Active Suspension System for Heavy Vehicles

A. Tahir, J. Yasin, M. Daneshtalab, and J. Plosila

***Abstract*—An Active Suspension System has the capacity to introduce, accumulate, and disperse energy to the system. Depending on the functional circumstances, the system may vary its parameters. This paper seeks to explain the designing of an Active Suspension System for heavy vehicles in the form of a case study and is focused on three methodological approaches: Proportional Integral Derivative control, Linear Quadratic Regulator control, and chattering free Sliding Mode Control. The findings should make an important contribution to the field of automation and control engineering. The upshots are also accentuated to evaluate the performances of control designs.**

***Index Terms*—Proportional Integral Derivative Control, Linear Quadratic Regulator, and Sliding Mode Control.**

I. INTRODUCTION

A suspension system is comprised of an arrangement of springs, shock absorbers, and linkages, which allow a vehicle's body to be controlled. If the suspension system of the vehicle is not robust, and the driver cannot control it, the power produced by the engine is worthless. Further, to ensure smooth movement, a vehicle needs to absorb all of the bumps and holes on the surface of a road using its suspension system. Without this system, even the smallest of bumps, which may appear negligible to a passenger, will disrupt the smoothness of a journey. There are two types of suspension system, i) passive, and ii) active. Conventionally automotive suspension, Passive Suspension System (PSS), designs have been compromised among the three incompatible criteria, namely road behavior, load transportation, and passenger relieve. In contrast, by directly controlling the suspension force actuators, Active Suspension System (ASS) allows to minimize the effect of conventional arrangement (PSS) as a compromise between behaviors and relieve.

Most studies in the field of suspension systems have focused solely on light vehicles such as motor bikes, and cars [1– 4]. However, far too little attention has been paid to the suspension systems of heavy vehicles. The aim of this study is to cast new light on the ASS for the model of a heavy vehicle by obtaining its' mathematical model, system analysis, and the feedback control implementation using three modi operandi; 1) Proportional Integral Derivative (PID) control, 2) Linear Quadratic Regulator (LQR) Control, and 3) Sliding Mode Control (SMC).

This paper is organized in six stages. Mathematical model of the ASS is described at stage II. Objectives of control setup are examined at stage III. The controller designs using PID, LQR, and SMC are developed at stage IV. Upshots are shown at stage V. Lastly; conclusions are emphasized at VI stage.

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II. THE ACTIVE SUSPENSION SYSTEM (ASS) MODEL

In order to design the ASS for the heavy vehicle, one of the four wheels (1/4) model is considered in Fig. 1 [5].

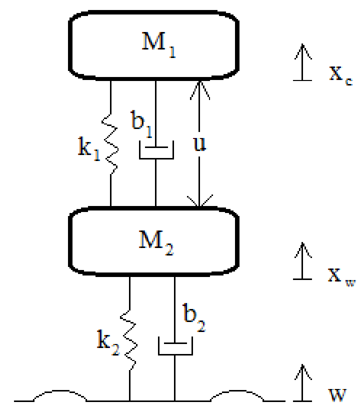

Fig. 1. ASS Model for ¼ Heavy Vehicles

In Fig. 1, '*u*' represents the control force, and '*w*' symbolizes as disturbance input.

The ASS is responsible for increasing the friction level between tires and a road by adsorbing the force, which is produced by bumps/holes or any other uneven surface. In Fig. 2, ASS that allows the relative movement between them connects wheels to the vehicle. Fig. 3 depicts the free body diagrams for the ASS composition that is given in Fig. 1.

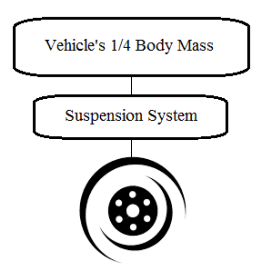

Fig. 2. Model within Vehicles

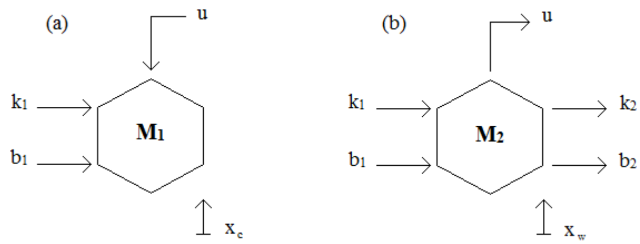

Fig. 3. Free Body Diagrams for the ASS

Using free body diagrams in Fig. 3, following differential equations for motion are derived.

$$M_1\ddot{x}_c = -k_1(x_c - x_w) - b_1(\dot{x}_c - \dot{x}_w) + u \quad (1a)$$

$$M_2\ddot{x}_w = -k_2(x_w - w) - b_2(\dot{x}_w - \dot{w}) - u - k_1(x_w - x_c) - b_1(\dot{x}_w - \dot{x}_c) \quad (1b)$$

The model exemplify in (1) be able to state in the standard form,

$$\dot{x} = f(x,u,t) \quad (2)$$

let,

$$x(t) = [x_1 \quad x_2 \quad x_3 \quad x_4]^T \quad (3)$$

setting (1) into (2), we obtain the following equation with respect to (3),

$$\dot{x} = f(x) + g(x)u + h(x)\dot{w} \quad (4)$$

where,

$$f(x) = \begin{bmatrix} x_2 - x_4 \\ -\frac{k_1}{M_1}x_1 - \frac{b_1}{M_1}x_2 + \frac{b_1}{M_1}x_4 \\ x_4 \\ \frac{k_1}{M_2}x_1 + \frac{b_1}{M_2}x_2 - \frac{k_2}{M_2}x_3 - \frac{(b_2 + b_1)}{M_2}x_4 \end{bmatrix}$$

and,

$$g(x) = \begin{bmatrix} 0 & \frac{1}{M_1} & 0 & \frac{-1}{M_2} \end{bmatrix}^T, \qquad h(x) = \begin{bmatrix} 0 & 0 & -1 & \frac{b_2}{M_2} \end{bmatrix}^T$$

Fig. 4 shows the overall plant system of the ASS, which will control by PID, LQR, and SMC techniques in section IV.

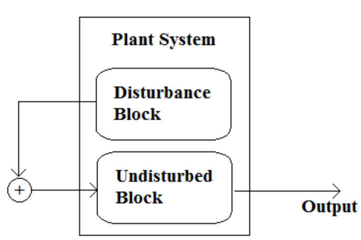

Fig. 4. Plant Model of the ASS

The overall transfer function using Fig. 4 is discovered as,

$$G_{plant}(s) = G_{disturbed}(s) \times G_{undisturbed}(s)$$

$$G_{plant}(s) = \frac{-0.1641s^4 - 6.35s^3 - 58.69s^2 - 976.6s}{0.0035s^6 + 0.1874s^5 + 8.009s^4 + 70.93s^3 + 1364s^2 + 2015s + 3.125e004} \quad (5)$$

To facilitate the analysis and feedback control designs at peak amplitude < 7 millimeter, and settling time ≤ 3 seconds for the ASS, the parameters of the bus vehicle are considered (as case study) in TABLE I [6].

TABLE I
SYSTEM PARAMETERS

Symbol	Quantity	Assigned Values [a]
M_1	¼ mass of the body	2500 Kg
M_2	suspension mass	320 Kg
k_1	spring constant of suspension system	80,000 N/m
k_2	spring constant of wheel and tire	500,000 N/m
b_1	damping constant of suspension system	350 N.s/m
b_2	damping constant of wheel and tire	15,020 N.s/m

[a] Units: Kg = Kilogram, N/m = Newton/meter, N.s/m = Newton. second/meter.

III. CONTROL DEVISE OBJECTIVES

The plant system of the ASS has four open loop poles at,

$$-23.9758 \pm 35.1869i, -0.1098 \pm 5.2504i \quad (6)$$

in which, these eigenvalues define the behavior of under damped system as, oscillations are produced in the presence of comfort level when bus rides over uneven roads/surface. This is described in Fig. 5.

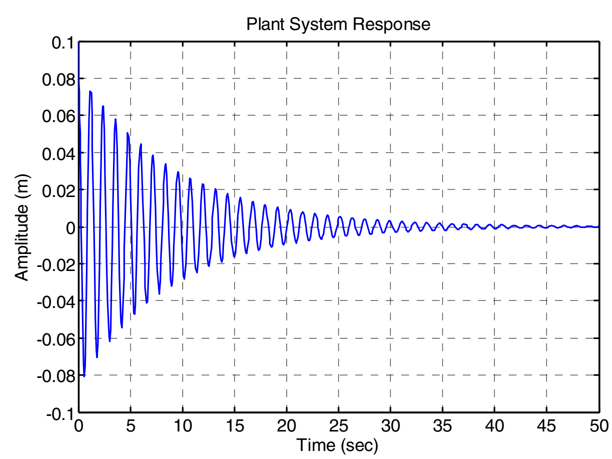

Fig. 5. Uncontrolled System Response

Fig. 5 shows the plant system swings with the amplitude slowly but surely decreasing to zero, known as under damped system.

In order to design the ASS, prior stage is to check the controllability of the scheme by executing controllability test to make sure whether the plant system is controllable or not. The state equation in (4) is completely controllable if the matrix [7],

$$S = \begin{bmatrix} B & AB & A^2B & A^3B & A^4B & A^5B \end{bmatrix}$$

is of rank 6, where S is called the controllability matrix. The computed controllability matrix using Matlab® for the ASS plant system is given in (7).

$$S = (1.0e{+}007)* \begin{bmatrix} 0 & 0 & 0.0001 & 0.0071 & -0.4445 & 8.3457 \\ 0 & 0 & 0 & 0.0001 & 0.0071 & -0.4445 \\ 0 & 0 & 0 & 0 & 0.0001 & 0.0071 \\ 0 & 0 & 0 & 0 & 0 & 0.0001 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

The uncontrollable states are = 0, S is non singular matrix as its determinant = 1, and its rank is equals to 6 that is equal to plant system's order. Therefore, the system is said to be controllable.

IV. CONTROL DESIGNS

In order to provide smooth running of vehicles under the effect of uncertainties, feedback control is designed using three control practices such as PID, LQR, and SMC. The block diagram of the control system is portrayed in Fig. 6.

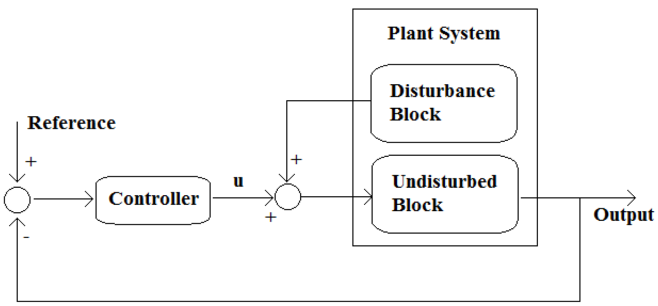

Fig. 6. Block Diagram of the ASS Control Design

In Fig. 6, the output of the plant is feedback to controller. Controller then computes the values under its vicinity, and generates the control force 'u' that is responsible to control the movement of a vehicle's body.

A. *Proportional Integral Derivative (PID)*

PID is a powerful technique in control programming with the ability to implement control designs via calculating errors [8]. This method authorizes the incorporation of several consecutive components in different capability, without upsetting core functionality. The PID control law is illustrated as,

$$u(t) = K_p e(t) + K_i \int e(t)\,dt + K_d \frac{d}{dt} e(t) \quad (8)$$

where, K_p, K_i, and K_d are known as proportional gain, integral gain, and derivative gain respectively. $e(t)$ is defined as the error term between set and measured points.

B. *Linear Quadratic Regulator (LQR)*

LQR is an automated way of finding an appropriate state feedback controller, in which system dynamics are based on a set of linear differential equations [9]. It uses a mathematical procedure that decreases the quadratic cost function by its considering factors. Quadratic cost function is defined as [9],

$$J = \frac{1}{2} x^T(t_1) F(t_1) x(t_1) + \int_{t0}^{t1} \left(x^T Q x + u^T R u \right) dt$$

The LQR control law is described as,

$$u(t) = -Kx(t) \quad (9)$$

where, K is the state feedback matrix gain, and it is computed as, $K=R^{-1}B^TP(t)$. Moreover, Q is an $i \times i$ symmetric positive semi definite matrix and R is an $j \times j$ symmetric positive definite matrix.

C. *Sliding Mode Control (SMC)*

Consider a changeable time surface, $s(t)$, in state space R^n by scalar equation $s(x, t) = 0$. The selected switching/sliding surface is as follows [10],

$$s = c_1 e_1 + e_2 \quad (10)$$

where, e_1 and e_2 are the tracking errors of position and velocity of the ASS respectively. c_1 is identified as control constriction that might be any Hurwitz scalar. The chattering free SMC control law using saturation function "*sat(s/□)*" is expressed as [11],

$$[u] = [\omega] - \left[\Omega sat\left(\frac{s}{\phi}\right) \right] \quad (11)$$

where, ω and Ω are the control blueprint factors, and □ is a scalar value that discovers the boundary layer thickness. The value of saturation function is,

$$sat(s) = \begin{cases} s & if \quad |s| \le 1 \\ sgn(s) & if \quad |s| > 1 \end{cases}$$

the control law in (11) ensures both the following reach-ability and sliding states respectively using (4).

$$s\dot{s} < 0 \quad (12)$$

$$\dot{V} = \frac{1}{2}\frac{d}{dt}s^2 = s\left(c_1\dot{e}_1 + \dot{e}_2\right) = -\eta|s| < 0 \quad (13)$$

Where, $\eta = 1$ in (13) that guarantees the system routes hits the sliding surface in a finite time. The method of finding "$\dot{V}$" is well known as Lyapunov's Stability Analysis. Hence, the ASS model is grown to be stable.

V. END RESULTS

The results are compiled in Matlab® at the initial condition of 0.1m high disturbance step. There are two design conditions, the resultant 1) settling time should be ≤ 3 seconds, and 2) peak amplitude should be < 7 millimeter. The final outcomes are shown below.

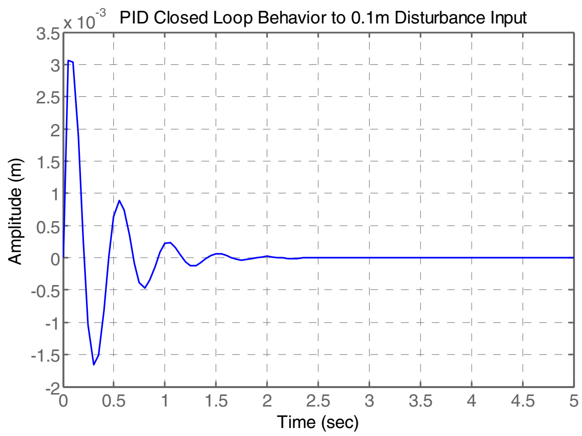

Fig. 7. The ASS for Bus using PID Control Technique

The graph in Fig. 7 depicts the satisfied behavior of 3 millimeter peak amplitude, and 2 seconds settling time for the ASS of heavy vehicle.

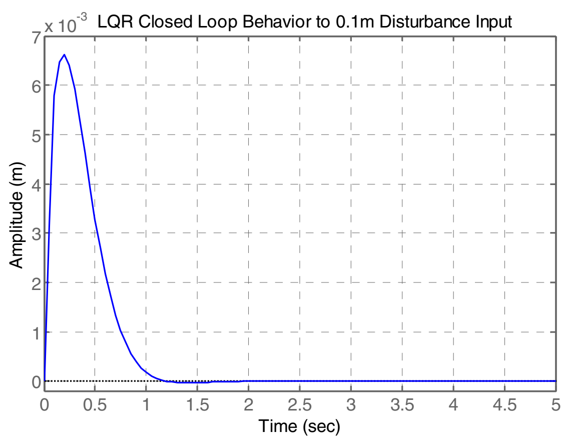

Fig. 8. The ASS for Bus using LQR Control Technique

In Fig. 8, the peak amplitude is about 6.6 millimeter, and settling time is equals to 1 second for the bus ASS.

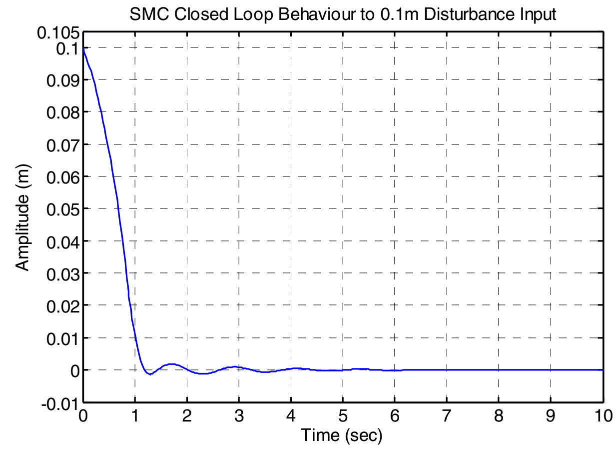

Fig. 9. The ASS for Bus using SMC Technique

The settling time of bus body is about 1.5 seconds in Fig. 9, whereas the peak amplitude is about 1.8 millimeter.

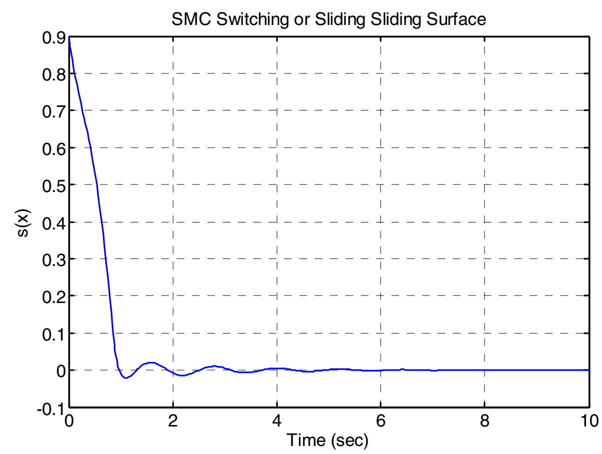

Fig. 10. Sliding Surface of the ASS using SMC Technique

Fig. 10 describes the behavior of sliding or switching surface of sliding mode that approaches to zero in about 2 seconds.

VI. CONCLUSION

With the intention of the ASS design for a heavy vehicle, three modi operandi such as PID control, LQR control, and SMC with saturation function are employed in this paper. Following is the performance evaluation using the implemented design systems.

TABLE II
PERFORMANCE EVALUATION

Projected Control Schemes	Settling Time (sec) [a]	Peak Amplitude (mm) [a]	Disturbance (m) [a]
PID	2.0567	3.06	0.1
LQR	1.0283	6.61	0.1
SMC	≈ 1.5	1.8441	0.1

[a] Units: sec = seconds, mm = millimeter, m = meter.

It is quite clear from TABLE II that there is a slight tradeoff between the results of PID and LQR performances. In contrast, the SMC procedure initiates the robust upshots. More to the point, all the executed control methodologies meet the design requirements of settling time ≤ 3 seconds, and peak amplitude < 7 millimeter.

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