y \geq 0, \qquad \mathrm{tr} Y T \geq 1 \quad \forall\ T \in J_F, \tag{12}$$

where Y is a dual variable constituting a witness for the set J_F. Once again, the Slater condition can be validated as in the case of state assemblages.

One can decompose the witness as $Y = d \sum_{ij} \omega_{ij} p(i) \varrho_i^T \otimes M_j$ for some state ensemble $\{p(i)\varrho_i\}$, POVM $\{M_j\}$, and set of real numbers $\{\omega_{ij}\}$. (The transpose is taken in the computational basis.) This decomposition shows that the weight $\mathcal{W}_F(\Lambda)$ is related to a payoff $P(\Lambda, \mathcal{G})$ of a specific input-output game:

$$\mathrm{tr} Y J_\Lambda = \sum_{ij} d\omega_{ij} p(i) \mathrm{tr}(\varrho_i^T \otimes M_j) J_\Lambda$$
$$= \sum_{ij} \omega_{ij} p(i) \mathrm{tr} \Lambda(\varrho_i) M_j = P(\Lambda, \mathcal{G}). \tag{13}$$

See Supplemental Material in [20] for more details.

To get our result for channels, note that an optimal decomposition for Λ from Eq. (1) with devices $D_F = \Gamma$ and $\tilde{D} = \tilde{\Lambda}$ gives a lower bound for the payoff of any canonical input-output game as

$$P(\Lambda, \mathcal{G}) = [1 - \mathcal{W}_F(\Lambda)] P(\Gamma, \mathcal{G}) + \mathcal{W}_F(\Lambda) P(\tilde{\Lambda}, \mathcal{G})$$
$$\geq [1 - \mathcal{W}_F(\Lambda)] \min_{\Gamma \in F} P(\Gamma, \mathcal{G}). \tag{14}$$

Note further that an input-output game given by an optimal witness Y is up to scaling in the canonical form. This can be seen by putting Eq. (1) into the Choi picture and applying an optimal witness on both sides of the resulting equation. It follows that the payoff for the channel $\tilde{\Lambda}$ is zero. Putting this together with Eqs. (12)–(14) and noting that the last equation is invariant under scaling of the games we get

$$\inf_{\mathcal{G}} \frac{P(\Lambda, \mathcal{G})}{\min_{\Gamma \in F} P(\Gamma, \mathcal{G})} = 1 - \mathcal{W}_F(\Lambda), \tag{15}$$

where the infimum is taken over all canonical input-output games. We are ready to state our third result.

Observation 3.—Let F be a convex subset of channels. For any channel $\Lambda \notin F$ there exists an input-output game in which the channel results in a lower payoff than any channel in F. Moreover, the relative advantage is exactly quantified by the convex weight of Λ with respect to F.

Note that this observation can be directly generalized to the level of sets of channels and sets of quantum instruments by considering the involved completely positive maps as a direct sum and having individual input-output games for each block. More precisely, for a set of instruments $\mathbf{I} \coloneqq \{\mathcal{I}_{i|x}\}_{i,x}$ one gets an extra coefficient $1/|X|$ and the Choi states become direct sums of the individual (subnormalized) ones, i.e., $\bigoplus_{i,x} J_{\mathcal{I}_{i|x}}$ in Eq. (11). The dual is simply a direct sum of the duals of the form Eq. (12) and the witnesses get the decomposition $Y_{i|x} = \sum_{a,b} p(a,i,x)\omega_{abix}\varrho_{a|i,x} \otimes M_{b|i,x}$. The payoff is then defined as the sum of all individual payoffs $P(\mathbf{I},\mathcal{G}) \coloneqq \sum_{a,b,i,x} p(a,i,x)\omega_{abix}\mathrm{tr}[\mathcal{I}_{a|x}(\varrho_{i|x,a})M_{j|x,a}]$. Using Eq. (12) it is straightforward to show that

$$\inf_{\mathcal{G}} \frac{P(\mathbf{I},\mathcal{G})}{\min_{\mathbf{\Gamma}\in F} P(\mathbf{\Gamma},\mathcal{G})} = 1 - \mathcal{W}_F(\mathbf{I}). \qquad (16)$$

As examples of free sets F we mention entanglement breaking channels, incompatibility breaking channels, compatible channels, compatible instruments, random unitaries, and finite rounds of LOCC protocols.

Conclusions.—We showed that the convex weight, a natural measure for quantum resrouces, has all the desirable properties. Besides the basic requirements of faithfulness, convexity, and monotonicity, the convex weight also exactly captures the relative advantage of a quantum resource in an exclusion (or anti-distinguishability) task. This correspondance is fully general and can be applied in principle to any type of quantum resource. As examples we have discussed the cases of state assemblages, sets of POVMs, and sets of transformations.

Moreover, these ideas could be directly applied to experiments (similarly to those of Refs. [28,29]), as the exclusion task requires only control of the input state and the output measurements.

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Note added.—Recently, we became aware of the related work by Ducuara and Skrzypczyk [30]. The authors prove a connection between exclusion tasks, convex weight, and single-shot information theory for resource theories of measurements and states.

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