



# Essays in game theory in tribute to Francesc Carreras – SING 15

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This special issue is a tribute to Professor Francesc Carreras on the occasion of his 70th birthday. The idea of a special issue dedicated to Francesc Carreras arose during the XXX-VIII Spanish Conference of Statistics and Operations Research (SEIO 2019) that took place in Alcoy in September 2019. The conference was held after Francesc's 70th birthday, so it was clear from the beginning that this volume would be too late for the original purpose. As we were preparing to work on a special issue for the 15th European Meeting on Game Theory (SING 15) celebrated in Turku in July 2019, we decided that it would be a good opportunity for this purpose. SING 15 was the first conference of the series held in Finland, organized by Hannu Salonen.

The special issue underlines the dedication, illusion, and enthusiasm that Francesc Carreras has put into every one of his research works. Throughout the years, another remarkable characteristic in his career is his love for the good work, his commitment to this discipline in his lectures, being an inspiration to and supporting his students, some of them following

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in his footsteps in this fascinating field of research in Game Theory. Some of the words that define him in his professional and personal life are fighter, humble, and honest.

We hope that those who read this collection of papers enjoys them and that these works can contribute to bring new and stimulating insights into the advancement of this exciting area of research. Above all, we would like that this volume serves to highlight the care and interesting ideas that Francesc Carreras brings us with each of his works.

We want to thank all the people who participated to make possible this special issue: Endre Boros, Editor-in-Chief of *Annals of Operations Research* for accepting our proposal and his cooperation; Publications Manager Ann Pulido for her efficiency and help throughout the whole editorial process; the 45 authors of the 18 published papers (including Francesc himself) for their inspiring and interesting contributions; those who submitted papers that for one reason or another could not be published; and the 58 referees who did a careful job during the revision process of each paper and helped to make this volume a success.

Next, we give a brief description of each of the accepted papers in this volume:

In “Multi-issue bankruptcy problems with crossed claims”, the authors introduce a novel model of the multi-issue bankruptcy problem inspired by a problem of abatement of emissions of different pollutants in which pollutants can have more than one effect on the atmosphere. Unlike the classical multi-issue bankruptcy problems already existent in the literature, they study situations with multi-dimensional states, one for each issue and where each agent claims the same to the different issues in which it participates. In this context, the authors generalize the well-known constrained equal awards rule for classical bankruptcy problems as the solution to a succession of linear programming problems. Conditional equal division and consistency determine uniquely the resulting rule.

In “Characterization of a value for games under restricted cooperation”, the authors consider a value for restricted cooperative games whose definition is based on the Harsanyi’s dividends approach extending the Shapley value. A characterization of this value similar to the one proposed by Shapley is given. In addition, this value is characterized on the subclass of restricted cooperative simple games. Finally, this value is applied to analyze the power distribution of the Catalanian Parliament in 1980.

In “Minimal winning coalitions and orders of criticality”, the authors deal with the question of how critical the players are in simple games, that is, how important they are to ensure that a coalition including them is winning. In particular, the authors show that the order of criticality of a player follows from the minimal hitting sets of the family of minimal winning coalitions that contain the player in question. The authors also explore a duality principle in which the dual game has a set of minimal winning coalitions that coincide with the original game’s minimal hitting sets. Finally, in their main result, the authors characterize the ordering of players resulting from applying the lexicographic order to the vector that follows the orders in which a given player is critical. The axioms used for the characterization demand anonymity and monotonicity (or lack thereof) properties.

In “Cooperative and axiomatic approaches to the knapsack allocation problem”, the authors propose three associated cooperative games for each knapsack problem. One of them (the pessimistic game) has already been considered in the literature. The other two (realistic and optimistic games) are defined in this paper. The pessimistic and realistic games have non-empty cores but the core of the optimistic game could be empty. Then two rules are proposed: the first, based on the optimal solution of the knapsack problem and the sec-

ond, the Shapley value of the optimistic game. Axiomatic characterizations of both rules are given.

In “An evolutionary game on compliant and non-compliant firms in ground water exploitation”, the authors study the exploitation of a common groundwater resource in an evolutionary game approach to consider the strategic interaction among the water resource users. Access to the shared resource is not free, and firms have to pay a royalty depending on the quantities of water pumped. We suppose to have compliant and non-compliant firms. The compliant ones pay a royalty to pump water, while the non-compliant ones pump water but do not pay royalty, facing the risk of being punished by public authority. If caught, the non-compliant firms pay fines and royalty proportional to their pumping extraction. The overall sanction comprises a fixed amount and the non-paid royalty. From the model analysis, it emerges that coexistence at the equilibrium between compliant and non-compliant firms is possible, and policy instruments can partially counter unauthorized exploitation. In particular, increasing the sanction level reduces the number of non-compliant firms but raises the incidence of illegal pumping. The opposite occurs if the regulatory authority increases the royalty price. To pursue both goals, applying a balance of policies is necessary.

In “The Italian Referendum: What can we get from Game Theory?”, the authors study the impact of a participation quorum threshold on a referendum where the outcome of the referendum is “approval” or “rejection”. They focus on three voting rules: a binary dichotomous voting rule where the electorate can vote “yes” or “no”, a 3-option dichotomous voting rule where the electorate has an additional choice to “stay at home”, and a quaternary dichotomous voting rule where there is an additional choice to “abstain”. The authors analyse the impact of the participation quorum threshold on the decisiveness and (individual and collective) blocking power of such voting situations. Using a graphical representation they provide a game theoretic analysis of these voting rules, and specifically consider the Italian referendum.

In “Characterizing NTU-bankruptcy rules using bargaining axioms”, the authors study bankruptcy rules in the class of bankruptcy problems with nontransferable utilities. They give axiomatic characterizations for three bankruptcy rules in this class: the proportional rule, the truncated proportional rule, and the constrained relative equal awards rule. The authors use properties of solutions for bargaining problems to formulate axioms for bankruptcy rules. The axioms state how the allocation corresponding to a rule changes as the claims vector or the estate changes.

In “Weak null, necessary defender and necessary detractor players: characterizations of the Banzhaf and the Shapley bisemivalues”, the authors introduce new properties related to three special classes of players: weak null, necessary defender, and necessary detractor for the class of bicooperative games providing axiomatic characterizations of the Banzhaf and Shapley semivalues. A computational procedure to calculate the allocations given by the Shapley bisemivalued through the generalized multilinear extension of the game is also given.

In “A critical analysis on the notion of power”, the authors deal with extensions of power indices defined on simple games with 2 possible outputs and  $j$  possible input levels. They show that different power indices can be obtained in these games depending on how the criticality of a player is measured. The combination of two particular probabilistic models and three collections of winning coalitions gives way to six well-known power indices for simple games as the Banzhaf and the Johnston power indices. Generalizations of these six

power indices for simple games with two possible outputs and at least three possible input levels are discussed.

In “Necessary players and values”, the authors introduce a new value for cooperative games with transferable utility providing an axiomatic characterization based on a property on necessary players. To illustrate the performance of this value, a particular cost allocation problem arises when the owners of apartments in a building plan to install an elevator and share its installation cost is analyzed. In addition, an extension of this value for cooperative games with transferable utility and a coalition structure is proposed. Finally, axiomatic characterizations of this coalitional value, the Owen value, and the Banzhaf-Owen value using alternative properties related to necessary players are obtained.

In “On the properties of weighted minimum colouring games”, the authors generalize the model of a minimum colouring game to a weighted minimum colouring (WMC) game which is induced by an undirected graph with positive weights on its vertices. The worth of a coalition in a WMC game associated to such a graph is determined by the weighted chromatic number of its induced subgraph. The authors characterize classes of graphs in which associated games admit certain population monotonic allocation schemes (PMAS). Specifically, they characterize classes of graphs that are WMC totally (respectively locally) balanced, meaning that for all (respectively at least one) positive integer weight vectors the corresponding WMC game is totally balanced, submodular, or admits a PMAS.

In “Electing a committee with dominance constraints”, the author analyses the problem of how to select a committee of  $k$  members, given that the composition of the committee has to satisfy some interval and dominance constraints based on the characteristics or labels of candidates. An interval constraint could demand that there should be at least three but at most five members in the committee with a label  $X$ . A dominance constraint could demand that there should be at least as many members with labels  $X$  as with label  $Y$ . A committee selection rule chooses the optimal committee satisfying the constraints, given the voters’ preferences over candidates. Optimality is defined with respect to a set-extension of the order underlying a best- $k$  committee selection rule. While the problem is NP-hard in general, it can be solved in polynomial time if the constraints form a tree-like structure over labels.

In “Core-stability over networks with widespread externalities”, the author studies stability of network structures in a model where coalitions may disrupt and create links. The solution concept, recursive core, has been previously used by the author in the context of cooperative games in partition function form. The stable network structures generated by this solution concept are studied in examples with negative and positive externalities.

In “Stackelberg-Nash equilibrium and quasi harmonic games”, the authors study the existence of subgame perfect equilibria in a two-stage, three-person almost perfect information game. In the first stage, the leader chooses a strategy from a compact set. In the second stage the two followers observe this choice and play a strategic form game with finite pure strategy sets. In cases where there exist numerous equilibria in the second stage game, the leader tries to choose optimally under the worst case scenario. It is known that under this criterion, an equilibrium may not exist, mainly because the Nash equilibrium correspondence need not be lower semicontinuous. The authors introduce a class of quasi harmonic second stage games, and show that at least an epsilon equilibrium exists in this class.

In “Some game theoretic marketing attribution models”, the authors introduce game theoretic approaches to design attribution mechanisms for multi-channel marketing campaigns.

The first type of attribution mechanism is based on the Shapley value of an associated cooperative transferable utility game (conversion game). They provide axiomatizations of two Shapley value based rules. The second type of mechanism is based on the constrained equal-losses rule of an associated bankruptcy problem (bankruptcy multi-channel attribution problem). An axiomatization of this mechanism is provided and properties of a mechanism based on the proportional rule are discussed. Besides situations where every position or channel appears only once, the authors also consider situations in which the positions or channels on the paths to conversion can appear more than once.

In “Some solutions for generalized games with restricted cooperation”, the author studies solutions for generalized games with restricted cooperation in the setting of public resources allocation. Several resources are distributed between points of a finite partitioned set. For each coalition of the partition, a fixed amount of a resource is distributed between its elements. Each coalition corresponding to a project has a gain/loss function which only depends on the restriction of an allocation on this coalition. The set of all the project-determined coalitions forms the set of vertices of a directed graph. Then, the author introduces an envy-stable solution for a fixed graph. The solution generalizes the equal sacrifice solution for claim problems and simplifies the generalized kernel of cooperative games. The paper also provides conditions that ensure the existence of the envy stable solution and under which situation it includes three familiar solutions, namely the generalized nucleolus, the generalized anti-nucleolus, and the Wardrop equilibria.

In “Spatial games and endogenous coalition formation”, the authors consider cooperative  $n$ -person games where players are located in some Euclidean space. These locations represent affinity between players determining, in such case, which coalitions are more likely to form. It is also modelled how players are likely to move in this space.

Finally, in “The nucleolus and inheritance of properties in communication situations”, the authors study conditions under which the nucleolus of a communication situation is invariant, meaning that the nucleolus of the graph-restricted game equals the nucleolus of the original game. They first characterize the families of graphs for which the graph-restricted game inherits the properties of strong compromise admissibility or compromise stability from the original game. Then, they use these characterizations to provide a direct, closed formula for the nucleolus of the graph-restricted game and, moreover, to characterize the family of graphs for which the nucleolus is invariant. Biconnected graphs and star graphs play a special role.

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