Quantum Reservoir Computing in bosonic networks

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ABSTRACT

Quantum reservoir computing is an unconventional computing approach that exploits the quantumness of physical systems used as reservoirs to process information, combined with an easy training strategy. An overview is presented about a range of possibilities including quantum inputs, quantum physical substrates and quantum tasks. Recently, the framework of quantum reservoir computing has been proposed using Gaussian quantum states that can be realized e.g. in linear quantum optical systems. The universality and versatility of the system makes it particularly interesting for optical implementations. In particular, full potential of the proposed model can be reached even by encoding into quantum fluctuations, such as squeezed vacuum, instead of classical intense fields or thermal fluctuations. Some examples of the performance of this linear quantum reservoir in temporal tasks are reported.

Keywords: Information processing, quantum reservoir computing, photonic systems, quantum optics, continuous variables.

1. INTRODUCTION

Machine learning methods represent a formidable opportunity and in some case a unique approach to process information in the big data era and in demanding tasks.[1–3] Beyond widespread present computers technologies, non-conventional methods[4,5] have been proposed in computing ranging from quantum computation and simulations[6] to neuromorphic computing.[7,8] In particular, neuro-inspired computing goes beyond von Neumann architectures, by physically co-locating processing and memory operations.[9,10] This computational paradigm is inspired by the computational power and energy efficiency of the human brain[11,12] and based on the dynamics of the considered physical substrate.

A prominent example of neuro-inspired computing is represented by artificial neural networks[13–15] where single dynamical units (e.g. perceptrons[16,17]) are combined in feed-forward or recurrent architectures to realize common tasks such as pattern recognition, image and speech processing or temporal series forecasting. To achieve a good performance, neural networks need to be tuned, i.e. optimized, with generally demanding procedures such as back-propagation.[18] Within this field, Reservoir Computing (RC) emerges as an easily trainable alternative, based on the observation that the optimization of the last (output) layer is often the most important ingredient to achieve the desired performance.[19–21] This alternative approach builds on echo state networks[22] or liquid state machines[23] proposed two decades ago and it can be generalized to physical reservoir computing, where computing substrates, beyond complex networks of neurons, can actually be implemented in a variety of physical systems and devices as recently reviewed in Refs. 20, 21. Experimental implementations have been recently reported in photonics[19] like in semiconductor lasers[24] or electronics circuits[25] with delayed feedback, in spintronics like spin-torque nano-oscillators,[26] mechanics[27,28] and even biological systems.[29] Reservoir computing extracts information from data inputs exploiting the rich dynamics of generally nonlinear physical systems and has been demonstrated to be successful in real-time data processing with state-of-the-art performance in tasks such as continuous speech recognition[30] and nonlinear time series prediction.[31] The main feature needed for

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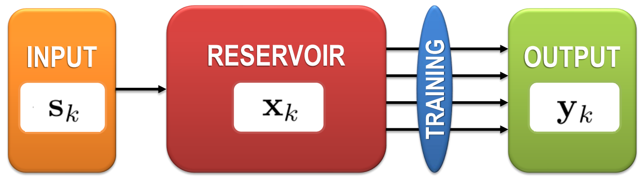

Figure 1: The basic structure for RC is formed by an input layer, a reservoir, and an output layer. The information contained in the input is introduced to the reservoir, whose internal connections act as a fixed hidden layer. Some of the responses of the reservoir are used to produce the desired output after an optimization procedure during the training process (only the output connections are adapted in the training).

on-line processing is the intrinsic memory of these systems, allowing one to execute temporal tasks and opening the possibility to embed them as components of more complex devices for edge computing.[32,33]

The potential of RC can be significantly broadened when considering quantum instead classical physical reservoirs, as recently reviewed in Ref. 34. A major advantage of quantum reservoir computing (QRC) is related to the possibility to access a larger Hilbert space even when considering quantum systems with few components.[35] Besides, also the opportunity of implementations of RC in NISQ devices[36] is particularly timely.[37] QRC indeed represents a promising avenue in the burgeoning field of quantum machine learning [38–43] exploring the potential for information processing in non-classical reservoirs, for either classical or quantum tasks and input data.[34] Recently, quantum reservoir computing proposals devoted to temporal tasks have been put forward considering complex networks of spins,[35,44,45] as well as bosonic networks,[46] and also with a single nonlinear oscillator.[47] Further examples of non-temporal tasks still based on the same architecture of QRC, as in Quantum Extreme Learning Machines (QELM), have been also reported for quantum chemistry calculations in spin systems[48] and Fermi-Hubbard[49–51] or Bose-Hubbard models.[52]

A proposal of interest in photonic realizations of QRC is based on optical networks. These can be engineered considering spatial, temporal or frequency light modes that interact in nonlinear devices and can exhibit non-classical features.[46] In Ref. 53 the possibility to create harmonic networks in optical parametric processes pumped by optical frequency combs has been reported. The flexibility in creating different complex networks is achieved by shaping the pump profile and multimode measurements, as already demonstrated in the context of linear optics quantum computing.[54–56] This physical platform can be considered either in the classical regime and beyond it, when quantum signatures such as squeezing and entanglement are present. Furthermore, these quantum networks are also of interest in the context of open quantum systems and emergent phenomena like synchronization.[57–59] Here we present the features of QRC in bosonic networks restricted to Gaussian states with a focus on their performance in temporal tasks and universality.

2. CLASSICAL AND QUANTUM RESERVOIR COMPUTING

2.1 Classical Reservoir Computing

The main idea behind classical RC is to make use of the intrinsic dynamics of a system, the reservoir, for information processing. The fundamental parts needed for RC are represented by the three-layer scheme in Fig. 1. The input at a given timestep k is mapped into the state of the reservoir, and, afterwards, the output is computed from observables of the reservoir via connections that are trained using some simple learning algorithm, as, for instance, a linear regression.

The classical information encoded in the input can be represented by a vector containing a series of real numbers in each of its components, $\{\mathbf{s}_k\}$. The state of the reservoir is given by a vector of N real numbers that represents its characteristic quantities, $\mathbf{x}_k$, which is updated after the injection of the input and also depends on the system's dynamics. In general, the state of the reservoir can be written as

$$\mathbf{x}_k = \mathbf{f}(\mathbf{s}_k, \mathbf{x}_{k-1}). \tag{1}$$

Notice that the function $\mathbf{f}$ depends on the previous state of the reservoir $\mathbf{x}_{k-1}$ and, implicitly, on previous inputs, so that the memory of the system is exploited. Moreover, $\mathbf{f}$ should be such that the system has the fading memory property (FMP), i.e. similar input sequences give rise to similar outputs, and also it has the echo state property (ESP), the state of the reservoir depends only on the recent input history and becomes independent of distant-past events. Finally, a good reservoir should also be equipped with the separability property, which ensures that different inputs are mapped into different outputs.

As a general output one can consider a series of vectors of real numbers, $\{\mathbf{y}_k\}$, that are computed from a selection of the reservoir variables, $\mathbf{x}_k^{\text{out}}$, as

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k^{\text{out}}), \tag{2}$$

where the trainability of the links between the reservoir and the output layer arises from free parameters present in $\mathbf{h}$. As usual in any supervised-learning technique, those free parameters are optimized giving training input examples to the system and minimizing a cost function between the known outputs and the predicted ones.

In addition to memory, a fundamental resource for computing is the existence of a nonlinear input-output map provided by the joint effect of $\mathbf{f}$ and $\mathbf{h}$. Therefore, if $\mathbf{f}$ guarantees nonlinearity, $\mathbf{h}$ could have a simple linear form, which facilitates the training process.

2.2 Quantum Reservoir Computing

The main motivation to study quantum systems for reservoir computing is the potential advantages coming from quantum resources. The principle of quantum superposition provides a large state space even for reservoirs consisting of few constituents.[35] Moreover, entanglement has proven to be a key ingredient for obtaining a quantum advantage for computing.[36] Consequently, there are great research efforts to use it to enhance the performance of computational tasks.

Common extensions of RC to the quantum domain are proposals inspired by the pioneering work of Fujii and Nakajima,[35] in which the classical reservoir of the scheme in Fig. 1 is replaced by a quantum reservoir. Furthermore, besides the quantization of the reservoir, a general overview leads to consider also the classical or quantum nature of the input and the computational task to be performed.[34] Instead of series of real numbers, quantum inputs could be consider as a series of general quantum states written as density matrices, $\{\rho_k^{\text{in}}\}$, that modify the state of the reservoir. Regarding the quantumness of tasks in the framework of RC, there is room for exploring the possibility of predicting the time-dependent evolution of quantum states or their properties.[60–62]

3. RESERVOIR COMPUTING WITH A LINEAR QUANTUM NETWORK

The system under consideration in the present manuscript is the bosonic platform studied in Ref. 46. This can be considered the simplest system to perform QRC being based on a set of coupled (quantum) harmonic oscillators and restricting to Gaussian states. Still, we will show that the system is powerful with the proper input encoding and also universal.

3.1 Quantum Reservoir of Harmonic Oscillators

The reservoir is a quantum system consisting of a network of N quantum harmonic oscillators. The oscillators interact with each other via spring-like links of strength g_{ij}, i.e. the interaction is described by the Laplacian matrix $\mathbf{L}$ with components $\mathbf{L}_{ij} = \delta_{ij}\sum_k g_{ik} - (1-\delta_{ij})g_{ij}$, and the Hamiltonian of the system is given by:

$$H = \frac{\mathbf{p}^{\mathrm{T}}\mathbf{p}}{2} + \frac{\mathbf{q}^{\mathrm{T}}(\mathbf{\Delta}_{\omega}^2 + \mathbf{L})\mathbf{q}}{2}, \tag{3}$$

where the momentum and position of the oscillators are, respectively, in the vectors $\mathbf{p}^{\mathrm{T}} = (p_1, ..., p_N)$, and $\mathbf{q}^{\mathrm{T}} = (q_1, ..., q_N)$. $\mathbf{\Delta}_{\boldsymbol{\omega}}$ is a diagonal matrix that contains the frequencies of the oscillators $\boldsymbol{\omega}^{\mathrm{T}} = (\omega_1, ..., \omega_N)$. For all quantities, we adopt the same system of units as in Ref. 46.

In this framework, N_a oscillators are arbitrarily chosen and referred to as the ancillae. Their states are reset at each timestep k in order to introduce the input, $\{\mathbf{s}_k\}$, to the system. The rest of the system constitutes the reservoir, whose state is characterized by the position and momentum operators of the $N - N_a$ oscillators, $(\mathbf{x}^{\mathrm{R}})^{\mathrm{T}} = (q_1, p_1, \ldots, q_{N-N_a}, p_{N-N_a})$.

For this system, the equation analogous to Eq. (1) takes the form

$$\mathbf{x}_k^{\mathrm{R}} = \mathbf{A}\mathbf{x}_{k-1}^{\mathrm{R}} + \mathbf{B}\mathbf{x}_k^{\mathrm{A}}, \tag{4}$$

where $\mathbf{A}$ and $\mathbf{B}$ depend on the dynamics arising from the Hamiltonian[46] and they act, respectively, on the state of the reservoir in the previous time, $\mathbf{x}_{k-1}^{\mathrm{R}}$, and on the state of the ancilla, $\mathbf{x}_k^{\mathrm{A}}$, which is fixed by the input. By encoding the input in the form of Gaussian states of the ancilla, it is sufficient to obtain the first moments $\langle \mathbf{x}_k^{\mathrm{R}} \rangle$, and covariances $\sigma(\mathbf{x}_k^{\mathrm{R}})$, of the reservoir observables (see Ref. 46 for their explicit form) in order to construct the output as

$$y_k = h\left(\langle \mathbf{x}_k^{\mathrm{R}} \rangle, \sigma(\mathbf{x}_k^{\mathrm{R}})\right). \tag{5}$$

3.2 Universality

Every time a new machine learning technique is proposed, there is a question that must be faced: what are the tasks that this technique can solve? From a theoretical point of view, one can tackle this problem with the help of universal approximation theorems. These theorems expose what are the conditions required to find examples of our class of models that can approximate elements of a given class of functions with arbitrary precision. A proposal that fulfills the requirements of one of these theorems is said to possess the universal approximation property, i.e., the model can solve (in theory) all the tasks considered in the framework of the theorem.

There are many well known results about the universal approximation property of neural networks, like the approximation of any continuous function by feed-forward neural networks.[63,64] In the RC field, universality has been shown for different systems such as liquid state machines[23] and echo state networks.[65] In the context of our work, a RC system is said to be universal when it can approximate any fading memory function,[23,65] which can be thought of as a continuous function of a finite number of past inputs. More specifically, we mean by universality as the possibility of finding elements of our RC class that approximate a given time-invariant, causal and fading memory map with arbitrary precision, as defined in Ref. 66.

Within the framework of QRC with continuous variables of Ref. 46, first the requirements for ESP and FMP were fully specified, showing that the condition $\rho(\mathbf{A}) < 1$ is a necessary and sufficient condition, where $\rho(\mathbf{A})$ is the spectral radius of matrix $\mathbf{A}$ in Eq. (4). Then, it was shown that the associated algebra has separability, which means that for any pair of different time series there is an instance of the model that can tell them apart. Finally, universality was proved invoking the validity of the Stone-Weierstrass theorem. Notice that unlike ESP and FMP, separability depends explicitly on the input encoding. Indeed, in Ref. 46 separability for three different kinds of encodings was shown: in thermal fluctuations, in magnitude of squeezing strength and in phase of squeezing. It should be pointed out that separability (and hence, universality) could be shown also for first moments encoding, like for coherent states. Importantly, noise does not necessarily rule out universal QRC[37] and it has been suggested[67] that a small amount of noise can prevent overfitting in NISQ systems, similarly to the classical neural-network case.

3.3 Temporal Tasks

In this Section, we present some examples of temporal tasks. The used network is completely connected with 15 unit mass quantum harmonic oscillators and identical bare frequencies $\omega_0 = 0.25$. The coupling strengths $g_{ij} = g_{ji}$ between network oscillators i and j are chosen uniformly at random such that $g_{ij} \in [0, 0.2]$. Five of the oscillators are randomly selected to play the role of ancillae, subject to periodic state resets according to the input. Product states of identical squeezed vacuum states are considered for simplicity. More explicitly, the covariance matrix for a squeezed vacuum state of frequency Ω, as the ones employed for the present tasks, is given by

$$\boldsymbol{\sigma}(\mathbf{x}) = \frac{1}{2}\begin{pmatrix} [\cosh(2r) + \cos(\varphi)\sinh(2r)]\,\Omega^{-1} & \sin(\varphi)\sinh(2r) \\ \sin(\varphi)\sinh(2r) & [\cosh(2r) - \cos(\varphi)\sinh(2r)]\,\Omega \end{pmatrix} \tag{6}$$

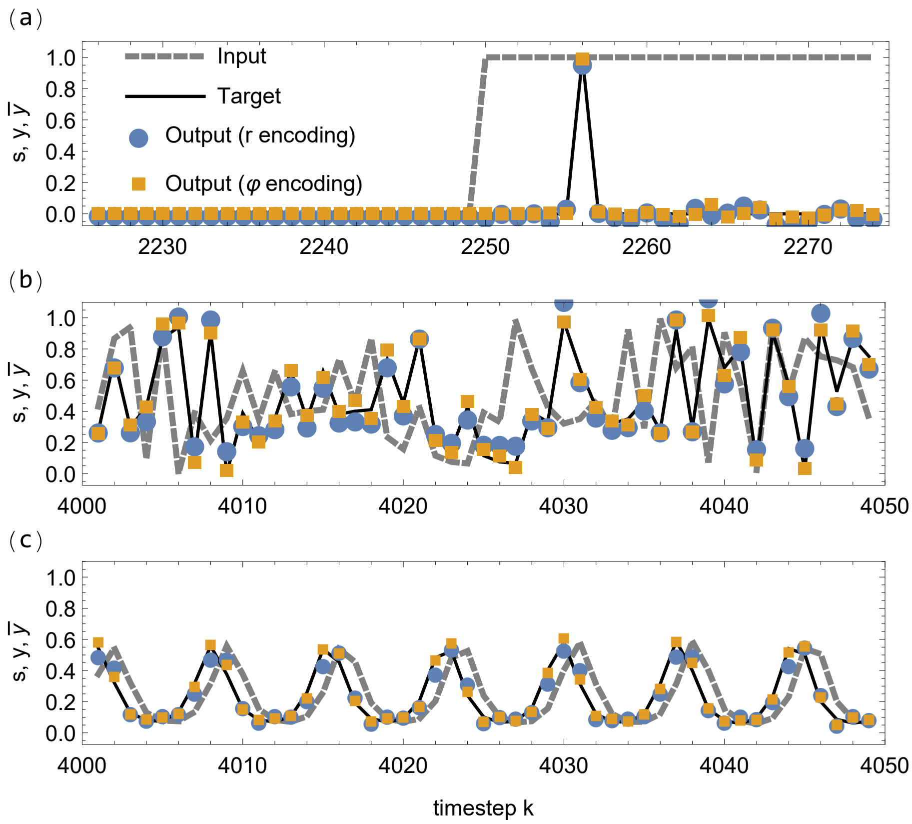

Figure 2: Temporal tasks solved with Gaussian states of a bosonic network. (a) In the timer task the reservoir is trained to respond to the change in the value of the input $\mathbf{s}$ (gray dashed line) with a given delay, here 6 timesteps, as shown by the target output $\bar{\mathbf{y}}$ (black solid line). Using squeezed vacuum states for the ancillae, this can be achieved by encoding the input either into the magnitude of squeezing r or its phase φ. The output $\mathbf{y}$ from the trained reservoir is indicated by circles and squares, respectively. (b) In the short term memory task the reservoir is trained to recall past inputs using its memory. Here the delay is 3 timesteps. (c) Santa Fe time series prediction task requires the reservoir to predict the next value of the input which follows the eponymous time series. See main text for more details.

In what follows, the input is either encoded on the magnitude of squeezing by setting $r = s_k$ whereas the phase is kept $\varphi = 0$, or on the phase by setting $r = 1$ and $\varphi = s_k$. The time Δt between state resets is chosen as in Ref. 46 for completely connected networks. The reservoir output at some timestep k is

$$y_k = w_0 + \sum_{i=1}^{20} w_i(\sigma(\mathbf{x}))_{ii} = w_0 + \sum_{i=1}^{20} w_i(\langle \mathbf{x}_i^2 \rangle - \langle \mathbf{x}_i \rangle^2), \tag{7}$$

where $\mathbf{x}_i$ are the position and momentum operators of the remaining 10 oscillators, playing the role of the reservoir, whereas w_i are weights trained to minimize the mean squared error between y_k and given target output $\bar{\mathbf{y}}$ as in Ref. 46. Due to FMP, the initial state of the network is irrelevant, however in all numerical experiments we have initialized the system from the ground state of the network Hamiltonian.

Here the network is trained to solve three different temporal tasks. Let us start with the timer task, a well-known benchmark task in RC literature which tests the memory of the reservoir by asking it to respond to the change in the input value with a given delay τ. The definition reads

$$s_k = \begin{cases} 1 & k \geq c, \\ 0 & k < c, \end{cases}$$
$$\bar{y}_k = \begin{cases} 1 & k = c + \tau, \\ 0 & \text{otherwise}, \end{cases} \tag{8}$$

where c is a given point in time where the input switches from one value to the other. Here we have first driven the reservoir for 2000 timesteps to get rid of the influence of the initial conditions. The switch happens at $c = 2250$ and we have chosen $\tau = 6$. The training procedure uses timesteps from 2001 to 4000.

The second task we consider is the short term memory task, another popular benchmark test that focuses on the linear memory of the reservoir, i.e. the ability of the reservoir to learn linear functions of the input. The definition is simply $\bar{y}_k = s_{k-\tau}$ where the delay $\tau \geq 0$. Here we use $s_k \in [0, 1]$, chosen uniformly at random, and $\tau = 3$. The reservoir is prepared with 2000 timesteps and trained with additional 2000 timesteps.

Finally, the Santa Fe time series prediction task is based on data recorded from a far-infrared laser in a chaotic state.[68,69] The full time series consists of 10093 non-negative real numbers, which for the sake of convenience we scale to lie in the interval $[0, 1]$. The input s_k is then the k-th element of the scaled time series whereas $\bar{y}_k = s_{k+1}$. Reservoir memory also plays a role here since the next value of the time series can be approximated by a function of the previous values. As in the short-term memory task, the reservoir is prepared with the first 2000 timesteps and trained with additional 2000 timesteps. Results for all three tasks are shown in Fig. 2, confirming that the network can achieve good performance with both of the considered encodings.

4. DISCUSSION AND OUTLOOK

Quantum reservoir computing is expected to combine the advantages of classical reservoir computing (easy and fast trainability, use of physical platforms to perform the computation) together with the potential opened by the reservoir quantumness (large Hilbert space, possible quantum advantage due to non-classical correlations). Here, we have delved into the case of QRC using Gaussian states of continuous variables, which can be seen as the quantum minimal model in terms of resources required.[70] Indeed, such a simple linear network already shows a remarkable advantage with respect to the use of purely classical resources, as detailed in Ref. 46. Furthermore, exploiting different forms of input encoding it allows for universal RC, i.e., it is able to approximate any fading memory function.

In this work, we have explicitly considered three different temporal tasks and shown the capability of our QRC to solve them. Remarkably, the only form of nonlinearity used to perform such information processing is represented by the way input is injected into the system,[46] as the encoding is in general nonlinear, for instance through the squeezing amplitude or the squeezing phase, as explicitly discussed in our examples. This alone provides nonlinearity for the overall input-output map (see also Ref. 71 for a recent discussion about this aspect). Furthermore, the input encoding can also be seen as a source of versatility for multi-purpose information processing. Indeed, depending on the task at hand, one may require different balances between linearity and nonlinearity, and also different degrees of nonlinearity, which can be achieved using the same reservoir and only changing the way input is injected.

Moreover, the versatility of the system is not limited to what explicitly discussed in this work, that is, to the solution of classical time-dependent tasks. Indeed, in spite of its simplicity, the model of QRC with Gaussian states is also suited to tackle purely quantum problems, i.e., tasks where also the input and the output are allowed to be quantum entities. An explicit example of such a potential is given in Ref. 34, where the system employed here was efficiently used, as a QELM, to classify the magnitude of squeezing of different input states.

To conclude, let us also comment on the potential for an implementation of continuous-variable QRC in physical systems, which lies at the hearth of any unconventional computing scheme. The protocol described here is especially suited for implementation in of photonic devices. As previously mentioned, large and complex networks of harmonic oscillators can be built using optical frequency combs and parametric processes, together with multimode homodyne or heterodyne detection.[53] Another possible implementation is based on the use of lattices of coplanar waveguide resonators, that have already been produced for photons in the microwave regime, as described in Ref. 72.

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REFERENCES

[1] Jordan, M. I. and Mitchell, T. M., "Machine learning: Trends, perspectives, and prospects," *Science* **349**(6245), 255–260 (2015).

[2] LeCun, Y., Bengio, Y., and Hinton, G., "Deep learning," *Nature* **521**(7553), 436–444 (2015).

[3] Silver, D., Schrittwieser, J., Simonyan, K., Antonoglou, I., Huang, A., Guez, A., Hubert, T., Baker, L., Lai, M., Bolton, A., et al., "Mastering the game of go without human knowledge," *Nature* **550**(7676), 354–359 (2017).

[4] Adamatzky, A., Bull, L., and Costello, B. D. L., [*Unconventional computing 2007*], Luniver Press (2007).

[5] Jaeger, H., "Toward a generalized theory comprising digital, neuromorphic, and unconventional computing," *Neuromorphic Computing and Engineering* (2021).

[6] Cirac, J. I., "Quantum computing and simulation," *Nanophotonics* **10**(1), 453–456 (2021).

[7] Marković, D., Mizrahi, A., Querlioz, D., and Grollier, J., "Physics for neuromorphic computing," *Nature Reviews Physics* **2**, 499–510 (Sep 2020).

[8] Zhu, J., Zhang, T., Yang, Y., and Huang, R., "A comprehensive review on emerging artificial neuromorphic devices," *Applied Physics Reviews* **7**(1), 011312 (2020).

[9] Wright, C. D., Hosseini, P., and Diosdado, J. A. V., "Beyond von-neumann computing with nanoscale phase-change memory devices," *Adv. Funct. Mater.* **23**(18), 2248–2254 (2013).

[10] Ielmini, D. and Wong, H.-S. P., "In-memory computing with resistive switching devices," *Nat. Electron.* **1**(6), 333–343 (2018).

[11] Monroe, D., "Neuromorphic computing gets ready for the (really) big time," *Commun. ACM* **57**, 13–15 (June 2014).

[12] Esser, S. K., Merolla, P. A., Arthur, J. V., Cassidy, A. S., Appuswamy, R., Andreopoulos, A., Berg, D. J., McKinstry, J. L., Melano, T., Barch, D. R., et al., "Convolutional networks for fast, energy-efficient neuromorphic computing," *Proc. Natl. Acad. Sci. U.S.A.* **113**(41), 11441–11446 (2016).

[13] Benjamin, B. V., Gao, P., McQuinn, E., Choudhary, S., Chandrasekaran, A. R., Bussat, J.-M., Alvarez-Icaza, R., Arthur, J. V., Merolla, P. A., and Boahen, K., "Neurogrid: A mixed-analog-digital multichip system for large-scale neural simulations," *Proc. IEEE* **102**(5), 699–716 (2014).

[14] Furber, S. B., Galluppi, F., Temple, S., and Plana, L. A., "The spinnaker project," *Proc. IEEE* **102**(5), 652–665 (2014).

[15] Indiveri, G. and Liu, S.-C., "Memory and information processing in neuromorphic systems," *Proc. IEEE* **103**(8), 1379–1397 (2015).

[16] Rosenblatt, F., "The perceptron: A probabilistic model for information storage and organization in the brain.," *Psychological Review* **65**(6), 386–408 (1958).

[17] Minsky, M. and Papert, S., [*Perceptrons: An Introduction to Computational Geometry*], MIT Press, Cambridge, MA, USA (1969).

[18] Williams, R. J. and Peng, J., "An efficient gradient-based algorithm for on-line training of recurrent network trajectories," *Neural Computation* **2**(4), 490–501 (1990).

[19] Brunner, D., Soriano, M. C., and der Sande, G. V., [*Photonic Reservoir Computing*], De Gruyter (2019).

[20] Tanaka, G., Yamane, T., Héroux, J. B., Nakane, R., Kanazawa, N., Takeda, S., Numata, H., Nakano, D., and Hirose, A., "Recent advances in physical reservoir computing: A review," *Neural Netw.* **115**, 100 – 123 (2019).

[21] Nakajima, K., "Physical reservoir computing—an introductory perspective," *Jpn. J. Appl. Phys* **59**(6), 060501 (2020).

[22] Jaeger, H. and Haas, H., "Harnessing nonlinearity: Predicting chaotic systems and saving energy in wireless communication," *Science* **304**(5667), 78–80 (2004).

[23] Maass, W. and Markram, H., "On the computational power of circuits of spiking neurons," *Journal of computer and system sciences* **69**(4), 593–616 (2004).

[24] Brunner, D., Soriano, M. C., Mirasso, C. R., and Fischer, I., "Parallel photonic information processing at gigabyte per second data rates using transient states," *Nature communications* **4**(1), 1–7 (2013).

[25] Appeltant, L., Soriano, M., Van Der Sande, G., Danckaert, J., Massar, S., Dambre, J., Schrauwen, B., Mirasso, C., and Fischer, I., "Information processing using a single dynamical node as complex system," *Nature Communications* **2**(1) (2011). cited By 542.

[26] Torrejon, J., Riou, M., Araujo, F. A., Tsunegi, S., Khalsa, G., Querlioz, D., Bortolotti, P., Cros, V., Yakushiji, K., Fukushima, A., Kubota, H., Yuasa, S., Stiles, M. D., and Grollier, J., "Neuromorphic computing with nanoscale spintronic oscillators," *Nature* **547**, 428–431 (Jul 2017).

[27] Fernando, C. and Sojakka, S., "Pattern recognition in a bucket," in [*European conference on artificial life*], 588–597, Springer (2003).

[28] Nakajima, K., Hauser, H., Li, T., and Pfeifer, R., "Information processing via physical soft body," *Scientific reports* **5**(1), 1–11 (2015).

[29] Nikolić, D., Häusler, S., Singer, W., and Maass, W., "Distributed fading memory for stimulus properties in the primary visual cortex," *PLoS Biology* **7**(12), e1000260 (2009).

[30] Triefenbach, F., Demuynck, K., and Martens, J.-P., "Large vocabulary continuous speech recognition with reservoir-based acoustic models," *IEEE Signal Processing Letters* **21**(3), 311–315 (2014).

[31] Pathak, J., Hunt, B., Girvan, M., Lu, Z., and Ott, E., "Model-free prediction of large spatiotemporally chaotic systems from data: A reservoir computing approach," *Phys. Rev. Lett.* **120**, 024102 (Jan 2018).

[32] Shi, W., Cao, J., Zhang, Q., Li, Y., and Xu, L., "Edge computing: Vision and challenges," *IEEE Internet of Things Journal* **3**(5), 637–646 (2016).

[33] Satyanarayanan, M., "The emergence of edge computing," *Computer* **50**(1), 30–39 (2017).

[34] Mujal, P., Martínez-Peña, R., Nokkala, J., García-Beni, J., Giorgi, G. L., Soriano, M. C., and Zambrini, R., "Opportunities in quantum reservoir computing and extreme learning machines," *Advanced Quantum Technologies* , 2100027 (2021).

[35] Fujii, K. and Nakajima, K., "Harnessing disordered-ensemble quantum dynamics for machine learning," *Phys. Rev. Applied* **8**, 024030 (Aug 2017).

[36] Preskill, J., "Quantum Computing in the NISQ era and beyond," *Quantum* **2**, 79 (Aug. 2018).

[37] Chen, J., Nurdin, H. I., and Yamamoto, N., "Temporal information processing on noisy quantum computers," *Phys. Rev. Applied* **14**, 024065 (Aug 2020).

[38] Biamonte, J., Wittek, P., Pancotti, N., Rebentrost, P., Wiebe, N., and Lloyd, S., "Quantum machine learning," *Nature* **549**(7671), 195–202 (2017).

[39] Dunjko, V. and Briegel, H. J., "Machine learning & artificial intelligence in the quantum domain: a review of recent progress," *Rep. Prog. Phys.* **81**, 074001 (jun 2018).

[40] Carleo, G., Cirac, I., Cranmer, K., Daudet, L., Schuld, M., Tishby, N., Vogt-Maranto, L., and Zdeborová, L., "Machine learning and the physical sciences," *Rev. Mod. Phys.* **91**, 045002 (Dec 2019).

[41] Marković, D. and Grollier, J., "Quantum neuromorphic computing," *Appl. Phys. Lett.* **117**(15), 150501 (2020).

[42] Sarma, S. D., Deng, D.-L., and Duan, L.-M., "Machine learning meets quantum physics," *Phys. Today* **72**(3), 48 (2019).

[43] Ciliberto, C., Herbster, M., Ialongo, A. D., Pontil, M., Rocchetto, A., Severini, S., and Wossnig, L., "Quantum machine learning: a classical perspective," *Proc. R. Soc. A.* **474**, 20170551 (2018).

[44] Martínez-Peña, R., Nokkala, J., Giorgi, G. L., Zambrini, R., and Soriano, M. C., "Information processing capacity of spin-based quantum reservoir computing systems," *Cognit. Comput.* , 1–12 (2020).

[45] Chen, J. and Nurdin, H. I., "Learning nonlinear input–output maps with dissipative quantum systems," *Quantum Information Processing* **18** (May 2019).

[46] Nokkala, J., Martínez-Peña, R., Giorgi, G. L., Parigi, V., Soriano, M. C., and Zambrini, R., "Gaussian states of continuous-variable quantum systems provide universal and versatile reservoir computing," *Communications Physics* **4**(1), 1–11 (2021).

[47] Govia, L. C. G., Ribeill, G. J., Rowlands, G. E., Krovi, H. K., and Ohki, T. A., "Quantum reservoir computing with a single nonlinear oscillator," *Phys. Rev. Research* **3**, 013077 (Jan 2021).

[48] Kawai, H. and Nakagawa, Y. O., "Predicting excited states from ground state wavefunction by supervised quantum machine learning," *Mach. Learn.: Sci. Technol.* **1**, 045027 (2020).

[49] Ghosh, S., Opala, A., Matuszewski, M., Paterek, T., and Liew, T. C. H., "Reconstructing quantum states with quantum reservoir networks," *IEEE Trans. Neural Netw. Learn. Syst.* , 1–8 (2020).

[50] Ghosh, S., Opala, A., Matuszewski, M., Paterek, T., and Liew, T. C. H., "Quantum reservoir processing," *npj Quantum Inf.* **5**, 35 (Apr 2019).

[51] Krisnanda, T., Ghosh, S., Paterek, T., and Liew, T. C., "Creating and concentrating quantum resource states in noisy environments using a quantum neural network," *Neural Netw.* **136**, 141–151 (2021).

[52] Ghosh, S., Paterek, T., and Liew, T. C. H., "Quantum neuromorphic platform for quantum state preparation," *Phys. Rev. Lett.* **123**, 260404 (Dec 2019).

[53] Nokkala, J., Arzani, F., Galve, F., Zambrini, R., Maniscalco, S., Piilo, J., Treps, N., and Parigi, V., "Reconfigurable optical implementation of quantum complex networks," *New Journal of Physics* **20**, 053024 (may 2018).

[54] Cai, Y., Roslund, J., Ferrini, G., Arzani, F., Xu, X., Fabre, C., and Treps, N., "Multimode entanglement in reconfigurable graph states using optical frequency combs," *Nature Communications* **8**, 15645 (Jun 2017).

[55] Chen, M., Menicucci, N. C., and Pfister, O., "Experimental realization of multipartite entanglement of 60 modes of a quantum optical frequency comb," *Phys. Rev. Lett.* **112**, 120505 (Mar 2014).

[56] Asavanant, W., Shiozawa, Y., Yokoyama, S., Charoensombutamon, B., Emura, H., Alexander, R. N., Takeda, S., Yoshikawa, J.-i., Menicucci, N. C., Yonezawa, H., and Furusawa, A., "Generation of time-domain-multiplexed two-dimensional cluster state," *Science* **366**(6463), 373–376 (2019).

[57] Cabot, A., Galve, F., Eguíluz, V. M., Klemm, K., Maniscalco, S., and Zambrini, R., "Unveiling noiseless clusters in complex quantum networks," *npj Quantum Information* **4**, 57 (Nov 2018).

[58] Nokkala, J., Galve, F., Zambrini, R., Maniscalco, S., and Piilo, J., "Complex quantum networks as structured environments: engineering and probing," *Scientific Reports* **6**, 26861 (May 2016).

[59] Manzano, G., Galve, F., Giorgi, G. L., Hernández-García, E., and Zambrini, R., "Synchronization, quantum correlations and entanglement in oscillator networks," *Scientific Reports* **3**, 1439 (Mar 2013).

[60] Angelatos, G., Khan, S., and Türeci, H. E., "Reservoir computing approach to quantum state measurement," (Preprint) arXiv:2011.09652 (2020).

[61] Flurin, E., Martin, L. S., Hacohen-Gourgy, S., and Siddiqi, I., "Using a recurrent neural network to reconstruct quantum dynamics of a superconducting qubit from physical observations," *Phys. Rev. X* **10**, 011006 (Jan 2020).

[62] Tran, Q. H. and Nakajima, K., "Learning temporal quantum tomography," (Preprint) arXiv:2103.13973 (2021).

[63] Cybenko, G., "Approximation by superpositions of a sigmoidal function," *Mathematics of control, signals and systems* **2**(4), 303–314 (1989).

[64] Hornik, K., Stinchcombe, M., and White, H., "Multilayer feedforward networks are universal approximators," *Neural networks* **2**(5), 359–366 (1989).

[65] Grigoryeva, L. and Ortega, J.-P., "Echo state networks are universal," *Neural Networks* **108**, 495–508 (2018).

[66] Boyd, S. and Chua, L., "Fading memory and the problem of approximating nonlinear operators with volterra series," *IEEE Transactions on circuits and systems* **32**(11), 1150–1161 (1985).

[67] Dasgupta, S., Hamilton, K. E., and Banerjee, A., "Designing a nisq reservoir with maximal memory capacity for volatility forecasting," (Preprint) arXiv:2004.08240 (2020).

[68] Huebner, U., Abraham, N., and Weiss, C., "Dimensions and entropies of chaotic intensity pulsations in a single-mode far-infrared NH_3 laser," *Physical Review A* **40**(11), 6354 (1989).

[69] Weigend, A. S. and Gershenfeld, N. A., "Results of the time series prediction competition at the santa fe institute," in [*IEEE international conference on neural networks*], 1786–1793, IEEE (1993).

[70] Weedbrook, C., Pirandola, S., García-Patrón, R., Cerf, N. J., Ralph, T. C., Shapiro, J. H., and Lloyd, S., "Gaussian quantum information," *Rev. Mod. Phys.* **84**, 621–669 (May 2012).

[71] Govia, L. C. G., Ribeill, G. J., Rowlands, G. E., and Ohki, T. A., "Nonlinear input transformations are ubiquitous in quantum reservoir computing," (Preprint) arXiv:2107.00147 (2021).

[72] Kollár, A. J., Fitzpatrick, M., and Houck, A. A., "Hyperbolic lattices in circuit quantum electrodynamics," *Nature* **571**, 45–50 (Jul 2019).

Pere Mujal, Johannes Nokkala, Rodrigo Martinez-Peña, Jorge García-Beni, Gian Luca Giorgi, Miguel C. Soriano, and Roberta Zambrini "Quantum reservoir computing in bosonic networks", Proc. SPIE 11804, Emerging Topics in Artificial Intelligence (ETAI) 2021, 118041J (2 September 2021); https://doi.org/10.1117/12.2596177