TURUN YLIOPISTO
UNIVERSITY OF TURKU

PROMOTING FLEXIBLE MATHEMATICAL THINKING WITH GROWTH MINDSET, DELIBERATE PRACTICE, AND SERIOUS GAMES

Phuong Bui

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Dedicated to my parents and my family.

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ABSTRACT

Adaptive expertise is a greatly appreciated, yet rarely achieved, goal of mathematics curricula because it is considered to typify high-level mathematical thinking. Adaptive expertise demonstrates knowledge and skills that can be dynamically implemented in uncommon situations, not just within highly defined tasks or sufficiently prepared contexts. To achieve adaptive expertise, students must be given occasions to practice solving open-ended mathematical tasks in unfamiliar circumstances, allowing them to contemplate, analyze, and explore different connections and alternative solutions to develop their emerging skills and knowledge structures. Traditional math classrooms are often equipped with textbooks and instructional approaches that focus on isolated, routine exercises, or *drill-and-practice*, which encourage students to master isolated procedural techniques to find the most or only efficient solution. Math teachers, therefore, employ teaching methods that emphasize speed and accuracy using these materials. The idea of mathematics as a "fixed" subject, which is full of rigid and absolute rules, unintentionally continues to be reinforced.

This doctoral dissertation aims to investigate design principles for learning environments that support flexible mathematical thinking in mathematics education. This thesis focuses on two objectives: first, it aspires to understand how adaptive expertise can be promoted with deliberate practice, and whether it can be done by using a mathematical game-based learning environment called the Number Navigation Game (NNG). The nature of deliberate practice is demanding and occurs just beyond one's abilities. It necessitates deep engagement, continuous efforts to enhance performance, and a positive attitude towards challenges—traits synonymous with a growth mindset. Given the association between a growth mindset and persistent learning behavior, the second objective explores ways to cultivate growth mindset in mathematics classrooms. This is vital for integrating game-based learning into conventional mathematics instruction and realizing the goal of adaptive expertise in mathematics.

This dissertation is divided into two parts, encompassing three sub-studies. Part one, comprising Studies I and II, focuses on the Number Navigation Game (NNG). Study I explores game experiences during the NNG development process and examines how different design choices influence students’ gaming experiences. The results provide insights into the iterative design process of a research-based serious game, shedding light on students' interactions with both learning and gaming components and their relation to novel mathematical learning objectives. Study II delves into various game performance profiles using gaming analytics and investigates the diverse ways students engage with the NNG. Utilizing log data from game performances in the energy mode, combined with measured mathematics learning outcomes, math interest, perceived challenge, and experienced flow during gameplay, Study II offers evidence on promoting adaptive expertise through deliberate practice, game-based learning environments, and learning outcomes. In essence, Studies I and II highlight how the NNG serves as a supportive platform for presenting students with novel contexts, challenging tasks, and immediate feedback, making it a viable tool for traditional classrooms.

Part two (Study III) investigates the current state of growth mindset interventions in mathematics education through a systematic review. The results show that when implicit theories of intelligence interventions were conducted specifically in the math domain, positive results were reported, whereas general implicit theories of intelligence interventions yielded mixed results. This indicates that to make the necessary behavioral changes based on changed beliefs, participants need to engage with mathematical content at a deeper level than the surface level. Most importantly, the learning environment must be embedded with elements that support *struggle* and *mistakes*, encourage effortful practices, and make progress visible to students. In this way, students will be provided with evidence of the development of their own mathematical skills as a result of practice.

KEYWORDS: adaptive expertise, game-based learning environment, growth mindset, deliberate practice, flexible mathematical thinking

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TIIVISTELMÄ

Adaptiivinen asiantuntijuus on yksi korkeatasoisen matemaattisen ajattelun taidoista ja sen kehittymistä on pidetty tärkeänä tavoitteena matematiikan opetussuunnitelmissa, vaikka käytännön opetustyössä sitä harvoin saavutetaankaan. Adaptiivinen asiantuntijuus kuvaa tietoja ja taitoja, joita voidaan soveltaa joustavasti uusissa tilanteissa, ei vain selkeästi ennalta määritellyissä tehtävissä tai konteksteissa. Tämän saavuttamiseksi on tärkeää, että oppilaille tarjotaan mahdollisuus harjoitella avoimien matemaattisten ongelmien ratkaisemista uusissa konteksteissa. Tällöin he voivat pohtia, analysoida, tutkia erilaisia yhteyksiä ja vaihtoehtoisia ratkaisuja, mikä kehittää heidän taitojaan. Perinteisessä matematiikan opetuksessa on usein käytössä oppikirjoja ja opetusmenetelmiä, jotka keskittyvät yksittäisiin, rutiininomaisiin harjoituksiin tai yksinkertaiseen toistoon perustavaan harjoitteluun. Nämä valmistavat oppilaita hallitsemaan mekaaniset laskutoimitukset ja proseduurit tehokkaimman tai ainoan ratkaisun löytämiseen. Tällaiset oppimateriaalit ja -menetelmät tähtäävät nopeuteen ja tarkkuuteen. Tällöin ajatus matematiikasta joustamattomana kouluaineena, joka on täynnä jäykkiä ja ehdottomia sääntöjä, jatkaa vahvistumistaan tahattomasti.

Tämän väitöskirjan tavoitteena on edistää joustavan matemaattisen ajattelun kehittämistä matematiikan opetuksessa. Väitöskirja keskittyy kahteen osatavoitteeseen. Ensinnäkin väitöskirjatutkimuksissa pyritään ymmärtämään, miten adaptiivista asiantuntijuutta voidaan edistää määrätietoisella harjoittelulla, ja voidaanko adaptiivista asiantuntijuutta kehittää käyttämällä matemaattista pelillistä Number Navigation Game oppimisympäristöä. Toiseksi määrätietoinen harjoittelu on vaativaa ja tapahtuu juuri oppijan kykyjen äärirajoilla; se vaatii syvää keskittymistä, sitoutumista, sinnikästä pyrkimystä suorituksen parantamiseen ja positiivista asennetta vaikeiden, epämiellyttäviäkin tunteita herättävien tehtävien edessä. Sekä sinnikkyys suoritusten parantamisessa että positiiviset asenteet haasteita kohtaan ovat myös kasvun asenteelle tunnusomaisia piirteitä. Useissa tutkimuksissa väitetään, että kasvun ajattelutavan tukeminen edistää sinnikästä oppimiskäyttäytymistä. Ymmärrystä siitä, kuinka kasvun ajattelutapaa voidaan tukea matematiikan tunneilla, voidaan hyödyntää, kun pelillistä oppimista integroidaan perinteiseen matematiikan opetukseen ja tavoitteena on adaptiivisen asiantuntijuuden taidot matematiikassa.

Väitöskirjassa on kaksi osaa, joihin kolme osatutkimusta jakautuu. Ensimmäinen osa sisältää Number Navigation Game -peliä koskevat tutkimukset I ja II. Tutkimuksessa I kartoitettiin oppilaiden pelikokemuksia pelin kehitysprosessin aikana, ja sekä sitä, kuinka erilaiset suunnitteluvalinnat vaikuttivat oppilaiden pelikokemuksiin. Tutkimukset tuottivat uutta tietoa tutkimuspohjaisen oppimispelin suunnittelusta ja muokkausprosessista, mikä puolestaan tuotti yleisempää tietoa oppilaiden ja pelin elementtien vuorovaikutuksesta, ja siitä miten tämä vuorovaikutus liittyy uudenlaisiin matemaattisiin oppimistavoitteisiin. Tutkimus II keskittyi erilaisiin pelaajien suoritusprofiileihin pelianalytiikan avulla ja tutki erilaisia tapoja, joilla oppilaat pelasivat Number Navigation Game -peliä. Tutkimuksessa hyödynnettiin lokidataa pelaajien suorituksista yhdessä mitattujen matematiikan oppimistulosten, matematiikan kiinnostuksen sekä pelaamisen aikana koetun haastavuuden ja flow-kokemuksen kanssa. Tutkimus tuotti tietoa siitä, millä tavalla adaptiivista asiantuntijuutta voidaan edistää tukemalla määrätietoista harjoittelua pelioppimisympäristössä. Yhteenvetona voidaan todeta, että tutkimukset I ja II tuottivat aikaisempaa tarkempaa tietoa siitä, kuinka Number Navigation Game -peli voi tarjota kannustavan oppimisalustan, joka tarjoaa avoimen oppimisympäristön, rutiinista poikkeavia ja haastavia tehtäviä, sekä pelidesignin, joka antaa oppijalle selkeää, välitöntä palautetta.

Väitöskirjan toisen osan (Tutkimus III) tavoitteena on tarkastella kasvun ajattelutavan interventioita matematiikan opetuksessa systemaattisen katsauksen avulla. Tulokset osoittivat, että kun älykkyyttä koskeviin uskomuksiin perustuvia kasvun ajattelutapaa tukevia interventioita toteutettiin erityisesti matematiikan alalla, raportoitiin positiivisia tuloksia. Kun kasvun ajattelutapaa tukevia interventioita toteutettiin yleisesti ilman erityistä kontekstia, tulokset olivat ristiriitaisia. Tämä osoittaa, että jotta tarvittavat käyttäytymismuutokset toteutuvat muuttuneiden uskomusten perusteella, osallistujien on uppouduttava matemaattiseen sisältöön pintatasoa syvällisemmin. On olennaista, että oppimisympäristöön on upotettu elementtejä, jotka tukevat "kamppailua ja virheitä", että ne kannustavat ponnisteluihin ja että edistyminen tehdään oppijalle näkyväksi. Näin oppija saa todisteita omien matemaattisten taitojensa kehittymisestä harjoittelun seurauksena.

ASIASANAT: adaptiivinen asiantuntijuus, pelioppimisympäristö, kasvun ajattelutapa, tarkoituksellinen harjoittelu, joustava matemaattinen ajattelu

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Lieto, 20th August 2023
Phuong Bui

Table of Contents

Tables

Figures

List of Original Publications

This dissertation is based on the following original publications, which are referred to in the text by their Roman numerals:

I Bui, P., Rodríguez-Aflecht, G., Brezovszky, B., Hannula-Sormunen, M. M., Laato, S., & Lehtinen, E. (2020). Understanding students' game experiences throughout the developmental process of the number navigation game. *Educational Technology Research and Development*, *68*(5), 2395-2421.

II Bui, P., Hannula-Sormunen, M. M., Brezovszky, B., Lehtinen, E., & McMullen, J. (2022). Promoting adaptive number knowledge through deliberate practice in the Number Navigation Game. In *Games and Learning Alliance: 11th International Conference, GALA 2022, Tampere, Finland, November 30–December 2, 2022, Proceedings* (pp. 127-136). Cham: Springer International Publishing.

IIII Bui, P., Pongsakdi, N., McMullen, J., Lehtinen, E., & Hannula-Sormunen, M. M. (2023). A systematic review of mindset interventions in mathematics classrooms: What works and what does not?. *Educational Research Review*, 100554.

In each of the three publications, Bui contributed to the conceptualization and design of the study. Bui was involved in the data collection process for Study I & Study III and was responsible for both data analysis and the writing of the manuscript in all cases.

1 Introduction

It is undeniable that mathematics is embedded in almost every facet of life. For young children to grow up and be fully capable of participating in society, they need to know and understand mathematics. In other words, children need to acquire sufficient mathematical skills to become competent citizens in this day and age.

Developing adaptive expertise for students is a valuable outcome of mathematics education, as it is considered to exemplify excellent mathematical thinking (Hatano & Oura, 2003). This is because adaptive expertise demonstrates acquired knowledge that can be creatively and flexibly applied in unfamiliar contexts, not just within common situations such as highly constrained textbook problems. In arithmetic, adaptive expertise is differentiated from "routine expertise," which implies the automatized use of previously known strategies in solving typical types of arithmetic tasks (Baroody, 2003; McMullen et al., 2020). However, fostering adaptive expertise in elementary mathematics education remains a challenge. How and when to teach for adaptive expertise, and whether this is an attainable and practical goal for all students, especially those at the lower-than-average math level (Verschaffel et al., 2009) are some of the most prevailing questions.

Regular mathematics lessons often focus on automating procedural skills through highly repetitive practice (Lehtinen et al., 2017; Verschaffel et al., 2009), which can lead to static routine expertise development (Baroody, 2003; Verschaffel et al., 2009). A hot issue during the past decades has been what types of problems and instructional practices are necessary for students to develop strong mathematical thinking skills. In 1992, Schoenfeld outlined and demonstrated a broad conceptualization of mathematical thinking and advocated for more non-routine problem-solving tasks, which aligns with what Hatano (1982) described as *adaptive expertise* as part of mathematic instructional goals. He also presented evidence that students' beliefs about mathematics as a discipline—such as "Mathematics problems have one and only one right answer"; "Ordinary students cannot expect to understand mathematics; they expect simply to memorize it and apply what they have learned mechanically and without understanding"; or "Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less"—were because mathematics curriculum and textbooks focused on the mastery of mechanical procedures as isolated parts (Schoenfeld, 1988,1992). Characteristics of a typical math classroom in which the instruction is predominantly *teacher-led* or *teacher-centered* (Anderson et al., 2018; Sun, 2018) heavily emphasize the development of routine expertise, as most problems are routine exercises (requiring isolated strategies and procedural knowledge) (Verschaffel et al., 2009). Prioritized speed and accuracy via timed tests and repeated stereotypical computational practices (Boaler, 2013) in the current literature match Schoenfeld's (1988, 1992) descriptions.

Schoenfeld (1992) suggested that the characteristics of the math classroom environment largely determine students' beliefs about the discipline and, consequently, their beliefs shape their behaviors. For instance, he argued that students who believe that if they struggle with a task for a few minutes, or cannot solve it immediately, they will not persist, even though they could likely solve the problem. Without preserving and reattempting to solve the task, students also limit themselves from discovering new connections or reflecting on alternative strategies—those that are less obvious and less straightforward. The reason for such behaviors is that students are used to working with procedural exercises, such as demonstrated routine calculations and the application of formulas from textbooks, worksheets, or homework. These exercises are often presented using a common structure: a task is presented to introduce a technique; then, such a technique is demonstrated as an example, and the following tasks are presented for students to practice "solving" the problems using the illustrated technique (Schoenfeld, 1992). Such exercises are supposed to be accomplished quickly. In class, textbooks often come with a guidance approach that provides repetitive exercises that mechanicalize basic techniques for choosing the "most" efficient solution for certain arithmetic problems (Verschaffel et al., 2009). Hence, teachers tend to implement an instructional approach that emphasizes the efficiency of choosing the "best" solution or the "only" solution using these materials (Anderson et al., 2018; Sun, 2018). Such norm-setting practices, if the knowledge quality is surface-level and task-performance oriented (Baroody et al., 2007), will reinforce the idea of mathematics being a one-dimensional, "fixed" subject that is suitable for a teacher-led learning environment in which students passively follow along (Anderson et al., 2018; Schoenfeld, 1992). Such descriptions of practice in math classroom were also matched with classrooms by teachers with a "fixed mindset" in math teaching (Sun, 2018). When the emphasis is on finding the one or the right solution, it leaves little room for discussion about alternative strategies, other justifications, learning from mistakes, or reflections on other possible connections to the real world.

Limited pedagogical frameworks foster adaptive expertise in school mathematics in practice and usually focus on the utilization of a few strategies (Verschaffel et al., 2009). Adaptive expertise requires a combination of conceptual and procedural knowledge, as conceptual knowledge is the underlying foundation for promoting procedural fluency (Baroody, 2003). Thus, it is argued that more complex practices that support flexibility and the development of conceptual knowledge in students are needed—those that are more abstract and richer in relationships (Lehtinen et al., 2017)—as both procedural and conceptual knowledge are necessary for success in mathematical learning (Baroody et al., 2007). Learners must be presented with more non-routine problem-solving activities, those that allow them to engage in opportunities to think and reflect on possible alternative strategies on their own, and those that give them the chance to contemplate and struggle. Such practices provide learners with situations in which to participate in higher-quality forms of training than merely repetitive practice, that allow them to establish their upcoming skills and knowledge foundation in addition to their fixed practice using their current skills (Lehtinen et al., 2017). In skill acquisition research, Ericsson et al. (1993) presented detailed research on expertise as a product of *deliberate practice*.

Deliberate practice has been demonstrated to be the principal component of extraordinary development in various domains (Ericsson et al., 2018). Deliberate practices are explicit and specific training activities, often proposed by external skilled agents such as teachers or mentors, with clear intentions and well-defined goals to enhance one's performance, gradually develop a better mental representation of previously acquired skills, and build more advanced skilled (Ericsson et al., 1993). In math education, Lehtinen and collaborators (2017) presented an integrated review of deliberate practice and how it can inform the design of more complex and quality practices for students. They presented a positive case to better understand complex levels of arithmetic tasks in an open-ended learning environment under the framework of deliberate practice, which allows students to engage in exploring different number operation combinations and strategies via a game-based learning environment called the Number Navigation Game (NNG). The NNG is a game-based learning environment designed to advance students' (aged 10 to 13) mathematical skills with whole-number arithmetic, focusing especially on flexibility and adaptivity (Brezovszky, 2019; Brezovszky et al., 2015; Lehtinen et al., 2015).Their research has indicated that playing the game as a supplemental part of regular math sessions led to learning gains in adaptive number knowledge and arithmetic fluency, which transferred to pre-algebra skills (Brezovszky et al., 2019; McMullen et al., 2017).

The present dissertation aimed to investigate design principles for learning environments that support flexible mathematical thinking in mathematics education via game-based learning environments, specifically via the NNG. Thus, the present thesis sought to explore different ways in which students engage with the NNG (e.g., game performances, game experiences, design choices, math interests) and a variety of learning measures to provide some evidence of how adaptive expertise can be promoted with deliberate practice. Furthermore, the nature of deliberate practice is demanding and requires students' acute concentration, persistent attempts to improve their performance, and a "can-do" attitude when facing challenging tasks. For this reason, it is often hypothesized that adopting a growth mindset would be beneficial in encouraging more persistent learning behavior (Blackwell et al., 2007; Dweck, 2006). However, a general intervention that teaches participants about the plasticity of the brain and principles of a growth mindset can be seen as too abstract, or "mysterious," since social-psychological interventions often fail to inform changes in practical educational settings (Yeager & Walton, 2011, p.275). To better understand how successful interventions that foster a growth mindset operate, especially those in the math domain, this dissertation also provides a systematic review of growth mindset interventions in math education. Subsequently, theoretical, and practical implications for the promotion of adaptive expertise in connection with deliberate practice and growth mindsets via game-based learning environments are provided. Finally, the various objectives of this thesis's studies were aimed at providing more detailed insights into the design and development process of a research-based serious game, which in turn offers a comprehensive overview of students' interaction with learning and gaming components, and how they are related to these new types of mathematical learning objectives.

1.1 Deliberate practice

Practice is considered the most popular approach to learning, as skills cannot be acquired without deliberate effort and repetition (Ohlsson, 2011). However, the quality (or efficiency) of practice, its mechanisms, task environments, or purposes when people practice vary greatly. In their seminal empirical research on expertise development, Ericsson et al. (1993) investigated the notion of deliberate practice and its fundamental role in skill acquisition. Violinist experts differed from other less accomplished individuals largely due to the amount and intensity of their practices, and the fact that these effortful practices were designed to pursue the explicit goal of performance improvement (Ericsson et al., 1993). Across domains, it is clear that expert performance levels are only possible after someone performs long, and challenging structured activities designed specifically to improve their skills and performance. Besides other relevant factors, critics also consider deliberate practice to be an undeniably important predictor of skill improvement (Macnamara et al., 2014).

Cognitive psychologists often discuss the results of long-term practice as *automaticity* and *expertise*, which contrast each other in some aspects (Ohlsson, 2008). While automaticity often means the ability to do certain tasks as an automatic response, entailing patterns or habits as a result of learning and repetition practice, it also entails rigidity in the execution (Schneider & Chein, 2003) and sometimes leads to other by-products such as Einstellung effects, which refers to the cognitive phenomenon of fixation on previously successful strategies and thereby preventing the contemplation of alternative approaches (Luchins & Luchins, 1959). Expertise, on the other hand, implies flexibility and adaptivity, especially in novel situations, and a high capacity for certain skills (Ericsson et al., 2018). Both automaticity and expertise insinuate training with the intent to improve; however, the difference between repetitive activities (which often leads to automaticity), and deliberate practice (expertise development) is how the practice is conducted. Repeated memorizing and regular practice might lead to exceptional procedural skills, whereas high-level expertise requires complex mental representations, which are possible only with extensive training that focuses on developing well-developed mental representations over a long period (Ericsson et al., 2018; Lehtinen et al., 2017). As illustrated by Ericsson (2008), when performance reaches a certain level of automaticity and effortless execution, additional practice or training of the same experience does not lead to a higher accuracy level in the behavior or refine the composition of the mechanisms. As a result, accumulating more of the same training or repeated practice does not elevate one's skill set. Aspiring experts can prevent automaticity by engaging in a practice that is currently outside of their capacity, thus gradually developing elaborate mental representations to attain prominent control over their performance (Ericsson, 2008).

There are ongoing debates regarding the definition of deliberate practice. In an attempt to replicate the seminal study on this subject by Ericsson, Krampe and Tesch-Römer in 1993, critics also discussed the issue using multiple definitions, especially regarding the position of teacher or coach in designing training programs in which an individual should engage (Macnamara & Maitra, 2019). Ericsson et al. (1993) suggested that deliberate practice includes practices that are intended solely to enhance a person's execution, either by a skilled teacher or the person themselves. While the exact definition of the term is somewhat unclear, the focal point is that training activities are designed with clear intentions and well-defined goals to enhance performance. Except for high-level performers, in most cases, deliberate practice requires external agents, such as teachers or skilled coaches, to provide support and guided activities for those whose levels are far from high-level skills. This is especially important in an educational setting with teachers and students (Lehtinen et al., 2017).

The principles of deliberate practice, according to Ericsson et al. (2018) and Lehtinen et al. (2017), are illustrated in Figure 1.

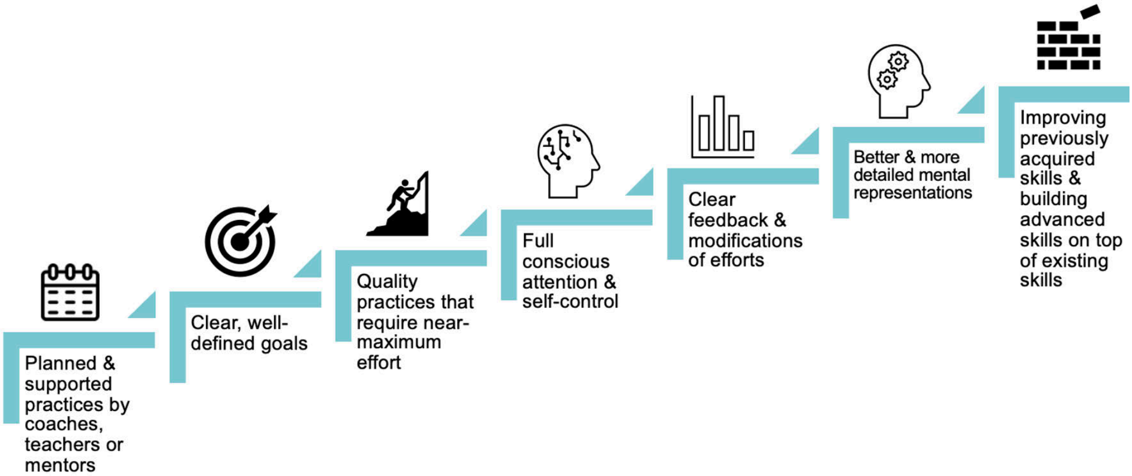

Figure 1 Deliberate practice's main characteristics adapted from Ericsson et al., 2018 (see also Lehtinen et al., 2017).

As demonstrated in Figure 1, deliberate practice is composed of several specific parts. First, before the practice, clear improvement goal(s) should be set. Often, with an instructor's guidance, training activities that help the individual to improve and attain a specific goal are designed. These training activities should be on the edge of the individual's current skills and require near-maximum effort. Such training requires the individual to try (and possibly fail) things that are demanding of their present abilities, which is not always pleasant. To do so, an environment in which attempts to try, and fail are common should be available. The practice demands full conscious attention and self-control from the individual, and such acute concentration is also limited in terms of time. Following practice, the individual's performance should receive clear feedback or modifications on aspects of practice that they need to repeat or improve. Similar to designing a training activity, unless the individual is a near-top-level expert, feedback on their performance is critical and should be presented by other skilled external agents. Finally, the gradual accumulation of such intensive training contributes to the development of better and more detailed mental representations, improvement of previously acquired skills, and the development of expertise by continuously forming advanced skills on top of strongly prevailing skills.

1.1.1 Deliberate practice in math education

Within school contexts, attempts to apply the concept of deliberate practice have been found effective in formal higher education, such as in medical education (McGaghie et al., 2011), or professional development for teachers (Dunn & Shriner, 1999; Han & Paine, 2010). Some researchers have tested whether deliberate practice can be motivated and taught, and subsequently improve students' achievements by teaching them about the tenets of deliberate practice and other motivational associations, such as expectancy-value or growth mindset (Balan & Sjöwall, 2022; Eskreis-Winkler et al., 2016). However, the results of these endeavors are mixed. Eskreis-Winkler et al.'s (2016) four longitudinal randomized-controlled trials (RCTs) "wise" deliberate practice interventions included expectancies, values, and aspects of deliberate practice, such as (a) setting clear goals outside of one's current ability, (b) getting prompt feedback on one's performance, (c) displaying acute concentration, and (d) repeating until mastery is achieved. The results from Study 2 suggested that occasional online interventions between 25 and 50 minutes had a positive impact on math achievement, deliberate practice behavior, and expectancy-value beliefs in fifth and sixth graders. In an attempt to replicate this study, Balan and Sjöwall (2022) conducted a school-based intervention session (involving eight 30-minute sessions over 14 weeks) to teach deliberate practice, grit, and growth mindset to seventh-grade students. Their reports are not aligned with findings from Eskreis-Winkler et al. (2016), as the intervention had no significant impact on students' attitudes in relation to mathematical performance, deliberate practice, or growth mindset.

One takeaway from this is that interventions aimed at teaching participants about concepts or phenomena have yielded inconsistent results (Yeager & Walton, 2011). Learning about a concept or phenomenon, such as a growth mindset or deliberate practice, in isolation from changes in classroom practices, learning environments, or contexts in which participants can exercise their newly acquired beliefs may not be sufficient for academic achievement. Social-psychological interventions have great potential to inform changes in educational settings; however, they are not "magic" (Yeager & Walton, 2011). Without concrete changes in classroom practices, for instance, where the nature of mathematical tasks chosen for daily lessons is rigid and heavily procedural, contains surface-level rules, and is traditionally performance oriented (Baroody et al., 2007), a brief general intervention teaching students a growth mindset and the plasticity of the brain might not have any lasting effects on students' beliefs or mathematical outcomes (Orosz et al., 2017). Such procedural practices are usually accompanied by outcome-oriented instruction, which potentially reinforces the idea of mathematics as a "fixed" subject. Learning about the concept alone might be too abstract for novice experts (such as students) to connect with mathematical beliefs in reality when learning environments and teaching practices communicate different messages. As discussed by Yeager and Walton (2011), if each social-psychological intervention is positioned through the force-field analysis framework (Lewin, 1952), they exist in a complex system of forces that comprise both the driving forces that promote the behavior and restraining forces that impede that behavior. This means that for an intervention that teaches students about the deliberate practice tenets and growth mindset concept to work, it also needs to remove potential “restraining forces” or provide concrete learning opportunities in educational environments. Another reason put forward by Yeager and Walton (2011) is that such social-psychological interventions often seem to be “mysterious,” as it is difficult to understand which “forces” (Lewin, 1952) these interventions operate on. They argued that it is not possible to directly see how certain beliefs about the nature of intelligence affect students (e.g., a fixed mindset). With the same logic, a general intervention that teaches students about the tenets of deliberate practice or a growth mindset can be seen as too general or too abstract of a concept, especially with a group of participants (students) who often receive many messages from different adult figures (e.g., parents, teachers, peers). Learning about how deep practice is frequently frustrating and involves failure and how soliciting feedback or talent is not all that matters (Balan & Sjöwall, 2022) is necessary, but not enough to engage students in deliberate practice activities and subsequently improve their mathematics achievements.

Discussions about adaptive learning systems and environments suggest that descriptions of these learning systems match some of the core principles of deliberate practice. Responding to the challenges of the traditional classroom with a teacher using similar limited learning materials (e.g., textbooks or practice) for all learners, adaptive learning systems or environments have been developed to cater to more individualized and tailored learning programs for all students using advanced technologies (Li et al., 2022; Oxman et al., 2014). According to Oxman et al. (2014), digital adaptive learning systems make use of information collected from learners during the learning process (when learners interact with the system) to provide better-suited activities, from difficulty levels to sequences of tasks, hints, or feedback. The main advantage of the adaptive learning system, which is similar to a core component of deliberate practice, is the possibility of identifying “knowledge gaps” based on a student’s answers or mistakes. The digital adaptive learning system subsequently supplies learning materials or practices according to the identified gaps (Oxman et al., 2014). Adaptive learning systems or environments vary from game-like learning environments to computer software or applications; however, most of those currently available are considered to be *rule-based* (Oxman et al., 2014), meaning that the instructional models comprise pre-defined *if-then* commands (Sjaastad & Tømte, 2018). Others make use of advancing machine-learning algorithms to continuously develop adaptive learning models and predict better probabilities of a learner being successful at learning particular content (Oxman et al., 2014). Despite their unprecedented advantages, there are several issues concerning the quality of the tasks or learning paths offered by current adaptive learning systems. Such systems are developed with a predetermined content model, which includes predefined questions, correct answers, and learning “paths.” The (usually limited) number of learning paths and how deep the content can go depend entirely on what is programmed in advance. In mathematics, this means a model of static, highly structured procedures and tasks that produce either “right” or “wrong” answers. Due to its nature, only narrowly defined mathematical problems can be included in the system, meaning excluding tasks that are more explorative (e.g., open-ended answers) or that require learners to make a deeper connection between different topics. From a deliberate practice perspective, this also means limiting the scope of complex practices that could be useful in advancing an individual’s current skill. Moreover, owing to the predetermined accuracy of an answer, the quality of feedback from such systems is often too general (e.g., either right or wrong). Findings from a review of adaptive feedback in computer-based learning platforms indicated that full adaptive feedback has not yet been achieved in non-procedural disciplines, and there is a lack of suitable frameworks that can include diverse criteria and assist with different feedback means, targets, goals, and strategies (Bimba et al., 2017). Learners still, of course, develop their skills and receive immediate feedback as they learn using such adaptive learning systems; however, their progress relies on predefined learning trajectories (Sjaastad & Tømte, 2018). The richness of practice is limited by a predetermined set of learning activities, which also restrains opportunities for learners to acquire the results of deliberate practice—flexibility and adaptivity in expertise development.

The past decade has seen serious gaming emerge as a complementary, if not revolutionary, pedagogical tool in education. Serious games, especially those designed for educational purposes, immerse students in interactive and problem-solving scenarios, making the learning process both engaging and contextual (Torbeyns et al., 2015). Particularly relevant is the concept of "stealth assessment" embedded within these games (Shute & Ventura, 2013). The idea behind stealth assessment is to seamlessly integrate evaluation within the gameplay and other interactive environments, allowing educators to gauge a student's abilities and problem-solving strategies without the overt pressures of traditional testing (Shute & Ventura, 2013). While not mentioning deliberate practice directly, "Newton's Playground", a physics-based educational game developed as part of the stealth assessment research project (Shute & Ventura, 2013), shares certain design elements and principles with the concept of deliberate practice. The game provides a learning environment where players can repetitively practice increasingly complex physics tasks, receive immediate feedback, and work on challenging tasks, all of which are crucial elements for skill development in line with the deliberate practice framework.

Only a few studies have looked into applying the principles of deliberate practice in investigating subjects in school settings. Lehtinen and collaborators (2017) briefly reviewed current attempts to apply the concept of deliberate practice in math education. For instance, in geometry subjects, Pachman et al. (2013) found that students with a higher-than-average level of mathematical skills, when given a choice, would choose comfortable tasks rather than demanding ones, which led to minimal improvement. When the deliberate practice model was implemented, these higher-than-average-level students were presented with more challenging tasks; thus, their skills were advanced. In another study, Pachman et al. (2014) assigned high school students to two groups: deliberate practice and free choice. Using worked examples in geometry, the deliberate practice group was given tasks related to their inadequate areas, while the free-choice group could choose the tasks themselves. As a result, higher-than-average-level students in the deliberate practice group improved their performance; however, this was too challenging for lower-than-average-level students. Pachman et al. (2014) suggested that while more advanced students only needed to work on a few weak areas, lower-level students had more weak areas and they could not successfully work on them all during the experimental session. The results indicated that when the core principles of deliberate practice were applied in a contextualized approach in which the intervention designed different ways for students to engage with the subject effortfully and meaningfully for each individual (e.g., students presented with more challenging tasks or tasks related to their specific weak areas), it resulted in clear learning improvement for more advanced participants.

However, there are some limitations to current applications. As discussed above, the nature of deliberate practice is demanding; hence, the goals should be clearly defined and targeted. If targeted with too many weak areas they need to improve, less advanced students have to focus on more problems compared to the more knowledgeable students. Moreover, the limited duration of laboratory experiments in these studies restricts their applicability in advancing the regular customs of teaching and learning mathematics in authentic settings. Time and human resource constraints are also challenging factors for implementing the deliberate practice concept, as individuals' practices and performance constantly require feedback and input from skillful agents, such as teachers. However, several studies have developed models in which these constraints in conventional teachings can be overcome using game-based learning environments, such as the NNG (see section 1.4.).

1.2 Adaptive expertise in arithmetic

Developing adaptive expertise for students is an important outcome of mathematics education (Baroody & Dowker, 2013; Hatano & Oura, 2003; Hickendorff et al., 2022; Verschaffel et al., 2009) as it is considered to exemplify outstanding mathematical thinking (Hatano & Oura, 2003). Adaptive expertise refers to learned knowledge that can be dynamically and creatively implemented in new or unfamiliar contexts, not only limited to highly controlled or substantially practiced settings but also "routine expertise," which implies the routine use of previously known strategies in solving typical types of arithmetic tasks (Baroody, 2003; McMullen et al., 2020).

Research has highlighted two of the most important components of adaptive expertise in arithmetic—adaptivity and flexibility (Baroody, 2003; Hatano & Oura, 2003). Even though they are often used interchangeably, adaptivity reflects the ability to make the most suitable choice regarding strategy, while flexibility primarily indicates the potential to switch and use different strategies in problem-solving (Verschaffel et al., 2009). Strategic flexibility is operationalized as using and switching smoothly between a variety of strategies with no further conditions, while task characteristics, subject variables, and sociocultural settings are some considerations that need to be examined when defining adaptivity (Verschaffel et al., 2009). Flexibility and adaptivity with arithmetic problem-solving have been intensively investigated in recent years, with emphasis on the importance of promoting arithmetic competencies through adaptive expertise, not just routine expertise (Hatano & Oura, 2003; Hickendorff et al., 2022; Leinwarnd, 2014; Verschaffel et al., 2009). However, striving for adaptive expertise in elementary mathematics instruction remains a challenge, specifically regarding how and when to teach for adaptive expertise, and whether it is an attainable goal for all students given their variable mathematical competences (Verschaffel et al., 2009).

While aiming for adaptive expertise is the broader outcome, developing one's proficiency in numerical characteristics and relations among numbers, which can be creatively implemented in unfamiliar situations, is a more concrete and attainable objective. Such proficiency is defined as adaptive number knowledge (ANK). More precisely, ANK indicates a rich, malleable, and well-connected network of knowledge of quantitative features and arithmetic relations, which is an important quality of adaptive expertise in arithmetic (McMullen et al., 2016). ANK is argued to be one of the underlying abilities that allows one to successfully execute a flexible and creative solution from an assortment of known solutions in a novel arithmetic task. ANK requires one to understand the nature of the natural number system to develop rich mental numerical relations based on those key characteristics (e.g., conducting estimations, finding "nice" numbers), and using this knowledge to solve arithmetic tasks in different contexts (McMullen et al., 2016). In other words, advancing the representation of numerical relations is important for recognizing and determining the appropriate strategy—at a specific moment—for the individual to be able to dynamically switch between numerous solution strategies. Together with strategy flexibility, ANK is considered to play a parallel and foundational role in developing adaptivity in arithmetic (McMullen et al., 2016; Verschaffel et al., 2009).

1.2.1 Promoting Adaptive Number Knowledge with deliberate practice

To cultivate students' adaptive expertise rather than merely the routine use of strategies in arithmetic problem-solving, opportunities to examine various number operation combinations and strategies in unknown arithmetical contexts are important (Rittle-Johnson & Star, 2009; Star & Seifert, 2006). However, there are limited pedagogical models and suggestions that can offer such opportunities to students in conventional classroom settings (Verschaffel et al., 2009). Practice in regular math lessons tends to emphasize the automatizing and efficiency of procedural abilities and static routine expertise outcomes (Lehtinen et al., 2017; McMullen et al., 2020), and textbooks are often accompanied by instructional approaches that provide drill-and-practice practices that automatize basic methods for choosing the most effective solutions for certain arithmetic problems (Verschaffel et al., 2009).

There is a lack of suitable frameworks for assisting in the development of adaptivity in arithmetic in school mathematics. Not all practice is equal. For instance, routine exercise with procedures for rational number arithmetic tasks in isolation does not result in sustainable learning gains (Moss & Case, 1999). As discussed above, deliberate practice to develop expertise in a domain is differentiated from repetitive training, which leads to the automatization of skills and inert routines. Lehtinen and collaborators (2017) presented an integrated review of students' self-initiated and deliberate practice in math education. They also offered an encouraging outlook to enrich the understanding of designing complex levels of arithmetic tasks in an open-ended learning environment through the lens of deliberate practice. This allows students to engage in exploring different number operation combinations and strategies via a game-based learning environment—the Number Navigation Game (NNG) (Brezovszky, 2019; Lehtinen et al., 2015; McMullen et al., 2016). Findings from a randomized controlled trial (RCT) indicated that playing the game as an extra part of daily mathematics sessions without special support from teachers resulted in the development of ANK and arithmetic fluency, which was passed on to pre-algebra skills (Brezovszky et al., 2019). Lehtinen and colleagues proposed that the design of NNG is embedded with some core principles of deliberate practice that allow for individually suited tasks that are suitably demanding for each student; therefore, by playing the NNG, students can continuously work on solving arithmetic problems with natural numbers (Lehtinen et al., 2015).

However, so far, there has not been direct evidence suggesting that students engage in deliberate practice while playing the NNG. Research on game-based learning has also highlighted the significance of gaming experiences, especially flow, as beneficial components during gameplay. These two concepts describe strong participation in a certain circumstance; however, there are principal dissimilarities between deliberate practice and flow.

When looking at flow in educational games, Kiili et al., (2014) discussed the following attributes: concentration, rewarding experience, loss of self-consciousness, and time distortion. They argued that when a person enters a flow state in a learning activity (for example, in educational games), they are pursuing an enjoyable activity at their optimal challenge level, and they can forget unpleasant things, ignore what others think of them, and lose self-consciousness (Csikszentmihalyi, 1990). In other words, during the flow state, the self seems to vanish from awareness. Moreover, as proposed by Csikszentmihalyi (1990), in the flow state, the sense of time also distorts because the experience can either pass quickly or feel slow. On the other hand, Ericsson, et al. (1993, 2018) provide evidence that immersing oneself in deliberate practice requires acute concentration and full-conscious, self-control attention because such training is a demanding experience, sometimes even unpleasant as one trains to go beyond their current skills level.

Therefore, when evaluating the evidence of deliberate practice in educational games, it is advisable to consider the gaming experience as part of the process (see Section 1.4 The case of the NNG).

1.3 Growth mindset in math education

A growing body of work has demonstrated that possessing a growth mindset is rewarding for students' academic achievement and that students potentially carry self-beliefs about the malleability of their intellectual capacities (Chen & Pajares, 2010; Yeager & Dweck, 2012). As reported by Dweck and colleagues (Dweck, 2006; Blackwell et al., 2007), students with a fixed mindset tend to accept that intelligence or ability is fixed and cannot be enhanced; moreover, they tend to view mistakes as consequences of their unchangeable ability or that they are made because they (i.e., the students) are not smart enough. In comparison, when students adopt a growth mindset, they are convinced that their intelligence or ability is influenceable and can grow and expand through learning and effort. They also demonstrate certain traits, such as embracing challenges and considering them opportunities to learn, practice, and improve; persisting when faced with setbacks; and considering effort as the key to mastery (Blackwell et al., 2007), whereas fixed-mindset individuals tend to shy away from challenges and consider mistakes as failures (Dweck, 2006).

Research has shown growing indications that growth mindset interventions can influence students' mindsets, and therefore improve their academic performance and motivation (Blackwell et al., 2007). Fostering a growth mindset significantly helps at-risk students to improve their average learning outcomes in core academic courses on a large scale (Paunesku et al., 2015), and aids students academically in their transition to high school (Yeager et al., 2016). Growth-mindset interventions demonstrate that *struggle* or *failure* is a learning opportunity, especially when learners take on challenging tasks; therefore, struggles, or mistakes should not be conceived as evidence of a continually incapable student. Thus, an extensive concentration on growth-mindset interventions in academic contexts attempts to encourage individuals to see failures or mistakes as opportunities to improve their abilities. However, there have been calls for greater attention to the soundness of the theory, the replicability of the original mindset studies (Blackwell et al., 2007; Yeager et al., 2019), and the effectiveness of mindset interventions in academic performances (Li & Bates, 2019; Macnamara & Burgoyne, 2022). The questions of why a growth mindset is only rewarding for students' academic achievements in some studies, and why mindset interventions seem to benefit only some groups of participants remain largely unsolved.

In the context of mathematics education, however, the idea of a *math brain*—something you either have or not—is an undeniably prevailing belief. Teachers and students often consider math achievement to represent an inborn ability rather than an achievement compared to achievements in other domains (Beach & Dovemark, 2007; Jonsson et al., 2012). Considering the prevalence of popular beliefs about the innateness of mathematical abilities, and since most growth-related research is domain-general, it is particularly important to investigate the domain-specific effects of growth mindset interventions in mathematics education. Moreover, as discussed above, examples of social-psychological interventions in which participants were taught about core principles of phenomena (Balan & Sjöwall, 2022; Yeager & Walton, 2011) were not as successful as hypothesized, nor were they consistent in results when replicated in different contexts. It is also strongly advisable to consider other variables within each context to understand possible patterns of interactions between the desirable factor (e.g., growth mindset), the individual, and the restraining forces in the environment (Lewin, 1952).

The majority of current systematic reviews and meta-analyses concerning growth mindset interventions include studies that evaluate the impact of a generic mindset intervention, and they do so across domains. In addition, most of the existing reviews and meta-analyses are synthesized purely from quantitative studies, neglecting qualitative research and mixed-method approaches. Hence, this dissertation aims to fill this gap in the literature by conducting a systematic review of different components of mindset interventions in mathematics classrooms in primary and secondary schools (e.g., targets, content, and delivery modes).

1.4 The case of the Number Navigation Game

The NNG is a serious mathematical game designed to advance students’ (aged 10 to 13) mathematical skills with whole-number arithmetic, focusing especially on flexibility and adaptivity (Brezovszky, 2019; Brezovszky et al., 2015; Lehtinen et al., 2015). Game mechanics are developed to trigger and maintain students’ interest in mathematics learning (Rodríguez-Aflecht et al., 2017). The design of the NNG is intrinsically integrated (Habgood et al., 2011) such that the main play components are merged into the educational content. This means that to progress in the game, one must engage in solving arithmetic problem-solving tasks. Game-based learning environments with intrinsic integration are considered more beneficial for accomplishing educational achievements and providing greater incentives than extrinsic designs (e.g., gamification). However, intrinsically integrated games can be more difficult and expensive to develop compared to those with extrinsic designs (Habgood et al., 2011). One of the main reasons for this is the *art* of balancing and integrating learning content and fun aspects in game design, and development is a complicated, challenging, and expensive task (Gaydos, 2015; Kiili et al., 2014). A motivating and beneficial game-based learning environment relies on a valid educational foundation and intriguing gaming components, which require various evaluation and development activities and substantial financial support (Gaydos, 2015).

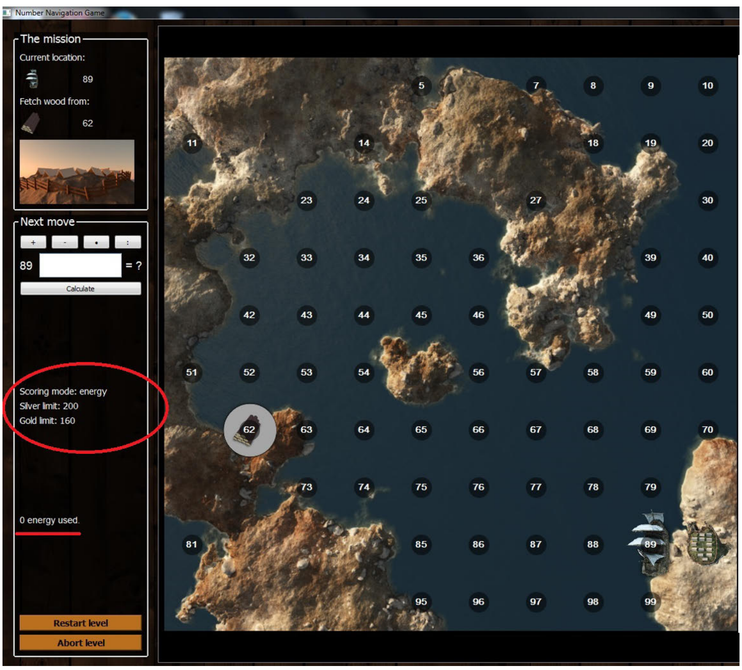

Figure 2. The layout of an Energy map – NNG version 1 (harbor at 89, target material at 62).

The main game components presented here are illustrated with an NNG version 1 map; however, the game mechanics and layouts remain consistent throughout the developmental process of the NNG. The map is 100 squares superimposed on different settings of an island and sea, in which the mission is to retrieve four raw materials to establish cities. Players advance in the gameplay by directing a ship from a starting position (the "harbor") to collect material at a specified number and then come back to the harbor by inputting different arithmetic combinations. For example, in Figure 2, the player begins the map from "number 89" and has to retrieve wood located at "number 62" while avoiding numbers that are covered by land and lines crossing the land. The player moves the ship from 89 to 62 by applying mathematical calculations in the operation box on the left side. A map is finished when all four of the items are redeemed.

There are two main scoring modes in the NNG: *moves* and *energy* scoring. In the moves mode, the items need to be collected and returned to the starting harbor with the least number of "moves"—or arithmetic calculations. In the energy mode, the goal is to utilize the least amount of "energy," which is equal to the total of all digits inserted into the calculation box. In the moves mode, the purpose is to trigger the use of basic arithmetic operations, such as addition and subtraction, with large two-digit numbers. In the energy mode, gameplay demands complicated computational relations that require the use of all four arithmetic calculations to solve the tasks. Therefore, energy levels are valuable for fostering the formation of ANK (McMullen et al., 2016). Besides the use of the 100 number-square and different game modes, other features in the NNG that are designed to enhance ANK are the maps' layouts (e.g., each map is different, including island sizes and shapes) and additional features during the more advanced game phases (e.g., the appearance of a pirate ship on the route that was previously chosen by the player, which prevents players from developing a practice of automatic repetition of reverse calculations on the returning route). For a detailed explanation of the theoretical background of the functions of the NNG characteristics in strengthening ANK, see Brezovszky (2019).

1.4.1 Number Navigation Game design and gaming experiences

The NNG design process was the result of collaborative work by an interdisciplinary group of researchers, designers, and programmers who have a variety of complementary backgrounds and expertise in various areas (e.g., mathematics education, interests, measurements, and game-based learning environment development). The development of the NNG involved constant trials and testing, from an applicability perspective for the first working model (Brezovszky et al., 2021) to a large-scale RCT for the finished release (NNG 1) with limited motivating components (Brezovszky et al., 2019; Rodríguez-Aflecht et al., 2015). Undergoing this process, NNG transformed from a two-dimensional (2D) graphic game-based learning environment with a narrow focus on asthetics to a three-dimensional (3D) graphic release, in which additional extrinsic motivating components and advanced tasks are available.

Deliberate practice principles in the NNG

Lehtinen and collaborators presented the models and design features on which NNG was based (Lehtinen et al., 2015), and suggested that some core aspects of deliberate practice are embedded in the gaming mechanism of the NNG learning environment (Lehtinen et al., 2017), which aimed at promoting students' ANK through gameplay (see Figure 3).

For students to engage in exploring different arithmetic operation strategies and combinations, the game challenges are positioned in an open-ended learning environment (e.g., navigating the ship among the islands). The game does not offer any clearly defined mathematical tasks. Thanks to intrinsic design, students have to create arithmetic calculations to advance in the game. The tasks are appropriately challenging and tailored to individual students, which provides circumstances for them to train a step beyond their current competencies because each map can be completed with different levels of achievement. As gameplay advances, maps slowly become more challenging, necessitating more complex numerical strategies. The game mechanics provide continuous feedback on students' performance (e.g., immediate changes in progress after each move), enabling them to make suitable actions. The open-ended learning environment also triggers the contemplation of alternative solutions to arithmetic tasks. It allows students to repeat each map without any time pressure (Lee et al., 2022) and gradually develop their comprehension of the essence of the base ten structure and natural number system, thus strengthening numerical relations and using this knowledge to solve arithmetic tasks in novel contexts (e.g., each game map in the NNG has a different layout) (Lehtinen et al., 2017; McMullen et al., 2016).

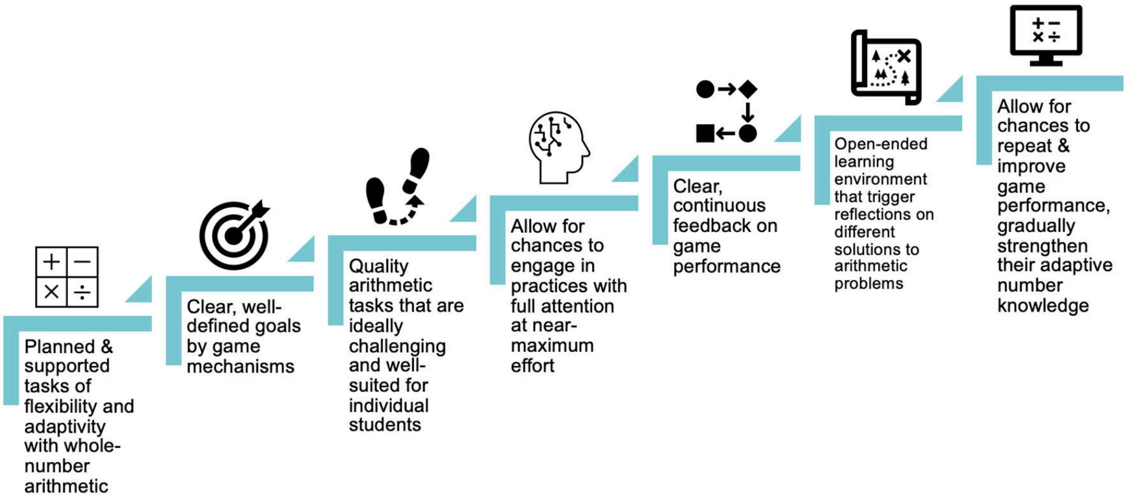

Figure 3. Core features of deliberate practice in the design of the Number Navigation Game (summarized and adapted from Lehtinen et al., 2015;2017).

Understanding serious game experience

Flow theory (Csikszentmihalyi, 1990) is considered the fundamental tenet of user experience, the groundwork of which is used to create engagement, enjoyment, and satisfaction. It is argued that when one engages in an optimal experience, it allows the person to enter the *flow state* where nothing else seems to matter (Csikszentmihalyi, 1990). Being in a flow state impacts learning (Kiili & Lainema, 2008), and affects players' enjoyment and accomplishment (Weibel & Wissmath, 2011). Another concept that is broadly considered as a good result of a gameplay experience is *immersion*, which is often related to engagement. Kiili et al.,(2012) explained that flow occurs when a participant pays all of their attention to an activity, and immersion happens when a participant immerses themselves in the experience. Both flow and immersion are important components of the complex and multifaceted gaming experience. Other factors that are often considered main components of the gaming experience are competence, challenge, positive affect, negative affect, and tension. Together with flow and immersion, these seven dimensions are important for exploring the nature of the gaming experience (Poels et al., 2007).

Another dimension that is argued to be essential in a game-based learning environment is *positive value* (Whitton, 2011), because players must believe that playing a serious game is beneficial to them before they can gain any benefit from the educational content of the game. Prior research on the NNG (Rodríguez-Aflecht et al., 2015) acknowledged that the addition of positive value to the model put forward by Poels et al. (2007), which assesses students' belief that the NNG is beneficial to their math learning, is complementary to the original framework (which was not developed to measure gaming experience specifically for serious games). Therefore, this dissertation will explore the gaming experience of students playing NNGs in Studies II and III through the revised framework with eight core elements: flow, immersion, competence, challenge, positive affect, negative affect, tension, and positive value.

Flow, challenge, and deliberate practice

The design of an NNG is proposed to be embedded with some core elements of deliberate practice (Lehtinen et al., 2015, 2017). Although the results (Brezovszky et al., 2015, 2019) suggested that students improved their mathematical skills after playing the NNG as part of their formal math sessions, there has not been direct evidence suggesting that students engage in deliberate practice (and not just repetitive training) while playing the NNG.

There have been several attempts to develop instruments to quantitatively measure deliberate practice and competence (Balan & Sjöwall, 2022; Bathish et al., 2016). Balan and Sjöwall (2022) developed questions related to their deliberate practice intervention, for instance, about how deep practice involves failure, or how to set a long-term goal. Bathish et al. (2016) developed a self-reported instrument that considered other characteristics, such as motivation, competence, and patient/clinic results to better examine the role of deliberate practice in nursing expertise. In this case, several aspects related to the core of deliberate practice are considered and this study proposes a multifaceted approach to providing insights into the practice of students using the NNG.

First, a core element of deliberate practice is acute concentration when one engages in a training activity that is slightly outside of one's current ability. Such concentration in game-based learning and gaming experiences is often discussed as "flow," which happens when one directs all of one's attention to playing (Kiili et al., 2012). Flow often implies a state of being in which high-level experts pursue enjoyable activities or optimal challenges (Csikszentmihalyi, 1990). During the flow state, the "self" seems to disappear from one's awareness (Kiili et al., 2014). Moreover, Csikszentmihalyi (1990) mentioned that the idea of time is distorted. Being in a flow state, one's experience can pass quickly or feel slow.

However, Ericsson et al. (1993, 2018) suggested that deliberate practice is a conscious and demanding experience; it requires full concentration and self-control attention as one engages in training that goes beyond their existing skill levels, which can be unpleasant and difficult. In contrast to being in the flow state, one needs to be fully aware of and present during deliberate activity. When engaging in a deliberate practice, one may take time to prepare mentally for such training. Such progress is often slow but persistent. As discussed in Lee et al. (2022), the slow-and-steady progressors (i.e., students who repeatedly replayed and reattempted to improve their solutions in an online mathematical game) (*Materials: From Here to There!*) showed the largest absolute learning gains compared to the other groups. Therefore, we speculate that in a mathematical game-based learning environment, evidence of deliberate practice may be linked to determined, constant but slow progress, modest flow experience, and a greater sense of challenge (Lee et al., 2022).

Other core elements of deliberate practice are strongly linked to the quality of the gameplay—in this case, the mathematical challenge presented in the NNG. As seen in Figure 3, the tasks are supposed to be well defined, ideally demanding, and appropriate for individual students. The tasks should strengthen one's knowledge about numerical relations and their ability to solve arithmetic tasks in novel contexts, thus developing strong ANK.

In the NNG, the energy game mode requires complex numerical knowledge and arithmetic relations, which necessitates more use of the sequences of all four arithmetic calculations to solve the tasks than in the moves mode. These are also challenging tasks for students, especially those with lower-than-average mathematical skills. Some students might need to spend more time playing and engaging in reattempts to try to solve these maps (Lee et al., 2022). Therefore, energy levels are argued to be specifically critical in promoting ANK (McMullen et al., 2016). Hence, we hypothesize that students who engage in deliberate practice while playing the NNG will gradually improve their performance in the energy game mode while experiencing a modest experience of flow and a greater-than-average feeling of challenge.

2 Aims and Structures of the Thesis

Adaptive expertise is a greatly appreciated, yet rarely achieved, goal of mathematics curricula because it is considered to typify high-level mathematical thinking (Hatano & Oura, 2003). Adaptive expertise demonstrates knowledge and skills that can be dynamically implemented in uncommon circumstances, not just within narrowly defined tasks or sufficiently prepared situations. In the context of primary school mathematics, students must be supported with occasions to practice finding answers to open-ended mathematical problems in novel circumstances. Traditional classrooms are often equipped with textbooks and instructional approaches that focus on isolated, repetitive route exercises, or *drill-and-practice*, which encourage students to master isolated techniques for finding the *most* efficient solution (Verschaffel et al., 2009). Therefore, math teachers should employ teaching methods that emphasize the speed and accuracy of these materials (Anderson et al., 2018; Sun, 2018). With a heavy focus on routine problem-solving (Schoenfeld, 1992) and task-performance-oriented assessment (Baroody et al., 2007), the idea of mathematics as a "fixed" subject, which is full of rigid and absolute rules that students should mechanically memorize and passively follow teacher-led instructions, will continue to be reinforced (Anderson et al., 2018; Schoenfeld, 1992).

This doctoral dissertation aims to investigate design principles for learning environments that support flexible mathematical thinking in mathematics education. To achieve this, this thesis focuses on two general objectives: first, it aspires to understand how adaptive expertise can be enhanced with deliberate practice, and whether it can be done using a serious game—the NNG. Deliberate practice is challenging, pushing individuals slightly past their current skill level. It requires deep involvement, consistent efforts for improvement, and an embracing approach to challenges—qualities that align with a growth mindset. Recognizing the link between a growth mindset and persistent learning behavior, the second aim investigates and assesses interventions implemented in math classrooms to influence the beliefs and mindsets of teachers and students. This is crucial for integrating game-based learning into conventional mathematics instruction and realizing the goal of adaptive expertise in mathematics.

To accomplish these aims, the present work comprises three sub-studies, which are divided into two parts, each focusing on one general aim. Briefly, part one (Studies I and II) involved studies of the NNG. Study I explored game experiences throughout the developmental process of the NNG, and how different design choices affect students' gaming experience. Study II focused on how different game performance profiles are related to mathematics learning outcomes—math interest, perceived challenge, and experienced flow during gameplay—to determine whether we could find evidence of deliberate practice through gaming analytics and learning outcomes. Part two (Study III) aimed to investigate the current state of growth mindset interventions in mathematics education through a systematic review study. Study III examined the types of interventions that have been implemented in math classrooms to change, shift, or foster the beliefs or mindsets of both teachers and students. The study also evaluated the reported effects of these interventions on students' beliefs, motivation, and engagement in math learning. Additionally, it looked into the reported influences of these interventions on math teachers' beliefs and practices.

Both parts of the present work aim to gain further insight into promoting flexible mathematics thinking via growth mindset interventions and serious games that allow for deliberate practice.

3 Methods

This dissertation collected data partly from one large research project and partly from one systematic review. In 2013, a large research project was implemented to design and examine different versions of the NNG in Finland. The systematic review was conducted to understand the current state of mindset interventions in mathematics education. In this section, more details about the methodology and common information in relation to the participants, measurements, and statistical analyses used in each study are presented.

3.1 Research context

This dissertation consists of three sub-studies that were conducted at the Centre for Research on Learning and Instruction within the Department of Teacher Education at the University of Turku in Finland. These sub-studies were administered as components of two large research projects: the Academy of Finland's CUMA project (principal investigator (PI): Professor Erno Lehtinen) and the Strategic Research Council's (SRC) Growing Mind project (established within the Academy of Finland) (PI: Professor Minna Hannula-Sormunen). In addition, this doctoral thesis was partially financed (from July 2020 to December 2021) by the Learning, Teaching, and Learning Environments Research (OPPI) doctoral program at the University of Turku Graduate School, Finland.

3.2 Ethical considerations

In all the sub-studies of this dissertation, the ethical guidelines used by the University of Turku were strictly followed. In sub-studies that collected information from individuals (Studies I and II), such as students and teachers, all participation in the research project was voluntary. Participants' consent and their parents' consent were obtained ahead of time.

In Studies I and II, the participants were primary school students; therefore, consent was secured not only from the participants themselves but also from their parents. The consent form was returned to the research project team prior to the beginning of the sub-studies. In both studies, the research project team carefully notified all parties involved in the interventions about the project's information, timeline, data collection procedures, and planned analyses. The teachers acted as connections between the research project and the participants. They distributed the consent forms to all parents through the students, and the students returned them. If there were any parties that did not provide their consent to participate in the research, the participants were given opportunities to be involved in other activities during the testing period or to take part in the study's programs, in addition to their regular school activities. However, any data from those who did not agree to participate were erased prior to the analysis process. Game analytics and measurement information, as well as any data related to the participants' identities, were stored in separate offline storage areas. Furthermore, in alignment with the global standards on data privacy, the provisions of the General Data Protection Regulation (GDPR) were also integrated. This ensures that the handling of personal information from participants, who act as data subjects, is done with the utmost care and within the confines of established regulations. The analysis process was carried out by parties other than the data coding personnel after the data were anonymized.

In Study III, there was no need to obtain consent, as the data collection did not involve any personal or confidential information from the participants. However, in Study III, a systematic review, issues related to ethical considerations were carefully examined to ensure that the frequent patterns of biases in searching—for instance, database bias, bias in citation, or process ambiguity—were limited as much as possible. Study III followed the preferred reporting items for systematic review and meta-analysis protocols (PRISMA) guidelines (Moher et al., 2009). Sixteen studies were included in Study III, of which two were mixed methods and qualitative, and the rest were quantitative. The included studies underwent an appraisal process in which two independent researchers used two appraisal instruments—the evidence-based librarianship critical appraisal checklist (EBLCAC) (Glynn, 2006) and the qualitative research checklist (QRC) (Critical Appraisal Skills Programme, 2018)—to assess the quality of each study.

There are different ways to synthesize and combine diverse forms of evidence, and the integrated design approach (Heyvaert et al., 2016) was chosen for Study III. All findings and intervention descriptions of studies were converted into qualitative form by reading the studies, formulating summary phrases that captured their characteristics and findings, and then performing a thematic analysis (Dixon-Woods et al., 2005) using the NVivo 12 software program. The reasons for choosing this approach are twofold. First, methodological differences in quantitative and mixed-method studies are minimized, given that both produce findings that can be synthesized qualitatively. Second, this approach allowed data related to the population and context of individual studies to be integrated and combined with findings that might help answer the question of whether the success of the intervention was context-dependent (Noyes et al., 2019), which was lacking in the current across-domains, general mindset meta-analyses.

In the last step, the summaries of the 16 included studies were transferred to NVivo 12 software to be thematically categorized and synthesized. To extract the relevant data and analyze the study summaries from the previous step, the categories were allowed to emerge inductively through codes from the data. At the same time, existing theoretical constructs and frameworks discovered during the literature search were used to guide the data extraction and to allow the researcher to unify codes into themes. Based on the research questions, preliminary coding schemes were created for the interventions (the independent variable) and for the outcomes and reported findings (the dependent variable). A distinction was made between "teacher-focused intervention" (in which the intervention targets teachers and the content is delivered to teachers directly) and "student-focused intervention" (in which the intervention targets students directly and delivers content directly to students without having classroom teachers as intermediaries) (Yeager & Dweck, 2020). Therefore, the outcomes and reported findings of the studies were coded according to the subsequent intervention target.

3.3 Participants

All participants in Studies I and II were primary school students in the southwest region of Finland. Except for the interviewed participants in Study I who attended Finnish-English classes, in which lessons were delivered in English half of the time, the rest of the participants attended classes in the Finnish language. Table 1 provides a detailed description of the participants.

Study I was a cross-sectional study, with samples consisting of three cross-sectional data collections retrieved at various periods during the development process of the NNG game. The selected participants in Study I came from a large-scale research study consisting of 1168 students from 61 public school fourth- to sixth-grade classrooms across southern Finland in spring 2014. The first version of the NNG was used by the experimental group, which comprised 642 (n = 299 girls) students. The 526 (n = 247 girls) student control group played the NNG 2 version after the post-test of the experimental group.

Since the core focus category for which the NNG was developed was fourth-grade students, it is important to consider the gaming experiences of those in the fourth graders' subgroups from all versions of the NNG. Fourth graders from subgroups NNG 1 and NNG 2 were selected from the above-mentioned large-scale research pool of 1168 students. In NNG 3, the fourth graders (N = 40, n = 13 girls) were recruited in 2015 from two public school classes in southwest Finland. The students' ethnic backgrounds and socio-economic status were comparable to subgroups that were involved in NNG 1 and NNG 2. Finally, this fourth-grade group was also selected to participate in the prototype implementation of NNG 4 in spring 2016, when they became fifth graders. Only 37 students were involved in playing the prototype because three were absent during the data collection period.

Participants in Study II were also selected from the same large-scale experimental group playing NNG 1 in spring 2014. Selected participants (N=214, M age = 11.37, SD = 7.13) completed at least five energy maps. Table 1 provides a detailed description of the participants.

3.4 Instruments

3.4.1 Game experience and math interest

Game experience: The Game Experience Questionnaire (GEQ) measures eight core components of game experience. These dimensions are competence, challenge, immersion, flow, negative affect, tension, positive affect, and positive values. The GEQ was used to assess students' game experiences in sub-studies I and II. In Study II, only the flow and challenge components were considered in the analysis process. The GEQ used in these studies was in Finnish, as translated by Oksanen (2013). The questionnaire was then simplified to be better suited to the participants. Of the 42 original items, 3 additional positive value items, and 1 challenge item were included, while 15 original items from Oksanen (2013) were removed. Each item pertains to a different aspect of the gaming experience. A scale from 1 to 5 designated a level of agreement, with options ranging from 1 (*not at all*) to 5 (*extremely*). A principal component analysis with varimax rotation was conducted by analyzing the factor structure of the 31 variables included in the GEQ. The data was considered adequate, as Bartlett's test of sphericity showed a significance of $p < 0.001$, while factor analysis was 0.95 in the Kaiser–Meyer–Olkin measure. The data showed eight separate factors and therefore utilized them as the standard for sub-scales.

Math interest: Math interest was assessed as a section of the Expectancy Value Math Motivation Questionnaire (Rodríguez-Aflecht et al., 2015) with three items (e.g., "I like math").

3.4.2 Related mathematical measures

Arithmetic procedural fluency: Basic arithmetic fluency was quantified using the Woodcock-Johnson Math Fluency sub-test, which comprises 160 items. This measurement was used in Study II. The Woodcock-Johnson IV Test of Achievement is a popular standardized and adequate measure with a high reliability score (Woodcock et al., 2001). Students were asked to complete as many arithmetic tasks (simple addition, subtraction, and multiplication) as possible in 3 minutes. Final scores comprised the total number of correct answers.

Adaptive Number Knowledge: The arithmetic sentence production task was implemented to measure students' ANK in Study II. This is a timed task conducted with paper and pencil that assesses participants' skills in realizing and making use of various numerical features and relations during their problem-solving. The instruction asked students to make as many different math problems as possible with the given numbers and a target number. For instance, a task unit comprises four to five specific numbers (e.g., 2, 4, 8, 12, 32) and the four basic arithmetic operations, and the objective is for participants to create as many arithmetic calculations that equal the target number (e.g., 16) as they can in 90 seconds. There were four units, and the Cronbach's alpha reliability value for the total number of correct results across all units was .70.

3.4.3 Game performances

Game performance was used in Study II as a measure of students' game performance on the energy maps. Only participants who completed at least five energy maps were included in the analysis process. Game log data in Study II provided a significant amount of reliable information on players' advancement during gameplay. In the energy maps, participants are supposed to expend the smallest amount of "energy" possible, which is calculated by totaling up all the digits inserted in the operation box. In the moves mode, gameplay often activates addition and subtraction operations with large two-digit numbers. The energy game mode necessitates more complex computational relations compared to the moves mode, and all four arithmetic operations are required. Therefore, energy maps are deemed to be especially beneficial for advancing ANK.

Students played NNG 1 on computers with personal USB drives, and the log data was stored on these drives. The data were then transported to Excel and tallied to calculate a "relative energy score" for game performance in Study II. Relative energy score was calculated as "map neutral measure of performance = score/gold standard" – which was computed from at least five completed energy maps. The first two maps completed were at times T1 and T2, the average of the middle maps was calculated at T3, and the last two finished maps at T4 and T5. To complete a map, participants need to collect all four materials. Based on their arithmetic solutions (game moves), their game performance (or scores) will result in bronze, silver, or gold coins. Solutions from gold-awarded maps are considered close to optimal solutions.

3.4.4 Semi-structured interview questions

Semi-structured interview was employed in Study I as a measure of students' gaming experiences regarding the development of NNG 3 and NNG 4. Interviews were implemented one at a time in English language with participants from an English-specialized classroom. Students in these classes split their formal study time equally in English and in Finnish. In total, six (n= 3 girls) students volunteered to participate in the interviews. These participants talked about their game experiences playing two versions of the NNG games without any language difficulties. The interview structure was developed partly based on the results of previous NNGs (Brezovszky et al., 2015; Lehtinen et al., 2015; Rodríguez-Aflecht et al., 2015) and partly on the continuous design of NNG 4. Participants in the semi-structured interviews also participated in the NNG 3 experiment. Therefore, these students had played the NNG previously and had already formed their own opinions about NNG 3. As a result, the semi-structured interview aimed to enrich the understanding of students' reactions to the development and changes in NNG 4, motivational components, and aesthetics, and how these changes affected students' attitudes and intentions to play NNG 4.

The interview structure was designed in three sections: (1) part one included opening statements and questions where participants were introduced to the interviewer, the purpose and information of the interview; (2) part two comprised warm-up questions, which connected to participants' gaming experiences previously with NNG 3, for instance, "Can you tell me something about your experience playing the game?" or "What did you like/dislike about that game?"; (3) part three contained substantive questions—in which participants were presented with questions connected to their gameplay with NNG 4, for example, "Do you see any changes of the game? What are they?", "Do you enjoy playing this game (NNG 4)? /What makes it enjoying/less enjoying to you?". Inquiries regarding students' practical game experience were supplemented with suitable follow-up questions.

Table 1. Overview of this dissertation's methods.

Study	Aims	Participants	Instruments	Procedures	Analyses
Study I	To better understand how different design choices and game elements support students' game experiences throughout the developmental process of the NNG	N= 1208 NNG 1 n=642 NNG 2 n= 526 NNG 3 n= 40 NNG 1 subsample 4th graders: N= 63 NNG 2 subsample 4th graders: N= 72 NNG 3 subsample 4 graders: N= 40 NNG 4 Same group as in NNG 3 now 5th graders: N= 37	Game Experience Questionnaire (GEQ) with N= 1208 participants Semi-structured interviews with NNG 4 subsample 5th graders: 6 participants	Quantitative data is collected via GEQ from previously done studies in 2014 & 2015. Qualitative data were collected via semi-structured interviews after students played NNG 4 in 2016.	NNG game and design process description of each version and summary of changes. Reliability test of Cronbach's alpha of 31 variables in GEQ. An independent-samples *t* test to test game experiences of participants from 3 grade levels (4th to 6th grade) between NNG 1 and NNG 2. A one-way ANOVA to compare subsample 4th graders game experiences playing three NNG versions from NNG 1 to NNG 3. Qualitative of interview analyses.
Study II	Understand various ways that students playing in NNG, and to establish whether through gaming analytics and learning outcomes there can be found evidence of deliberate practice.	N= 214 M(age)= 11.37, SD = 7.13 Participant selected from the larger sample of 642 student who took part in a large-scale RCT over a ten-week period playing NNG 1 in 2014.	*Pre and post-test measurements*: ANK with Arithmetic Sentence Production Task. Arithmetic Procedural Fluency (Woodcock-Johnson Math Fluency sub-test). Pre-algebra Knowledge. Math interest.	A series of growth mixture models were calculated. Individuals' initial scores of Energy map performance and their linear and quadradic slopes were estimated and applied as benchmarks for defining a categorical variable of profile	Growth mixture modelling analysis of Energy map performance. Analysis of profile memberships with external variables (ANK, math interest, perceived challenge, and flow).

		Participants were those who completed at least 5 Energy maps.	*Post-test measurements:* Game Experience Questionnaire. Game performance (NNG game logs of Energy maps) Mplus program version 8.4	membership. When the most suitable number of profiles is set on, we applied a 3-step method to investigate the connection between profile memberships and other variables (i.e., flow, ANK, math interest, etc.).	
Study III	Provide a systematic review of mindset interventions in math classrooms in primary and secondary schools	Participants from 16 studies.	Appraisal checklists: EBLCAC; QRC Thematic analysis software: NVivo 12 program. Mindset Interventions in Math Classrooms Review Coding Book	Preferred reporting items for systematic review and meta-analysis protocols (PRISMA)	Descriptive analysis of the included articles. Thematic analysis of included articles regarding intervention targets, intervention content, mode of delivery and outcomes of interventions.

4 Overview and Main Findings of Studies

Study I

Bui, P., Rodríguez-Aflecht, G., Brezovszky, B., Hannula-Sormunen, M. M., Laato, S., & Lehtinen, E. (2020). Understanding students' game experiences throughout the developmental process of the number navigation game. *Educational Technology Research and Development*, 68(5), 2395-2421.

The NNG is a serious game designed to enhance mathematical skills with whole-number arithmetic for students at the primary school level, distinctively concentrating on adaptivity and flexibility (Brezovszky et al., 2015; McMullen et al., 2017). The general aim of the NNG is to foster students' ANK by supplying various opportunities for students to be involved in deliberate problem-solving with different numerical combinations and operations using the 100-square of whole numbers (1–100). The NNG is an example of an open-learning environment embedded with elements that support and encourage effortful practices, and progress is made visible to students via different in-game feedback components (Lehtinen et al., 2015).

To better understand how different design choices and game elements support gaming experiences during the developing process of the NNG, Study I aimed to analyze students' gameplay experiences of four consecutive versions of the NNG from 2014 to 2016 by answering two research questions: 1) How do changes in the different releases of the NNG influence students' gaming experiences? 2) What are the students' attitudes and preferences toward new changes and elements in NNG 4 compared to NNG 3? How are the new features in NNG 4 connected to students' attitudes and determination to play the NNG?

Study I was conducted using a mixed-method design of both quantitative and qualitative research. Regarding quantitative measures, Study I made use of the GEQ, comprising eight game-experience components, to survey different cohorts of primary school students in southwest Finland in three separate experiments (Lehtinen et al., 2015; Rodríguez-Aflecht et al., 2015). The participants played four different versions of the NNG from 2014 to 2016. Regarding qualitative measures, participants' gaming experiences, preferences, and game features of NNG 4 were recorded via six semi-structured interviews after students played NNG 4 in 2016.

Table 2. Information about the participants.

2014	2014	2015	2016
NNG 1 4th to 6th graders N= 642	NNG 2 4th to 6th graders N= 526		
NNG1 4th graders' subsample N= 63	NNG2 4th graders' subsample N= 72	NNG3 New 4th graders sample N= 40	
			NNG4 Same cohort in 2015 now became 5th graders. N= 37

The results indicated that improvements in the NNG's usability and clarity in the player interface were successful in sustaining more beneficial, smooth, and immersive gaming experiences since students' playing experiences in successive versions of the NNG were significantly enhanced compared to their experiences with NNG 1. Additionally, it appears that there is a distinct benefit in providing better aesthetics in game-based learning platforms, as players favored and valued more attractive design and graphic improvements in NNG 4 compared to NNG 3. Future work is required to understand the precise merit of extrinsic elements in maintaining players' enthusiasm, motivation, and situational interest in game-based learning environments, as not all motivating components provide predetermined influences on players' gaming experiences. Motivating elements were emphasized in both NNG 3 and NNG 4 compared to the previous versions. In NNG 3, players could use the coins they earned from finishing the maps to purchase ships on the "shop" page. Similarly, in NNG 4, players could construct and build ships with settlements and gold coins earned from finishing maps on separate islands. While the extrinsic components in NNG 3 and NNG 4 were essentially designed with the same intention, which is to provide a use for the earned coins and materials in the maps, how they were executed made them different from one another. While additional elements in NNG 3 did not have an effective impact on students' game experiences compared to NNG 2, qualitative data indicated that extrinsic features in NNG 4 succeeded in making the game more stimulating or compelling. Above all, Study I provided a broad description and analysis of the development process of creating the NNG. This can be used as a future reference to decide which game elements are essential in creating relevant serious games and how development in game design can have an impact on students' game experiences.

Study II

Bui, P., Hannula-Sormunen, M. M., Brezovszky, B., Lehtinen, E., & McMullen, J. (2022). Promoting Adaptive Number Knowledge Through Deliberate Practice in the Number Navigation Game. In *Games and Learning Alliance*: 11th International Conference, GALA 2022, Tampere, Finland, November 30–December 2, 2022, Proceedings (pp. 127-136). Cham: Springer International Publishing.

In mathematics education, it is particularly essential to supply students with opportunities to take part in abstract and quality-rich forms of problem-solving that nudge them to improve their unfolding skills and knowledge foundation (i.e., deliberate practice) rather than just regular sequences of static training using their current abilities (drill-and-practice) (Lehtinen et al., 2017). Because of the traditional constraints and challenging specifications of deliberate practice, it is laborious to systematically apply its principles in conventional classrooms.

Adaptive expertise is a greatly appreciated expectation of mathematics syllabuses (Watt, 2011) and is considered to exemplify outstanding mathematical thinking (Hatano & Oura, 2003). Adaptive expertise demonstrates acquired knowledge that can be creatively implemented in new or unusual circumstances, not just limited to well-practiced situations. ANK indicates a well-connected network of knowledge of numerical features and arithmetic relations, which is a desirable quality of arithmetic expertise (McMullen et al., 2017). To develop ANK, students must be supplied with regular opportunities to engage in solving non-routine problem-solving tasks in unfamiliar situations, preferably situated in an open-ended learning context. In the larger context of traditional schooling, it is often too demanding to sustain such conditions for all students.

Both deliberate practice and adaptive expertise in mathematics are crucial yet difficult concepts to directly implement in authentic settings. However, it is possible to overcome these limitations with the affordances of game-based learning. In the NNG, the mechanics are designed with deliberate practice's principles in mind (Lehtinen et al., 2015) while providing an open-ended learning environment that activates contemplation of various solutions to arithmetic problems, especially in the energy mode, which demands more complicated arithmetic relations and possibly leads to improvements in ANK (McMullen et al., 2017).

The design of the NNG reinforces deliberate practice (Lehtinen et al., 2015). However, no evidence indicates that students engage in deliberate practice while playing the NNG. Studies on games have highlighted the role of "flow" as a beneficial and engaging experience during gameplay. The concepts of “deliberate practice” and “flow” both indicate acute engagement in a circumstance. However, the two phenomena are fundamentally different. When analyzing the flow notion, Csikszentmihalyi (1990) explained that the features of the duration when experts (e.g., chess players and rock climbers) experienced sought pleasant training at the most idea amount of challenge. On the other hand, Ericson et al. (1993) indicated that deliberate practice is a mindful practice that often causes discomfort when people attempt to improve a certain skill that past their current ability. Therefore, we can suppose that in a mathematical game-based learning environment, deliberate practice could be linked to a determined, but not a brisk, process, low flow occurrence, and a high level of challenge. In Study II, we worked to better acknowledge distinct aspects of students’ engagement in playing the NNG and to decide if there was evidence of deliberate practice from gaming analytics and learning gains. Therefore, we asked two research questions: 1) What are the different profiles of game performance? 2) How are the profile memberships of game performance related to mathematics learning outcomes, experienced flow, challenge during gameplay, and math interest?

Participants in Study II were students from fourth to sixth grade (N = 214, M age = 11.37, SD = 7.13) chosen from the large pool of 642 students who participated in a large-scale RCT over 10 weeks (Brezovszky et al., 2019). Participants in Study II were selected based on their energy game performance (i.e., those who completed at least five energy maps). Students completed the following measures: ANK, arithmetic procedural fluency (Woodcock-Johnson Math Fluency sub-test), and math interest before and after the intervention. The post-test also comprised the GEQ, which measured core dimensions of gaming experience (also used in Study I). Students’ performance on the energy maps was assessed based on a relative energy score. This relative energy score was calculated based on at least five completed energy maps (map neutral measure of performance = score/gold standard). The first two maps were calculated as times T1 and T2, while T3 was the average of the middle maps, and subsequently, the later maps were completed at times T4 and T5. When players completed a map by retrieving four proposed materials, their game performances were rewarded with either bronze, silver, or gold coins, depending on their arithmetic solutions. Gold-coin rewarded maps are supposed to have close-to-optimal solutions.

A sequence of growth mixture models was conducted with Mplus software 8.4 to record patterns of performance across the recorded energy maps. Growth mixture models were employed in this analysis because they are a clustering course of action that determines coherent memberships derived from participants’ estimated growth curves. To determine a categorical variable of profile membership (e.g., a cluster membership), individuals' initial scores of energy map performance and their linear and quadradic slopes were calculated and used as benchmarks.

In the model, the number of profiles was set to start with one, then gradually increase. Each following model was evaluated with a set of statistical guidelines (Nylund et al., 2007) and theoretical considerations (Hickendorff et al., 2018). When the most suitable number of profiles was established, we employed the three-step approach (Asparouhov & Muthén, 2014) to investigate the relationships between profile membership and external variables (i.e., ANK, math interest). Based on the BIC value of the two- to seven-profile models, the four-profile growth mixture model was the most suitable as it had the lowest value (Nylund et al., 2007). Figure 4 presents the mean scores and standard errors for each profile throughout the five-time points energy map.

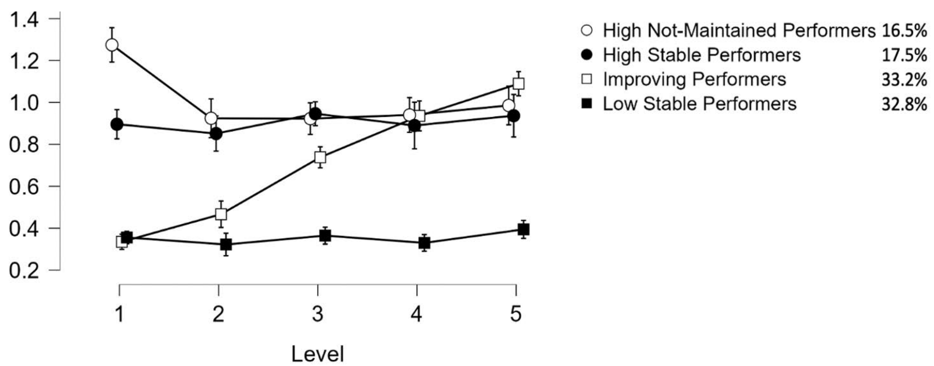

Figure 4. Mean scores of relative game performance scores by growth mixture model profile for each game map. Error bars represent 95% confidence intervals.

Our results indicate that the NNG offers some participants opportunities to be involved in deliberate practice in the best interest of their ANK. Based on the four profiles, it is noticeable that the improving performers group consistently improved their gameplay at every measured point in spite of their subpar initial performance. This group also showed strong academic outcomes in ANK and arithmetic fluency and recorded a lower-level flow experience. Based on these statistical results and theoretical considerations (e.g., flow states vs. deliberate practice experience), our data provided direct evidence that the improving performers group participated in deliberate practice during the NNG energy game maps. Additionally, this version was the first finished working release of the NNG, and at the time of intervention, its usability and clarity were far from optimal. As presented in Study I, game experience in succeeding versions of the NNG was significantly enhanced compared to those in this version. Therefore, students who were committed to advancing their game performance despite these usability deficiencies intentionally played the maps more than required. In regard to the challenge dimension, no significant differences were detected within the groups, apart from the fact that the high stable performers group reported a higher level of perceived challenge than the improving performers group. Reasons for this difference can be explained from the retrospective measure, as the improving performers group consistently advanced their game performance; therefore, in the post-test, their perceived challenge was reported to be lower. Overall, Study II findings suggested that although adaptive expertise in mathematics and deliberate practice are very difficult and demanding concepts to implement at the same time in regular classrooms, it is possible with well-designed serious games as they can provide unprecedented opportunities to overcome these shortcomings.

Study III

Bui, P., Pongsakdi, N., McMullen, J., Lehtinen, E., & Hannula-Sormunen, M. M. (2023). A systematic review of mindset interventions in mathematics classrooms: What works and what does not?. *Educational Research Review*, 100554.

In the domain of mathematics education, a belief in the "math brain"—as something you do or do not possess—is widely prevalent. Studies have shown that teachers and students are likely to feel that math achievement is due to inborn ability more so than other academic domains (Beach & Dovemark, 2007; Jonsson et al., 2012). Given this prevalence of strong beliefs about the innateness of mathematical ability, possible idiosyncratic effects of mindset interventions in the math domain may have been overlooked by research reviews and meta-analyses that do not examine domain-specific effects, as the majority of growth mindset-related research in schools is domain-general.

Study III provides a systematic review of mindset interventions in primary and secondary school math classrooms to map the scientific research on mindset interventions and to provide a body of scientific research on mindset interventions in math learning and teaching that captures the diversity of research perspectives and methodologies by including both quantitative and qualitative studies. Study III focused on the following questions: 1) What kinds of interventions aiming at changing, shifting, or fostering teachers' and students' beliefs or mindsets have been carried out in math classrooms? 2) What are the reported impacts of these interventions on students' beliefs, motivation, and engagement in math learning? 3) What are the reported impacts of these interventions on math teachers' beliefs and practices?

The study is designed, and the results are reported following the PRISMA guidelines (Moher et al., 2009). The review included 16 empirical investigations assessing growth-mindset interventions in mathematics classrooms. These studies were published in peer-reviewed journals and conference proceedings between 1998 and May 2023. Each of the retrieved studies was carefully assessed based on a quality appraisal process to examine possible dissimilarities in the included studies regarding validity and rigor. For quantitative studies, the EBLCAC (Glynn, 2006) was chosen; for qualitative studies, the QRC was used (Critical Appraisal Skills Programme, 2018). Then, the integrated design approach was chosen to synthesize and combine both the qualitative and quantitative evidence of selected studies (Heyvaert et al., 2016). All related data and findings were converted into qualitative form, and subsequently, a thematic analysis was performed (Dixon-Woods et al., 2005) using the NVivo 12 program. This approach was chosen because the methodological differences in quantitative and mixed-method studies are minimized, as both produce findings that can be synthesized qualitatively. It also allowed data related to the population and context of individual studies to be integrated and synthesized and considered findings that might help answer the question of whether the success of the intervention was context-dependent. Eligible studies were coded to summarize the information from each study into two broad categories: 1) features of the study and intervention characteristics, and 2) results and discussion of the findings. Then, the features of the study were organized into groups and clusters based on the type of intervention (generic or domain-specific), target population (teacher-focused or student-focused), and intervention content (implicit theories of intelligence-related concepts, math content, or professional development content). Next, data were tabulated to prepare for the vote-counting process, which involved the tabulation of statistically significant and nonsignificant findings in quantitative studies and the reported impacts in qualitative and mixed-method studies. Finally, the thematic analysis technique was employed to consistently recognize the core, recurrent, or most important (guided by the research questions) themes or notions across these selected studies.

In general, results suggest that the potential for positive effects of mindset interventions on students' learning in the math classroom may have been overlooked in previous research. This study offers a comprehensive systematic review of mindset interventions within primary and secondary mathematics classrooms. A clear distinction is made between general interventions, which emphasize the malleability of intelligence broadly, and mathematics-specific interventions. These are finely tuned to address the unique beliefs and misconceptions surrounding mathematical abilities.

The findings indicate that math-specific interventions, especially those that integrate a growth mindset directly into mathematical content, are notably more successful in enhancing students' performance than broader general interventions. Such results underscore the importance of tailoring interventions to the nuances of a domain, particularly in mathematics, where entrenched beliefs about an innate "math brain" can hinder learning. On the educator side, while teacher-focused interventions were less frequent in the literature, those that deeply and longitudinally embedded a growth mindset within the mathematics education context were particularly effective in transforming teachers' beliefs and classroom practices.

For optimal outcomes, mindset interventions in mathematics should be domain specific. Additionally, educators benefit from sustained and immersive training that blends growth mindset principles with practical math teaching strategies. As highlighted by Orosz et al. (2017), without tangible changes in classroom practice, a mere temporary shift in general beliefs might not significantly influence learning outcomes. Deep engagement with math content, beyond a superficial level, is essential, and in certain scenarios, a growth-mindset narrative might not even be needed if the learning environment inherently encourages effortful practices and showcases progress to students, as suggested by O'Rourke et al. (2016). Therefore, future research should carefully weigh domain-specific effects and intervention targets when crafting mindset interventions.

5 Main contributions and General Discussion

This doctoral dissertation aims to investigate design principles for learning environments that support flexible mathematical thinking in mathematics education via game-based learning environments. Thus, this dissertation sought to investigate how adaptive expertise can be promoted with deliberate practice via a serious game called the NNG, and how successful growth mindset interventions in math education operate. Three studies provide theoretical and practical implications for the promotion of adaptive expertise through deliberate practice and growth mindset via game-based learning environments.

Study I explored students' gaming experiences throughout the developmental process of the NNG and how different design choices affected students' gaming experiences and attitudes. Study I's results provided insights into the design and iteration process of a well-designed research-based serious game, which in turn offers a comprehensive overview of students' interaction with learning and gaming components, and how they are related to these new types of mathematical learning objectives. Study II focused on game performance profiles using gaming analytics and explored the different ways students engaged with the NNG. By leveraging log data of game performances in the energy mode, together with measured mathematics learning outcomes, math interest, perceived challenge, and flow during gameplay, Study II provided evidence of how adaptive expertise can be promoted with deliberate practice through gaming analytics and learning outcomes. In short, Studies I and II provided concrete evidence of how the NNG—a mathematical game-based learning environment—offers an alternative learning platform in which contexts are novel, open-ended environments trigger non-routine and challenging tasks, game design provides clear and immediate feedback, and the game can be implemented in conventional classrooms for all students.

Study III provided a systematic review of mindset interventions in mathematics classrooms in primary and secondary schools. Results from this study emphasize the overlooked potential of mindset interventions in enhancing students' learning in math classrooms. Through a systematic review, a notable distinction emerged between general and math-specific interventions. Math-specific interventions, particularly those embedding growth mindset principles directly into content, proved more effective in boosting students' performance than their general counterparts. This highlights the critical nature of domain-specificity in mindset interventions, especially in fields like mathematics where deep-seated beliefs can act as barriers. While teacher-focused interventions were less prevalent, those that merged growth mindset principles with extended math teaching practices showcased promising shifts in educators' beliefs and methodologies. This suggests that to make behavioral changes based on changed beliefs, participants need to engage with math content at more than just the surface level. Most importantly, it is important that the learning environment is embedded with elements that support and encourage effortful practices and that progress is made visible to students.

5.1 Theoretical and methodological implications

Firstly, this dissertation contributes to the theoretical discussion of developing mathematical thinking via the practical application of deliberate practice in game-based learning environments.

A significant component of developing mathematical thinking (Schoenfeld, 1992) is opportunities for students to engage in non-routine problem-solving tasks, which allow them to think “mathematically” by reflecting on possible alternative strategies on their own. In other words, learning environments should foster students’ ability to dynamically navigate and flexibly apply acquired strategies or adaptive expertise to problem-solve in novel contexts (Hatano, 1982). Instead of mastering static, procedural, and stereotypically narrowed calculations, students should be encouraged to work on open-ended, non-routine problems and those with less straightforward answers. Students should be required to analyze, explore, and reflect until they find a suitable (to them) strategy. However, typical math tasks in traditional classrooms and coursebooks often represent a fixed set of formulas and rules to strengthen the automatization of procedural skills (Lehtinen et al., 2017; Verschaffel et al., 2009). Math teachers, therefore, would be inclined to follow instructional approaches that focus on choosing the “correct” or the “only” solution using these materials (Anderson et al., 2018; Sun, 2018). Thus, there is a need for a pedagogical model of non-routine practice that is more complex, abstract, and richer in relationships—practices that allow students to contemplate, analyze, and explore different connections and alternative solutions to develop their unfolding skills and knowledge formation—such as deliberate practice (Schoenfeld, 1992; Lehtinen et al., 2017). However, it is challenging to systematically apply the deliberate practice approach in mathematics learning in traditional classrooms due to its demanding focus on individuals’ competence levels. Nevertheless, with the affordances of game-based learning, game mechanics can be designed with core embedded principles of deliberate practice, such as is done in the NNG.

The current set of two studies (Studies I and II) is related to the promotion of adaptive expertise via the practical application of deliberate practice in the development of the NNG. Study I focused on the students' gaming experiences during the development process of the NNG through a mixed-method study, and Study II made use of gaming analytics and various measures of learning outcomes. Both quantitative and qualitative study designs were employed (Study I) to gain more in-depth knowledge of students' game experiences playing subsequent versions of the NNG. Quantitative measures (e.g., GEQ) allowed for comparisons of game experience between different versions of the NNG, while qualitative data shed light on how design choices could have varying impacts on learning and gaming experience. In Study II, a quantitative approach was more suitable to provide visibility of changes in game performance throughout the duration of gameplay and the relationships of participants' game performance with other measured learning outcomes, both directly through paper-and-pencil tests (e.g., ANK measurement) and indirectly through game-log data (e.g., game performances of energy maps). The design of the game scoring encouraged students to find closer-to-optimal mathematical solutions through replaying, thus presenting circumstances for students to reflect on their solutions and the underlying relations with varying numbers and operations. Our findings suggested that the NNG provides some students opportunities to engage in deliberate practice in support of their ANK development.

The present thesis contributes to the understanding of growth mindset interventions in mathematics education. Study III—a systematic review—contributed to the literature based on features and findings that were coded and patterns that were reported during the analysis process. Most previous studies have focused on delivering mindset interventions directly to students (Zhang et al., 2017). Interestingly, while all student-focused interventions were quantitative studies, teacher-focused interventions were approached using more diverse methods (qualitative, mixed-method, and quantitative studies). For student-focused interventions, while general mindset interventions have shown mixed results, math-specific interventions demonstrated more positive outcomes in improving students' mathematical performance. The limited success of general interventions, such as the one by Blackwell et al. (2007), contrasts with the consistent positive impacts of math-specific approaches. Moreover, this review also highlights a significant gap in the literature: the absence of follow-up data, which raises questions about the sustainability of these interventions. The prevailing "math brain" belief, which posits that individuals inherently possess or lack mathematical abilities, underscores the need for domain-specific mindset interventions. While math-specific interventions have shown promise, the long-term effects, best practices, and potential for application in other domains require further exploration. The findings suggest that the idiosyncratic nature of beliefs about mathematical ability requires targeted interventions, and further research is needed to provide a comprehensive view of whether, and how best, to employ mindset interventions in math classrooms for students.

Concerning teacher-focused interventions, Study III found mixed effects on students' achievement (Yeager & Dweck, 2020). However, when looking into the domain-general and domain-specific issues, positive effects were found in those in which the interventions' content was specific to teaching and learning math. A possible explanation could be that while the general mindset interventions provided training that was beneficial for introducing a growth mindset and reinforcing the growth mindset message in lessons through communication with teachers, they did not have any specific focus on math (or any subjects). They also did not incorporate (mathematical) teaching practices or supportive mathematical learning environments. It is difficult to imagine or evaluate the way teachers made connections between the highly broad content of the growth mindset that they received and the subject in practice. It is also possible that any behavioral changes that occurred were not sustained. Another possible explanation for this is differences in terms of the length and intensiveness of the interventions, which supported identity-shifting and behavioral changes in classroom practice. Similar to student-focused interventions, when the interventions' content was a general domain, the core content teaches the incremental view of ability—the concept that academic and intellectual abilities can be improved with effort, strategies, and some concrete actions that participants can take to execute the growth mindset that they learn (Yeager & Dweck, 2020).

General domain interventions have no specific focus on any subject, nor do they incorporate any specific type of problem-solving. In the mathematics-specific domain interventions, the core content varies greatly. The growth mindset message was integrated with mathematical content, and depending on the aims, participants, and focuses, the intervention and implementation methods were executed differently. Therefore, in some domain-specific teacher-focused interventions, teachers were offered opportunities to learn, discuss, reflect, or share with other participants either via online platforms or face-to-face meetings (Anderson et al., 2018). Results found a clear shift not only in teachers' beliefs about mathematics and themselves but also in their expectations of students and instructional practices. Teachers were able to reflect on and examine their own beliefs about math, their relationships, and their experiences with math both as learners and as teachers. Such possibilities for changes in belief were made possible when the highly broad content of the growth mindset was grounded in the mathematical context, and the interventions' activities allowed participants both the time and space for it to happen. Mindset interventions that happen in isolation from changes in classroom practices or other interactions might not result in sustainable effects (Orosz et al., 2017).A temporary change in beliefs cannot have large or sustained impacts on learning outcomes without concrete steps grounded in the subject or a supportive learning environment in which learning progress is made clear and practices are encouraged.

Revisiting the broad conceptualization of mathematical thinking put forward by Schoenfeld (1992), it is worth noting that teachers' and students' "fixed" beliefs about mathematics are not new. Schoenfeld (1982) emphasized how the instructional roots of students' (and teachers') beliefs about mathematics are systematic. When the curricula and textbooks are divided into smaller, isolated sections and focus on the mastery of algorithmic procedures using routine problems, instructional approaches will then emphasize guiding students to apply the procedures to solve problems correctly, without thinking about the underlying knowledge or attempting to "make sense" of the mathematics underneath (Schoenfeld, 1992). The ways in which math textbooks are structured shape the nature of math classrooms and teachers' instructional practices, which then determine how students interact with mathematics and their beliefs about the subject. A few situations in which students are presented with non-routine problems are also not enough to change their beliefs about mathematics or improve their adaptive expertise (Schoenfeld, 1992; Verschaffel et al., 2009). Attempting to foster teachers' and students' beliefs about the subject independent of changes in instructional approaches or the nature of math tasks and learning environments cannot have sustained impacts. Hence, learning environments should encourage struggles and effortful practices, and tasks should be open-ended, non-routine problems, which allow students to explore, reflect, discuss, and share their ideas until they find a suitable strategy.

The integration of advanced technological tools, including digital platforms and serious games, can offer students a more tailored learning experience, allowing for real-time adaptability based on individual needs. Furthermore, a shift in assessment strategies is required, moving towards tools that evaluate not just content knowledge but the adaptability and flexibility of mathematical thinking (Hulse et al., 2019). Stealth assessment (Shute & Ventura, 2013), for instance, could provide a non-intrusive means of gauging student progress and allow for a deeper understanding of students' interactions with learning components in a game-based learning environment. Thus, it is essential to intertwine pedagogical strategies, technological advancements, and innovative assessment methods to truly cultivate an environment where students can develop both a deep understanding and adaptive expertise in mathematics.

5.2 Practical implications

The affordances of game-based learning bring forth different alternative training opportunities that allow learning to happen despite traditional classrooms' challenges, such as time or resource constraints. Previously, traditional practices that attempted to foster the understanding and representation of numerical features and relations required one-on-one teacher support and scaffolding (Heirdsfield, 2011). Even current mathematical practices that make use of materials in textbooks, which not only emphasize drill-and-practice activities that are heavily procedural and routine oriented but also have little to no "replay value" (Phoenix, 2014) once the problems are solved, are unlike well-designed educational games.

Thanks to the affordances of technological advancement and game-based learning, it is possible to provide tasks that are suitably demanding yet individually tailored to students; these tasks require students to train at the edge of their ability, and continuous feedback and learning progress are made visible to help develop their current skill levels. The game tasks are positioned in an open-ended learning environment that triggers mental reflections on possible alternative solutions to arithmetic tasks. These tasks are not "one dimensional," as there are varying possible solution paths. Furthermore, as there are no clear ready-made mathematical tasks, the design of the NNG tasks offers great "replay value," with many different options for practice (Phoenix, 2014). As opposed to textbooks, the NNG allows for multiple trials, errors, and replays, as students can decide to restart a map or replay a previously finished map to obtain higher scores with new mathematical solutions compared to their previous problem-solving strategies based on clear visual feedback (e.g., the total number of gold coins). As examined in Study III, learning environments that support struggles and mistakes foster a positive learning culture in which students feel more comfortable exploring, engaging, and trying new things without fear of making mistakes or failing. Consequently, such environments are critical in "norm-setting practices" that communicate growth mindset messages (Sun, 2019). Game-based learning environments offer unprecedented opportunities for teachers and students to engage with mathematical tasks in a more relaxing way, where "mistakes" are mere "trials," and effortful practices are rewarded with advancement in the game.

One important practical implication of the NNG is that it serves as an example of future practical applications of deliberate practice in game-based learning environments and adaptive learning systems. Practices in current adaptive learning systems share several similar components with deliberate practice, such as learners being provided with suitable learning paths based on their previous actions, and tasks being modified to fit with individuals' learning goals. However, the task design and adaptive learning system feedback are still limited by their predetermined content model, and suitable mathematics tasks for adaptive learning systems can be highly static and structured procedures (Bimba et al., 2017). At the moment, the integration of open-ended math tasks and non-routine problems is partly limited by technology, and partly due to the design models, which closely follow textbook content. Hence, the design of an open-ended learning environment integrated with deliberate practice from the NNG can inform future studies on the possibilities of improving adaptive learning systems and environments.

Another practical implication of embedding deliberate practice in game-based learning environments, such as the NNG, is that through the game design process, it is possible to manipulate game design choices to fit with certain learning environments, thus achieving possible different results in terms of motivational aspects, game experience, or the intensity and challenge level of practice. Despite the strong interest in serious games, much focus has been on whether game-based learning products fulfill their educational promises, rather than how the development decisions were made, and how these decisions affect the game experience (Gaydos, 2015). Rapid technology development has led the way for the popularity of mathematical content game-based products (Laato et al., 2020) and it also changes how students interact with learning content (Bray & Tangney, 2017). The quality of these freely available math games and apps, and how often these games are used in purposeful ways, are up for debate (Laato et al., 2020). From a game-design viewpoint, it is essential to understand how even small alterations in the game can have certain outcomes on others (Hunicke et al., 2004), as these differences have consequences for learning and players' experiences. Furthermore, learning technologies based on gaming hold promise in evaluating student learning throughout the problem-solving journey. Although numerous instructional technologies can document each student's action, there is a research gap on how to effectively utilize this data in the context of mathematical practice learning constructs during instruction (Hulse et al., 2019). Thus, while game-based learning technologies present vast potential for enhancing educational outcomes, there is an imperative need to bridge the gap between game design, data utilization, and instructional practices to fully harness their benefits.

Last but not least, the NNG can be used on a laptop, personal computer, or downloaded to a smartphone or tablet. It can also be installed as part of online learning platform hubs (e.g., the ViLLE system). If it is to be installed on a device, it does not require a Wi-Fi connection to run. Therefore, it is convenient for teachers to include NNG practices together with normal mathematical exercises. Teachers can make use of NNG applications in different ways. Students can either play in pairs or individually once they are accustomed to the game rules and mechanics.

Games are inherently motivating and have great potential to engage students through challenging and exciting tasks (Whitton, 2011). However, to maximize the impact of games such as the NNG, teachers need to provide timely support and guidance, especially for those at lower-than-average levels of mathematics. As discussed above, it is not enough to tell students to "make an effort" or to "try harder" when they are playing a challenge map in the hope of engaging them in deliberate practice and fostering their growth mindsets. It might also be necessary for teachers to create and lead constructive discussions about the different strategies for solving a task and provide opportunities for students to verbalize and engage in such mathematical discussions. Encouraging students to share and reflect on their solutions and challenges with others is also important to break the barrier of mathematics being a "fixed" and one-dimensional subject. Normalizing math as a fun, interesting, and explorative discipline through the use of games such as the NNG also enables students to have a different perception of mathematics and what it entails.

5.3 Limitations and future studies

One of the general limitations of this dissertation is its lack of empirical evidence to examine whether fostering a growth mindset supports and sustains students' engagement in such intensive deliberate practice. Previous research suggests that maintaining students' interest while playing the NNG is difficult, especially for those who have low initial motivation when tasks become challenging (Rodríguez-Aflecht et al., 2017). The nature of deliberate practice is demanding and takes place just beyond learners' abilities; hence, it requires deep engagement, persistent attempts, and a supportive, mistake-friendly learning environment when facing difficult tasks, or when trying to improve one's weak skill. These are the distinguished traits of having a growth mindset. Study III offered the desirable characteristics of a mindset intervention in math education, such as attention to intervention targets, grounded in mathematical content, emphasizing the importance of a mistake-friendly learning environment, and accompanying tasks. Future studies can investigate whether fostering a growth mindset specifically in the math domain would be beneficial for more students to engage in extended periods of deliberate practice in mathematics classrooms, and ways to embed motivating, mindset elements in game-based environments to trigger and sustain students' deliberate training, especially those who have low mathematical interest and lower-than-average math levels. Teachers' roles and mindsets are also critical for promoting flexible mathematics learning and creating an open, mistake-friendly learning environment in which mistakes and errors are normalized and considered part of the learning process. Future research could investigate the roles of teachers' support and teachers' mindsets when using the NNG, especially focusing on other kinds of data (teaching reflection, journals, observations) with regard to the application and daily practices with the NNG in a math classroom. These supplementary sources of data supplied in-depth data regarding contexts, implementation fidelity, and, most essentially, insights concerning the execution process of game-based learning accompanied by support from teachers in a regular classroom.

The NNG provides a rich amount of analytics (e.g., game logs, movement patterns, tasks finished, time spent on tasks/non-tasks) that could inform future research about students' practices, game performance development, game experience, and situational interests. Other kinds of gaming analytics, such as measures of in-app interactions (Hulse et al., 2019; Ottmar et al., 2015), or stealth assessment (Shute & Ventura, 2013) also offer insights into students' learning during the problem-solving process and their in-game experiences, along with situational interests (Kiili et al., 2021). Moreover, we are uninformed about why the NNG only enabled deliberate practice for one group (improving performer), and little is known about students with other profiles. The low stable group, for instance, had lower prior knowledge than the improving performers group. Hence, it possibly limited their capacities for self-initiating concentrated practice or sustaining intense sessions when the mathematical tasks were becoming more challenging. Observations of young children suggested that from the early ages of three to four years old, children can be spontaneously engaged in mathematically relevant practices such as focusing on numerosity (Hannula & Lehtinen, 2005). Little is known about the connection between "early forms of deliberate practice," such as spontaneous mathematical tendencies, and self-initiated practice, and later, (coached) deliberate practice (Lehtinen et al., 2017). Therefore, prior knowledge of different profiles and how they are related to their self-initiating capacities is a subject worthy of future investigation.

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