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BELIEFS ABOUT MATHEMATICS LEARNING DIFFICULTIES: OPPORTUNITIES FOR LEARNING – A NAMIBIAN STUDY

Shemunyenge Taleiko Hamukwaya

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SARJA – SER. AI OSA – TOM. 700 | ASTRONOMICA – CHEMICA – PHYSICA – MATHEMATICA | TURKU 2023

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This doctoral thesis is dedicated in thanksgiving to my dearest parents,
Stefanus Hamukwaya and Johanna Nandigolo Shatika-Hamukwaya.
Their love for education made me who I am today.
May their souls continue resting in eternal peace.

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ABSTRACT

This dissertation used a phenomenological approach to explore beliefs regarding indicators of MLD that mathematics teachers, teacher educators, pre-service teachers, and high school students hold. In addition, the dissertation looks into the beliefs about practices in Namibian high school mathematics classrooms that may contribute to mathematics learning difficulties (MLD). The common beliefs regarding MLD that were discovered address numerous factors: (i) systemic factors (ii) emotional dispositions, (iii) students' difficulties in solving basic mathematics tasks because of several different reasons, (iv) students' knowledge and beliefs and (v) students' poor learning habits.

This dissertation is based on the following four original publications. The notations and results of these studies have been modified and unified for the presentation of this dissertation.

Study I examined Namibian K–12 mathematics teachers' (n = 231) beliefs regarding MLD as a reflection of their teaching practice. Teachers related students' difficulties to students' lack of mathematical understanding (knowledge and skills). These beliefs addressed a negative impact on classroom teaching practices, which can obstruct access to supportive learning opportunities.

Study II explored beliefs regarding the teaching and learning of mathematics by Namibian Grade 11 students (n = 27), who were considered by their teachers as experiencing MLD. The students' perceptions provided an in-depth perspective on learning mathematics and the best way to assist students in studying mathematics. Despite their learning difficulties, students believed that they had the ability and potential to learn mathematics, but only if the educational system supported their learning.

Study III aimed to address one of the limitations of Study II, which was that the interviews with teachers were not conducted to obtain the reasons for identifying the characteristics of students experiencing MLD. In study III, the terminology shifted from *beliefs* about MLD to *perceptions* about the difficulties as the perceived difficulties in learning mathematics were explored from the perspective of Namibian Grade 12 mathematics teachers (n = 6) and compared to those of their Grade 12 students (n = 23). The findings of this study provided essential information for teaching practices and fostering students' mathematics education.

Study IV aimed to investigate beliefs about MLD that had developed during a Namibian teacher education programme, the views and practices that might be emphasised in the programme, and any change in beliefs experienced in the first year of teaching. Pre-service teachers (n = 4) and teacher educators (n = 3) believed that high school students' knowledge and beliefs are related to MLD. Teacher educators remarked the influence of teachers' knowledge and practices concerning MLD, whereas pre-service teachers did not. The participants deemed individualised student learning and teaching to be necessary to minimise MLD among students in mixed ability groups.

KEYWORDS: Namibia, mathematics-related beliefs, mathematics learning difficulties, high school students, mathematics teachers, pre-service mathematics teachers, mathematics teacher educators

TURUN YLIOPISTO
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TIIVISTELMÄ

Tässä väitöskirjassa käytettiin fenomenologista lähestymistapaa matematiikan opettajien, opettajankouluttajien, opettajaopiskelijoiden ja lukiolaisten matematiikan oppimisvaikeuksista (MLD)-indikaattoreihin liittyvien uskomusten tutkimiseen. Lisäksi tässä väitöskirjassa tarkastellaan uskomuksia Namibian lukion matematiikan luokkahuoneiden käytännöistä, jotka voivat edistää MLD:tä.

Tässä tutkimuksessa nousi esille Namibian koulutusjärjestelmää koskevia kriittisiä kysymyksiä. MLD:tä koskevat yleiset uskomukset, jotka löydettiin, koskevat useita tekijöitä: (i) systeemiset tekijät, (ii) emotionaaliset taipumukset, (iii) opiskelijoiden vaikeudet ratkaista matematiikan perustehtäviä useista eri syistä, (iv) opiskelijoiden tiedoista ja uskomuksista sekä (v) opiskelijoiden huonoista oppimistottumuksista.

Tämä väitöskirja perustuu seuraaviin neljään alkuperäiseen julkaisuun. Näiden tutkimusten merkintöjä ja tuloksia on muokattu ja yhtenäistetty tämän väitöskirjan esittämistä varten.

Tutkimuksessa I tarkasteltiin Namibian ensimmäisen ja toisen asteen matematiikan opettajien (n = 231) uskomuksia MLD:stä heidän opetuskäytäntönsä heijastuksena. Opettajat liittivät opiskelijoiden vaikeudet oppilaiden matemaattisen ymmärryksen (tietojen ja taitojen) puutteeseen. Nämä uskomukset käsittelivät kielteistä vaikutusta luokkahuoneen opetuskäytäntöihin, mikä voi estää pääsyn hyödyllisiin oppimismahdollisuuksiin.

Tutkimuksessa II tutkittiin namibialaisten 11. luokan oppilaiden (n = 27), jotka heidän opettajansa ajattelivat kokevan MLD:tä, uskomuksia matematiikan opettamisesta ja oppimisesta. Opiskelijoiden havainnot antoivat syvällisen näkökulman matematiikan oppimiseen ja parhaan tavan auttaa opiskelijoita matematiikan opiskelussa. Oppimisvaikeuksistaan huolimatta opiskelijat uskoivat, että heillä oli kyky ja potentiaali oppia matematiikkaa, mutta vain jos koulutusjärjestelmä tuki heidän oppimistaan.

Tutkimuksessa III pyrittiin käsittelemään yksi tutkimuksen II rajoituksista, joka oli se, että opettajien haastatteluja ei tehty selvittääksemme, miten he pitivät MLD:tä kokevien opiskelijoiden ominaispiirteinä. Tutkimuksessa III terminologia siirtyi MLD:tä koskevista uskomuksista käsityksiin vaikeuksista, kun matematiikan oppimisen havaittuja vaikeuksia tutkittiin Namibian luokan 12 matematiikan opettajien (n = 6) näkökulmasta ja verrattiin heidän 12 (n = 23). Tämän tutkimuksen

tulokset antoivat olennaista tietoa opetuskäytäntöihin ja opiskelijoiden matematiikan koulutuksen edistämiseen.

Tutkimuksen IV tavoitteena oli selvittää namibialaisen opettajankoulutus-ohjelman aikana syntyneitä uskomuksia MLD:stä, näkemyksiä ja käytäntöjä, joita ohjelmassa saatetaan korostaa, sekä ensimmäisen opetusvuoden aikana koettua muutosta uskomuksissa. Esiopettajat (n = 4) ja opettajankouluttajat (n = 3) uskoivat, että lukiolaisten tiedot ja uskomukset liittyvät MLD:hen. Opettajakouluttajat huomauttivat opettajien tietämyksen ja käytäntöjen vaikutuksen MLD:hen, kun taas opettajaopiskelijat eivät. Osallistujat pitivät yksilöllistä opiskelijoiden oppimista ja opetusta välttämättömänä MLD:n minimoimiseksi sekakykyryhmien opiskelijoiden keskuudessa.

AVAINSANAT: Namibia, matematiikkaan liittyvät uskomukset, matematiikan oppimisvaikeudet, lukiolaiset, matematiikan opettajat, matematiikan opettaja-opiskelijat, matematiikan opettajakouluttajat

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List of Original Publications

This dissertation is based on the following original publications that are referred to in the text with roman numerals I, II, III and IV:

I. S.T. Hamukwaya (2019). K-12 Namibian teachers' beliefs on learning difficulties in mathematics: Reflections on teachers' practice. In Jankvist, U. T., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (Eds.). (2019). Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (pp. 4652–59, CREME 11, February 6–10, 2019). Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME. https://hal.science/hal-02431483

II. S.T. Hamukwaya & R. Ruttenberg-Rozen (2023). Believing in Your Own Abilities: What Namibian High School Students Experiencing Mathematics Difficulties Can Teach Us. *Canadian Journal of Science, Mathematics and Technology Education*, 1–19. https://doi.org/10.1007/s42330-023-00260-7

III. S.T. Hamukwaya (2022). A Comparison of the Perceived Difficulties in Learning High School Mathematics from the Perspective of Teachers and Students. *International Journal for Mathematics Teaching and Learning*, *23*(1), 46–63. https://doi.org/10.4256/ijmtl.v23i1.317

IV. S.T. Hamukwaya & C. Haser (2021). "It does not mean that they cannot do mathematics": Beliefs about Mathematics Learning Difficulties. International *Electronic Journal of Mathematics Education*. 16(1), em0622. https://doi.org/10.29333/iejme/9569

Author contribution

Shemunyenge T. Hamukwaya is the first author for two manuscripts (II and IV) that are included in this doctoral thesis. Hamukwaya contributed to the study design, data collection, and transcribing under the supervision of Professor Peter Hästö and

Associate Professor Cigdem Haser. The analysis, interpretation and writing of the manuscripts were made collaboratively with co-authors. In II all authors analysed the data. The author actively took part in finishing the paper, and at the same time, mentored by the second author. In IV the second author acted as a peer reviewer, and monitored the coding process of the data. Careful and closely monitored coding process and critical discussion among the authors.

1 Introduction

Mathematics is a powerful language that enables people to view the world through numbers, shapes, and measures; it is also a science that contributes to solving real-life problems. It is a discipline in which new knowledge is built on prior learning, and learning mathematics without prior knowledge is difficult, particularly at higher grade levels (Gafoor & Kurukkan, 2015). Moreover, mathematics is an entry requirement for many post-secondary programmes and the careers they support, including the natural sciences, engineering, business and architecture (Ampandu, 2013; Siyepu, 2013). This characterisation positions mathematics as a foundation for many scientific and technological studies (Mundia, 2010), with a thematic link to other subjects across the school curriculum (Gafoor & Kurukkan, 2015; Mundia, 2010). A general perception surrounding mathematics is that achieving satisfactory results in mathematics serves as an adequate measure of higher education and a requirement in order to be accepted into university degree studies (Ampandu, 2013). At the same time, many students experience difficulties while developing competence in mathematics in primary and secondary schooling, which may then create barriers when seeking admission to higher education institutions. Subsequently, when provided a choice, some students may withdraw from studying mathematics because of their perceived difficulties in learning mathematics (Hofer, 2000; Ndlovu & Ngcobo, 2018).

Some students also lack confidence in their mathematical abilities. Thus, they may develop negative attitudes towards mathematics, which can impact their career potential in the science field (Brown, Brown, & Bibby, 2008). It is challenging for primary and secondary teachers to encourage students with these perspectives to learn mathematics, particularly if the students have already lost confidence in their mathematical abilities at lower grade levels. Students may even experience increased difficulty in catching up as they progress to upper secondary school, and mathematics might become an impasse to these students, despite its importance in working life and the job market (OECD, 2014).

Additionally, there are normal variations in the way students learn mathematics which are not disabilities (Lewis, 2014). It is expected that mathematics teachers should adapt their teaching practices to match the variability in the learning needs of

individual students. Considering the difficulties and variations in learning mathematics, the current study did not rely on a label of "*disability*" in relation to mathematics as a characteristic of learner difficulty. Underlying the goals of this study is that access to learning opportunities may allow students experiencing mathematics learning difficulties (MLD) to study mathematics and find it meaningful. This factor implicates the teaching and learning processes for learners experiencing MLD. Therefore, this study and its arguments are based on the literature about learning difficulties, learning opportunities, support for mathematics education, teachers' knowledge about students experiencing MLD and low achievement in mathematics, and teachers' and learners' beliefs about learning mathematics.

1.1 The problem of the study

Research investigating beliefs in mathematics-related contexts has been extended enormously since the 1990s (e.g., Pehkonen, 2001). The current study extends such research, narrowing the focus to beliefs regarding MLD and factors that cause difficulty in students' learning of upper secondary school mathematics. Furthermore, according to Rubie-Davies, Flint and McDonald (2012), studying those beliefs could be important for students' understanding and learning, as well as for increasing diversity and inclusion in mathematics education. Although numerous studies have been conducted on mathematics-related beliefs and significant findings have been obtained, literature indicates that little is known concerning beliefs about MLD.

This study is situated in the African country of Namibia. Similar to the limited research on mathematics beliefs for learners experiencing MLD, Namibia has gained scant attention from academic communities researching mathematics education. Most research pertaining to mathematics and Namibia focuses on the problems of poor performance of learners and low achievement as well as teachers' and learners' perceptions and attitudes and teacher training in mathematics education. The limited research conducted in this area of study in Namibia has created the need to investigate the beliefs about MLD in detail in order to provide research-based knowledge for improving students' opportunities for the learning of mathematics and inform policies and practices, primarily in the Namibian context where this study was conducted. Additionally, several lessons can be learned from the Namibian education system and the teaching and learning of mathematics within it, especially from the reform of the system after Namibia gained its political independence. These perspectives constitute the underlying motivation for this study.

It is essential to understand what people believe (Cooney, Shealy, & Arvold, 1998). Thus, the purpose of this study was to understand what teachers, pre-service teachers, and students believed regarding MLD. In this research, I did not observe

the participants, I did not talk to policy makers, nor did I know the home situations of the students. Rather, based on the study findings, I tried to gain an understanding of beliefs about MLD in relation to systemic factors, and to teaching and learning. I did not only explore and document the participants' beliefs, but based on those beliefs, I tried to examine what was happening in the classrooms, in teacher education, and in the education system, so that I could better understand the beliefs about MLD. Thus, in Chapter 6, I explored the beliefs about MLD based on the findings, drew conclusions, (in relation to the literature), and suggested possible further research and how the findings could be used to influence teaching practices, education policies, learning habits, and teacher education. At the same time, I tried to understand the participants' beliefs, building on Muis's (2004) definitions, which I then explained later in the literature chapter. Hence, using a phenomenological approach, four studies were conducted, each exploring MLD from a slightly different perspective. The beliefs of Namibian mathematics educators (namely teachers, pre-service teachers and teacher educators) and students were considered in this study. In order to support students experiencing MLD, teachers must understand beliefs about MLD and, subsequently, must implement effective instructional strategies that support the learning of students experiencing MLD. Although the research lacks supporting evidence from classroom practice, the findings from this study regarding the beliefs about MLD support the change for classroom practice, future practice, and mathematics curriculum reform.

1.2 Namibian mathematics education: Reform and challenges

Namibia is an African country with a population of 2.5 million. According to the fifteenth school day report of 2022, the total number of learners and teachers nationally is 839 579 and 31 421 respectively (Ministry of Education Art and Culture, 2022). Annually, the learners and teachers' populations in grades 0–12 are approximately 800 000 and 30 000, respectively. Although this study was conducted on a smaller population, it reached an important portion of the potential participants, therefore this could make a meaningful contribution to the Namibian mathematics education.

Even if countries may have different education systems, their classroom practices may be similar. The Namibian mathematics education system has a complicated history involving unequal outcomes for Black and White learners (Ottevanger, Macfarlane, & Clegg, 2005). These inequalities have included an overall lack of access to mathematics education for Black learners (Alausa, 2000; Ottevanger et al., 2005). In the past, teachers in Namibia have been regarded

culturally as experts in their subject, and learners have been thought of as empty vessels passively participating in the learning process (Iipinge & Kasanda, 2013).

After a lengthy national liberation struggle against Germany and then South Africa, Namibia gained its independence in 1990. The Namibian government declared education to be the top priority in the country. Thus, education has received the largest share of the national budget since the country's political independence (Makuwa, 2005). The Ministry of Education has subsequently reviewed its education curriculum and policies to create a better education system, and has made changes such as switching the medium of instruction from Afrikaans to mostly English. Specifically, in primary schools from pre-primary to Grade 3, the recommended medium of instruction is the mother tongue of the students. Only from Grade 4 does English become the main medium of instruction.

Additionally, after Namibian independence in 1990 there was a shift from traditional classroom practice to a learner-centred approach. This enabled learners to become active participants, responsible for their own learning (Ministry of Basic Education, 1993). However, in practice, very little learner-centred education takes place because of challenges facing the education sector. In some schools, there is a large number of learners in classes and a shortage of teaching and learning materials.

Furthermore, compulsory, free access to quality, inclusive schooling until the age of 18 has been introduced. Access to free education eases the burden of families by covering the costs of books and equipment. Such educational reform is historic. In Namibia, the rights of each child are protected, including children with learning difficulties and disabilities. Schools are required to organise special learning programmes to meet the needs of individual learners when their needs cannot be met in a subject-organised approach, that is, additional in-class help or after-school help is offered (Ministry of Education, 2010). However, the inclusive education policies do not specifically mention MLD, and thus no clear definition or direction have been provided.

Notably, the government and its educational institutions have striven to equalise access to mathematics education. Subsequently, all students must take mathematics as part of their school curriculum in K–12, which was not the case in pre-colonial mathematics education. The curriculum includes a provision for Namibian high school students to study the core component of the mathematics syllabus or study the extended or higher component of the syllabus. The core level of the mathematics syllabus ensures that every student can become numerate. The extended and higher levels focus on pure mathematics and abstractions.

Namibia has developed policies to support its mathematics teachers' practices in improving their mathematical knowledge and pedagogies for better teaching. Numerous continuous professional development programmes have been implemented, and many of the mathematics teachers have attended these

programmes. Such programmes aim to contribute to the effective teaching and learning of mathematics (Kasanda, 2015). They include, among others, In-service Training and Assistance to Namibian Teachers, Mathematics and Science Teachers' Extension Project and Basic Education Support. As a result, when compared to other African countries, Namibia presents the relative proportional growth in mathematics learning (UNESCO, 2014). Although Namibia performs poorly in mathematics, according to all Southern and Eastern Africa Consortium for Monitoring Educational Quality reports, there has been an incremental improvement when compared to other countries in Africa (UNESCO, 2014).

In addition to government efforts and educational reforms as far as mathematics teaching and learning is concerned, many challenges still affect the Namibian education sector, specifically teaching and learning. These challenges include overpopulation in classrooms and the poor mathematics skills of students due to a lack of appropriate educational materials. Furthermore, there are vast differences in the quality of mathematics teaching and learning between private and public schools. Mathematics teaching and learning support materials are inadequate, especially in rural schools (Makuwa, 2005). These limitations can affect mathematics education negatively and may even result in some Namibian students experiencing difficulties in learning mathematics.

It is worth noting that this study, carried out in Namibia, is based on a phased-out high school mathematics curriculum which is a two-year programme targeting Grades 11 and 12. A new programme, Namibia Senior Secondary Certificate Advanced Subsidiary, was implemented in 2020. In the old programme, students were required to pass their mathematics courses before they entered Grade 10. They could continue their education even if they failed Grade 10 mathematics. The Namibian curriculum guidelines stated that students had to obtain a score of 23 out of a possible 42 in the six highest scoring subjects out of nine core subjects, including English, to progress to Grade 11. Subsequently, a student could have proceeded from Grade 10 to Grade 11 without meeting many or any of the mathematics requirements in the previous grade. However, building on prior knowledge is embedded in the Namibian high school curriculum (Ministry of Education, Sport, Arts and Culture, 2015). The curriculum emphasises adding value to the knowledge students have already acquired, which is critically important in learning and developing in mathematics. However, this aspect is somewhat contrary to what is occurring in practice since, based on the phased-out curriculum for instance, mathematics was not required to be among the best six subjects in order to proceed to Grade 11. Thus, mathematical difficulties would often compound as Namibian students moved to higher grade levels (Nambira et al., 2009). When students commence with Grade 11, they are expected to have problem-solving skills and knowledge of a wide range of mathematical concepts and basic statistics to advance in mathematics. Although

every Namibian student is required to have mathematical literacy, satisfying this requirement is a challenge for students with poor mathematical abilities or low achievement in mathematics.

The results of continuous assessment in mathematics in Grades 11 and 12 indicate whether a student should take the final examination in Grade 12 at the core, extended or higher level. Moreover, the national examinations that include mathematics are taken at the end of Grades 10 and 12, and they serve as gatekeepers for entry into post-secondary education. To gain admission to the teacher education programme, prospective mathematics teachers must earn 25 points, which are gathered from the best five Grade 12 subjects on the University of Namibia's evaluation scale. They must, furthermore, have reached a good level of achievement in mathematics (at least a C), and have an acceptable pass in Grade 12 English (at least a C). The training programme is a four-year Bachelor Honours degree programme focusing on connecting content and pedagogical knowledge. In the programme, pre-service teachers are introduced to the theoretical and practical aspects of the teaching profession. The programme addresses a range of mathematics teaching methods because pre-service teachers will meet students with different mathematics learning needs. Teacher educators and teachers from practicum schools observe and evaluate the teaching practices of pre-service teachers in three phases.

1.3 Research aims

This study documents beliefs about MLD from the standpoint of both high school students and mathematics educators, namely teachers, pre-service teachers and teacher educators, to provide research-based knowledge. The study aims to contribute to the ongoing international debate on mathematics-related beliefs. The study also contributes to the development of research-based mathematics education in the African context. Furthermore, it serves as a lens for understanding Namibian students and teachers' beliefs about MLD to enhance the mathematics learning opportunities for students experiencing MLD. A summary of the research questions, aims and knowledge gaps is illustrated in Table 1. The beliefs and knowledge that were revealed in this study can influence teachers' practices, the pre-service preparation of teachers and curriculum development. This study can also provide teachers and teacher educators with opportunities to reflect on their teaching practices in order to improve students' mathematics education. It may also contribute to theories that support the learning of students experiencing MLD, especially in Grades 11 and 12, when students are approaching their mathematics exit examinations and high school graduation.

This study is important as it acknowledges the voices of Namibians students and teachers about mathematics education through a qualitative approach. The approach

involved the interpretation of data gathered from semi-structured interviews and a survey. In support of this choice, Hannula (2011, p. 50) states that "researchers who emphasise the individual nature of affect tend to rely on psychological theories and in-depth interviews or surveys" as the approaches evaluate the origins of affect. Thus, the findings provide a better understanding of the concepts from participants' perspectives, the beliefs and experiences of students who are observed to experience MLD and the way in which mathematics educators view MLD.

The following research aims constitute the basis of this study:

1. Examine the beliefs about MLD (Studies I, II, III, and IV).
2. Investigate the beliefs about the practices which may contribute to MLD in Namibian high school mathematics classes (Studies II, III, and IV).
3. Understand beliefs about mathematics teaching and learning experiences that would support students experiencing MLD in the Namibian context (Studies II and IV).
4. Document the beliefs about MLD that had developed by the end of the teacher education programme, the practices related to MLD that might be emphasised in the programme and the possible changes in these beliefs during a teacher's first year of teaching (Study IV).

As previously stated, the findings are published in four studies in which the participants shared their beliefs about MLD; furthermore, the requisites for addressing and reducing MLD to improve learning opportunities were explored. Through a survey, Study I investigated primary and secondary school mathematics teachers' beliefs about the concept, MLD, as a reflection of their teaching practices (they taught 7–13-year-olds and 14–19-year-olds, respectively). In Studies II, III, and IV data were collected through semi-structured interviews. Study II examines the beliefs of Grade 11 students experiencing MLD. Study III focuses on a comparison of the perspectives of high school students and their teachers regarding MLD. Study IV documents the beliefs about MLD developed by the end of the teacher education programme, and possible changes in these beliefs that occurred during teachers' first year of teaching.

In these studies, self-reported data were employed. A survey and semi-structured interviews were conducted as they were the most compelling data colletion methods.

Table 1. Summary of the research questions, aims and gaps.

Study and Title	Research questions	Aim	Research gaps
Study I K–12 Namibian teachers' beliefs on learning difficulties in mathematics: Reflections on teachers' practices	1. What are Namibian mathematics teachers' beliefs about MLD? 2. What challenges do mathematics teachers experience when teaching students experiencing MLD in an inclusive classroom, and what potential measures do they employ?	To explore primary and secondary school mathematics teachers' beliefs associated with learning difficulties and their perceptions of the problems leading to MLD	Little is known about beliefs about MLD in the Namibian context.
Study II Believing in your own abilities: What Namibian high school students experiencing mathematical difficulties can teach us	1. What are the beliefs about teaching and learning mathematics of Namibian students experiencing MLD? 2. How do those beliefs about teaching and learning interact with their beliefs of support that would enable their mathematics learning?	To understand the beliefs and experiences of learners experiencing MLD	(i) Insufficient knowledge about beliefs of students experiencing MLD (ii) Lack of research about students' beliefs in Namibian mathematics classes
Study III A comparison of the perceived difficulties in learning high school mathematics from the perspective of teachers and students	1. What descriptions do teachers employ when identifying students who have MLD? 2. What are the views of students who are identified as having MLD? 3. What is the nature of the relationship between teachers and their students' views in classifying students experiencing MLD?	To examine descriptions of identifying students experiencing MLD	(i) Insufficient knowledge about teachers' knowledge and beliefs about students experiencing MLD and their characteristics (ii) Lack of research about students' beliefs about MLD in mathematics classes
Study IV "It does not mean that they cannot do mathematics": Beliefs about Mathematics Learning Difficulties	1. What are pre-service secondary mathematics teachers' and teacher educators' beliefs about MLD at the high school level? 2. To what extent do pre-service high school mathematics teachers' beliefs about MLD change in their first year of teaching?	To investigate beliefs about MLD and document the beliefs developed by the end of the teacher education programme, the views and practices that might be emphasised in the programme and possible changes during the first year of teaching	(i) Insufficient knowledge of pre-service teachers' beliefs and teacher educators' beliefs about MLD and beliefs developed at the end of the teacher education programme (ii) Lack of knowledge of possible changes in beliefs about MLD during the first year of teaching (iii) Lack of knowledge about views and practices that are emphasised in the teacher education programme concerning MLD

The literature of the previous studies that have led me towards the above research objectives are presented in Section 2.

This dissertation presents beliefs about MLD based on four studies:

Study I. Hamukwaya, S. T. (2019). K–12 Namibian teachers' beliefs on learning difficulties in mathematics: Reflections on teachers' practice. In U. T. Jankvist, M. Van den Heuvel-Panhuizen & M. Veldhuis (Eds.). *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 4652–4659). Utrecht, the Netherlands: Freudenthal Group & Freudenthal, Institute, Utrecht University and ERME. https://hal.science/hal-02431483

Study II. Hamukwaya, S. T. & Ruttenberg-Rozen, R (2023). Believing in your own abilities: What Namibian high school students experiencing mathematical difficulties can teach us. *Canadian Journal of Science, Mathematics, and Technology Education*, 1-19. https://doi.org/10.1007/s42330-023-00260-7

Study III. Hamukwaya, S. T. (2022) A comparison of the perceived difficulties in learning high school mathematics from the perspective of the teachers and the students. *International Journal for Mathematics Teaching and Learning*, *23*(1), 46-63. https://doi.org/10.4256/ijmtl.v23i1.317

Study IV. Hamukwaya, S. T. & Haser, Ç. (2021). "It does not mean that they cannot do mathematics": Beliefs about mathematics learning difficulties. *International Electronic Journal of Mathematics Education*, *16*(1), em9569. https://doi.org/10.29333/iejme/9569

The first study investigated mathematics teachers' beliefs about MLD regarding their own teaching practices. The second study explored students' beliefs about teaching and learning practices that could contribute to their experiences of MLD. The third study compared the perceived difficulties in learning upper secondary school mathematics from the standpoints of teachers and their students. The final study investigated pre-service teachers' beliefs about MLD that were developed by the end of their teacher education programme, and changes in these beliefs as the pre-service teachers entered their first year of teaching. These four studies contribute to the current debate regarding beliefs in mathematics education and provide readers with a picture of beliefs about MLD within a Namibian context. The studies also suggest areas of need within the Namibian mathematics education system vis-á-vis educating future teachers to support learners experiencing MLD. Furthermore, the beliefs presented could be considered for effective professional, continuous development programmes to support the practices of mathematics teachers. The study also documents the beliefs of a relatively under-represented group in mathematics education, namely students experiencing MLD.

Studies indicate that MLD is a common problem throughout all grades of comprehensive school (Miranda, Amadhila, Dengeinge, & Shikongo, 2007). During my fifteen-year career as a mathematics teacher in Namibia, both at primary and secondary levels, I recognised the important role and responsibilities of mathematics teachers in supporting students who experience MLD, since teachers have the potential to develop their students' mathematics learning and improve their mathematical skills. I also noticed that the difficulties in learning mathematics at higher grade levels seemed to be more complex than those experienced at the lower grade levels. As Nambira, Kapenda, Tjipueja and Sichombe (2009) report in their Namibian study, learners' difficulties in mathematics increased as they progressed to higher grade levels. Similarly, a Nigerian study found that many secondary students experienced difficulties in mathematics and performed poorly in the subject (Tella, 2007). It became important for me to explore the impacts of and beliefs about difficulties in learning mathematics in upper secondary school.

These factors underpinned my intentions, research questions and motivation in conducting this study. Therefore, this research also reflects my personal experiences in teaching mathematics in relation to MLD as an aspect of mathematics-related beliefs.

1.4 Definition of the key terms

While writing this doctoral dissertation, I developed new perspectives and understanding about the use and meaning of some concepts I had used in the four studies (I, II, III, and IV). For example, (1) I critically looked at students *with* MLD. The word *with* labels the learner and conveys the existence of a *diagnosis*. I felt that using *with* could be construed as researcher bias. Therefore, in this dissertation, I changed *with MLD* to *experiencing MLD*. (2) In the first studies, I had used the term *cognitive disabilities*; however, as my thinking developed, I had concerns about the use of this term. *Disability* has been demonstrated by other researchers to be a strong and sensitive word to use. Students may not solve basic mathematics tasks for several reasons, none of which may be related to a cognitive disability. (3) Also, in the studies, I had used the two terms *mathematics learning difficulty* and *difficulty in learning mathematics* interchangeably without giving an explanation or reason—there was no consistency. (4) Moreover, I reviewed my usage of the terms *negative beliefs* and *positive beliefs* in the studies. I realised that these terms are very subjective because one belief may be considered negative in one educational aspect but positive in another. Thus, in this dissertation, I adopted terms used by Muis (2004) when she talks about *availing beliefs* and *non-availing beliefs* as I described them later below and when I explained the key terminologies used in this study. However, where appropriate, I used positive or negative terminology if the research

I cited used these distinctions. (5) Furthermore, the other important terminology that I use in the studies is emotional disposition.

Therefore, I would like to clarify the key concepts utilised in this dissertation as they are essential and support readers' understanding and appreciation of the subject matter. These terms are defined in Table 1 below.

Table 2. Key terms of the study.

Key terms	Definition and other additional information
Belief (in general)	In this study, *belief* is considered as an individual construct based on personal knowledge, experience and thinking (Kaasila, 2007; Kuhn & Weinstock, 2002; Martin, 2000; McLeod, 1992; Nespor, 1987; Roesken, et al., 2011). Beliefs are important in the teaching and learning process; they impact the desire to learn, achieve and participate, as well as a person's career path (Markovits & Forgasz, 2017). Furthermore, they influence the success and/or failure of the individual (Goldin, Rösken, & Törner, 2009).
Availing and non-availing beliefs	In this study, I situated the two terminologies availing and non-availing beliefs in relation to MLD – for students and teachers, building on the previous literature (Muis, 2004; Haser & Işler, 2017; Alpaslan et al., 2014). For example: For student A, an availing belief could be "even though mathematics is difficult, I can still study and understand mathematics." But for student B, a non-availing belief could be "I cannot make sense of mathematics, I am not good at it, even if I work hard, I won't be able to do it." This belief is not helping students to learn mathematics or to make an effort to study the subject. However, an availing belief helps the students be perseverant to overcome certain difficulties and learn mathematics. For teacher A, an availing belief could be "if I can identify the difficulties that my students have, then I can help them to make sense of mathematics better." This belief helps the teacher to organise his/her teaching practices or to change his/her practices or to better understand the students and organise his/her teaching accordingly. A non-availing belief for teacher B in the case of MLD could be "these students do not know anything, and there is nothing much I can do about it." This type of belief is not helping the teacher address MLD or the student to learn mathematics better.
MLD	The construct of MLD refers to any difficulty that may hinder students' mathematical learning, which is a result of insufficient opportunities to learn mathematics, the lack of adequate prior mathematics knowledge and teaching practices (Chinn, 2015), ineffective instruction (Carnine et al., 1997) and systemic factors.
Beliefs regarding MLD	These are specific types of beliefs regarding teaching and learning practices that are considered as reasons of experiencing MLD. They also denote the links to beliefs concerning the possible reasons for MLD and the possible means of overcoming it (Beswick, 2007/2008; Hassan, 2014).
Students experiencing MLD	This study considers students experiencing MLD to be individuals who are struggling to grasp the basic knowledge and skills of mathematics that are required to comprehend the subject effectively (Jitendra et al., 2013). These students often do not perform as expected. They also experience difficulties with mathematical concepts and procedures; however, if they receive learning support, they can improve their understanding of mathematical concepts. In certain studies, such as that of Glynis (2013), MLD has been considered to be disability/difficulty in acquiring essential skills in performing mathematical procedures. However, that is not the stance that the current study takes as it does not automatically consider low achieving students to be students experiencing MLD. The rationale for this is that students might be unable to solve basic mathematics tasks for a variety of reasons.

Key terms	Definition and other additional information
Perceptions	In this study, the term perception refers to how MLD is viewed, interpreted, or understood as a result of experiences such as education and knowledge (Mutodi & Ngirande, 2014). In the context of this study, these experiences influence the way educators or students perceive MLD. For example, a student may have a positive perception towards mathematics as it is important in his/her daily life, even though he/she views the subject as difficult to learn.
Emotional Disposition	In this study, disposition refers to an interplay between the students' learning background and foreground (Alrø, 2009). The students' disposition is how they position themselves when learning mathematics based on their previous learning experiences. Based on students' prior experiences, some may experience *emotional* dispositions towards mathematics such as anxiety, shyness, fear, or panic on which they may base their lack of success or understanding (Colomeischi & Colomeischi, 2015; Di Martino & Zan, 2011). Such students may find reasons to disengage in mathematics learning because of their background (Alrø, 2009), which may prevent them from learning mathematics (Geisler & Rolka, 2021; Roesken et al., 2011).

1.5 Structure of the dissertation

This dissertation contributes to studies conducted in mathematics education and extends the current knowledge base regarding mathematics education in the Namibian context. It comprises six chapters, including the synthesis of related studies.

This first chapter discusses the scope of the study and the context of Namibian mathematics education. In the second chapter, the literature that underpins this study is discussed. This literature review presents studies conducted by other researchers and identifies the research gap regarding beliefs about MLD. It discusses beliefs about mathematics education and MLD, specifically its causes and characteristics, as well as teachers' knowledge of students and their characteristics. In the third chapter, the aims of the four studies that the dissertation is based on are presented. The fourth chapter discusses the research process, namely the research design and methodology of this study. It also discusses research ethics and the trustworthiness of the study. Following this, in the fifth chapter, a summary of each of the four studies (articles) is presented. Lastly, in the sixth chapter, the study findings and their practical contributions to mathematics education, including the implications of the findings in the Namibian context, are critically discussed. The dissertation concludes with suggestions for future research based on the limitations of the study.

2 Literature Review

The literature review provides a firm foundation for the study and makes a case for the investigated problem. This chapter provides the essential basis for a better understanding of the context that frames this study and determines the extent to which the study findings are relevant to other educational contexts. It presents studies conducted by other researchers and identifies a research gap regarding beliefs about MLD. Therefore, this chapter discusses the clarification of the nature of the beliefs construct, followed by a discussion on beliefs about mathematics education and MLD, and lastly, a scrutinisation of the relationship between teachers' beliefs and their knowledge about students experiencing MLD and their characteristics.

2.1 Clarification of the nature of the beliefs construct

To clarify the nature of the beliefs construct, I will, firstly, review research in mathematics education and discuss the ways in which beliefs differ from and relate to one another. In mathematics education research, attitudes, beliefs, emotions, and values are the main concepts used in the literature on mathematics-related affect to understand individual thinking regarding teaching and learning. The work of scholars such as McLeod (1992) and Hannula (e.g., 2004, 2014) contributes significantly to understanding these concepts. These scholars discuss the role of the concepts in mathematics teaching and learning from different perspectives. They consider how the concepts differ and relate to one another and what role and influence the concepts have in mathematics education. According to the literature, these concepts cannot be understood in isolation, as they are intertwined (Hannula, 2004). Although I am specifically interested in *beliefs*, it is clear that it is necessary for the reader to also understand the other affects in mathematics education and how they interact. Thus, in this thesis, I focus on *beliefs* and briefly discuss the concepts of *emotion* and *motivation*.

There is currently no agreed-upon definition of *beliefs* in general, nor is there in mathematics education specifically, which has been indicated by scholars in the previous decades (Beswick, 2012; Ernest, 1989; Furinghetti & Pehkonen, 2002; Goldin et al., 2009; Schoen & LaVenia, 2019). In this field, researchers have

employed different definitions, depending on the contexts of their studies. Some of the definitions found in the literature are related to experience (Bendixen, 2002; Pajares, 1992), attitude, opinion, disposition, perception, philosophy, value (Leder & Forgasz, 2002), and emotions (McLeod, 1992). For example, a belief is a piece of knowledge to which a person attributes truth; beliefs are highly cognitive, may be highly constructed (Goldin, 2002), and they are usually stable (Goldin, 2002; Hannula, 2004).

Moreover, beliefs vary in strength where some beliefs are more resistant to change (Green, 1971). For instance, the cultural belief that 'maths is an innate ability' is considered to be more resistant to change than beliefs formed after attempts at some experience such as 'I am not a maths person' (Brown, 2004; Gabillon, 2012). However, since beliefs are personal constructs, the likelihood of change depends on how strong the individual's beliefs are (Hannula, 2011; Calderhead, 1995; Rott, 2020). They cannot be observed or measured directly; rather, they are reflected in what people express, intend, and act (Pajares, 1992).

People generally hold beliefs that are attributed to previous individual experiences (Bendixen, 2002; Haser & Star, 2009; Leatham, 2006). Thus, social context, cultural influences, and individual past experiences or events are considered primary factors in shaping beliefs (McLeod, 1992). For example, if a teacher concludes that Student A has MLD or is not good at mathematics, the teacher is unlikely to believe that Student A will perform well, according to Hannula, Evans, Phillippou and Zan (2004). Moreover, Muis (2004) talks about availing and non-availing beliefs for students. She defined availing beliefs as those beliefs that may contribute to the process of learning and are associated with better learning outcomes. They are positively related to quality learning and achievement; in other words, they advance learning. In contrast, non-availing beliefs are believed to negatively influence learning outcomes. A non-availing belief does not affect learning or achievement in a positive way. Besides, some studies that investigated preservice mathematics teachers' mathematics-related beliefs also used availing and non-availing beliefs terms (e.g., Haser & Işler, 2017; Alpaslan et al., 2014). These labels are simply descriptive of relationships, eliminating negative connotations associated with teaching and learning (Muis, 2004). Thus, I considered these terms to better fit this dissertation, although other scholars may talk about beliefs in terms of negative and positive beliefs (e.g., Gafoor, & Kurukkan, 2015; Maasepp, & Bobis, 2015; Uusimaki & Nason, 2004).

Green (1971) talks about beliefs as a system. Within a belief system, beliefs are organised in a cluster around specific situations and contexts (Green, 1971). For instance, learners can have beliefs about mathematics itself or beliefs that are clustered around learning situations, just like teachers can have beliefs about learning mathematics or beliefs organised around teaching mathematics. These clusters may have contradictory beliefs that the individuals may not be aware of. According to

McLeod (1992), a belief system about mathematics education is made up of elements, including beliefs about oneself, the nature of mathematics, teaching mathematics, and learning mathematics. Furthermore, one may have beliefs about a specific situation (e.g., beliefs about or regarding homework, beliefs related to taking a test, using mathematics in everyday situations, or taking exams). According to Hannula et al. (2004), when one refers to the system or the systemic nature of beliefs, one also addresses emotion, motivation, and affect as they influence and are influenced by beliefs. Thus, beliefs are often related to emotions, motivation, and cognition—the three dimensions of mathematics-related affects (see Figure 1).

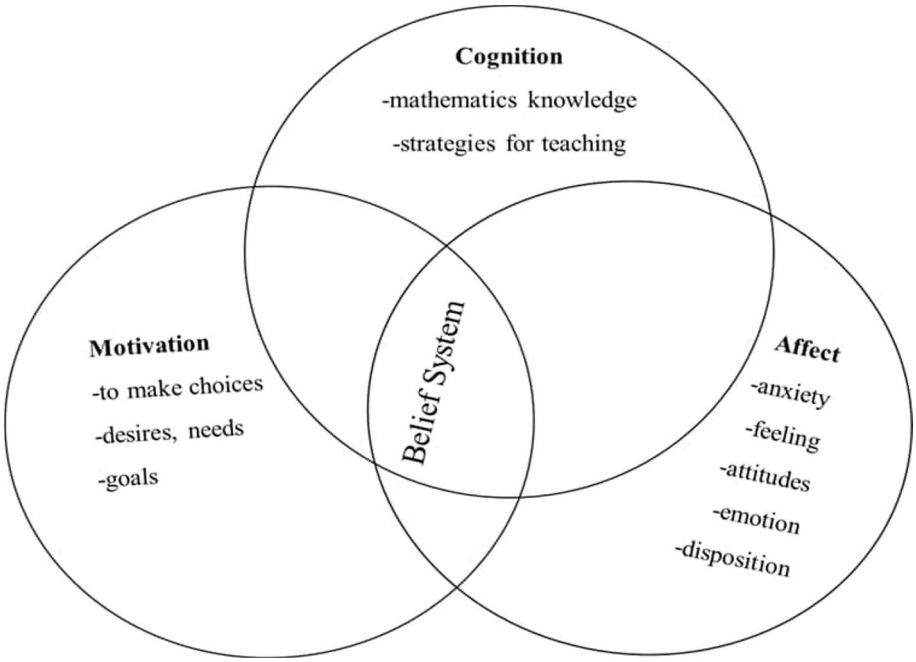

Figure 1. Mathematics-related affects and the relationship between them (adapted from Hannula, 2011; Hannula, Op't Eynde, Schlöglmann, & Wedege et al., 2007).

Figure 1, adapted from Hannula et al., (2007), illustrates the interaction between three elements of mathematics-related affects (motivation, cognition, and affect) in the belief system. The figure shows how the concepts relate to each other and indicates the importance of interaction between beliefs and emotion, beliefs and cognition, and beliefs and affect. It is important to understand the interaction and relationship between the three main elements of mathematics-related affect (motivation, cognition, and affect), and how they differ and relate to one another. According to Hannula (2004), the three elements are the main concepts for the construction of the human mind. For instance, on one hand, basic emotions

(emotional feelings) are understood as being connected to motivation, such as personal goals, and involve physiological reactions (Hannula et al., 2004). On the other hand, emotions are interconnected with cognition (Goldin, 2000).

This thesis uses the term *emotion* (in Study II) to refer to an emotional state. For example, the students may feel a certain (positive or negative) way about experiencing MLD and express this experience in ways that they and their mathematics teachers may or may not be able to observe. These include fear of failure based on past experiences, otherwise known as mathematics anxiety (Aldrup, Klusmann, & Lüdtke, 2020; Namkung, Peng, & Lin, 2019), which can be related to test anxiety and low mathematics achievement (negative affect, unpleasant emotion of fear). This experience is associated with emotion and produces emotional experiences (Hannula, 2004). For example, student A may experience mathematics anxiety which prohibits him/her from studying mathematics. However, student B may not experience anxiety, and this will motivate him/her to learn mathematics, change his/her way of studying, or encourage him/her to ask for help. This example shows that positive and negative emotional beliefs interplay with cognition and influence students' motivation. These associations can be indicators of students' future success and failure.

Furthermore, when students are involved in mathematics learning, they can experience positive or negative emotional reactions (McLeod, 1992; Tuohilampi, 2016). These reactions may involve emotions that regulate the body, behaviour, or social coordination (Schwarz & Skrunik, 2003). Moreover, emotions are affect (e.g., frustrations, anxiety, feelings, and emotional reactions, such as fear and shame) and are considered unstable (McLeod, 1992). In contrast to McLeod, Hannula states that people can have stable patterns for emotional responses across similar situations; repeated emotional reactions may result in certain attitudes (Hannula, 2011). Thus, beliefs influence emotions (Mandler, 1989), and both beliefs and emotions may also influence individual choices, which may influence the teaching and learning of mathematics (Hannula, 2011).

2.2 Beliefs about mathematics education

Beliefs about mathematics education have long been of interest in the teaching and learning of mathematics (Ernest, 1989; McLeod, 1992; Ndlovu & Ngcobo, 2018; Philipp, 2007; Thompson, 1992; Rupnow, 2021; Xie & Cai, 2021; Voss, Kleickmann, Kunter, & Hachfeld, 2013) as beliefs have strong relationships to affective and cognitive processes (McLeod & McLeod, 2002). Figure 2 illustrates the way in which researchers have classified beliefs about mathematics education. For instance, McLeod (1992) identifies aspects of beliefs: beliefs about mathematics as a school subject, self and context. The following sub-chapter discusses these beliefs held by mathematics teachers (including pre-service teachers) and students.

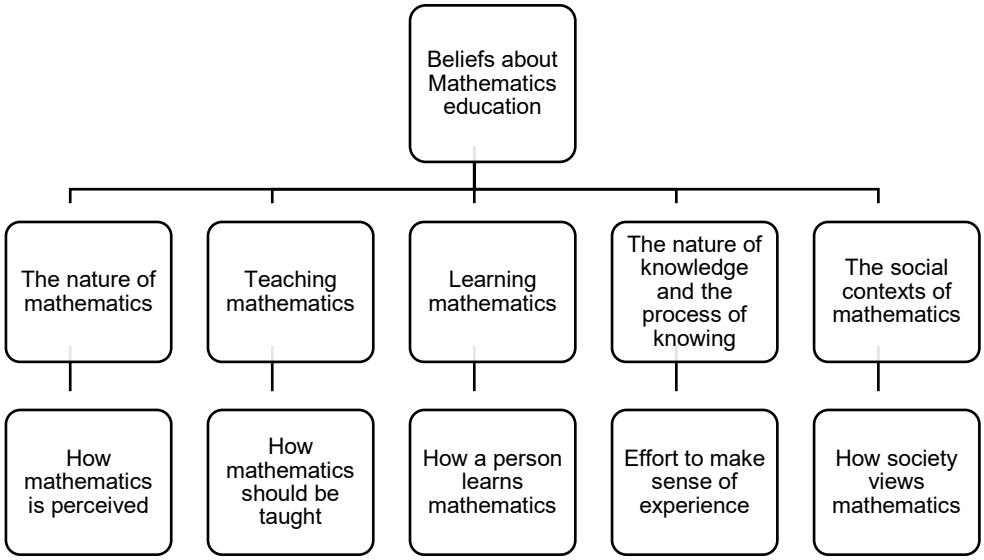

Figure 2. Beliefs about mathematics education.

2.2.1 The beliefs of teachers and pre-service teachers regarding mathematics

According to Pajares (1992), teachers' beliefs vary, depending on the way they associate these beliefs with their work, responsibilities, roles, subject or lesson, and students. Furthermore, Pajares (1992) states that teachers' beliefs depend on their subject knowledge and confidence in task performance. As previously stated, teachers' beliefs are often based on individual experiences, which form the foundation of their teaching practice (Fosnot, 1989; Skott, 2001). Personal experiences, experiences with schooling and instruction, and experiences with formal knowledge (both school subjects and pedagogical knowledge) constitute a base for teachers' beliefs (Richardson, 1996).

Teachers' mathematical beliefs are personal (Brown & Rose, 1995), and they influence mathematics teaching (Cooney et al., 1998; Ernest, 1989) and classroom practices (Philipp, 2007; Schommer-Aikins et al., 2005). These practices reflect their teaching performance (Kagan, 1992) and have a significant impact on students' learning (Stuart & Thurlow, 2000). Thus, teachers' beliefs can encourage or hinder students from recognising the importance of mathematics (Depaepe et al., 2015). Below are some of the mathematics-related beliefs that teachers may have:

- Reaching the correct answers is only possible by memorising, remembering and following procedures and rules or completing a mathematics task (Carter & Norwood, 1997). If students do not follow the

given procedures and rules, it is likely that they will not arrive at the correct answers. This, consequently, affects their performance. This type of belief is an instrumentalist view—mathematics is seen as a set of rules and facts (Thompson, 1984).

- To avoid learning difficulties, students need to know what procedures they can utilise, how to utilise them, and understand why they perform these procedures (Salihu & Räsänen, 2018).
- If students do not understand a specific concept or content area, teachers should repeat that lesson at a slower pace (Carter & Norwood, 1997).
- Good mathematics teaching allows students to understand and master concepts and solve mathematics problems (Ndlovu & Ngcobo, 2018).
- If students fail to learn mathematics, their overall performance will be affected (Kislenko, Grevholm, & Lepik, 2007).

The above beliefs are influenced by other factors, such as curriculum structure, subject content, students' expectations, the national curriculum, teacher training programmes and instruction (Ernest, 1989; Handal & Herrington, 2003). Hence, identifying teachers' beliefs about teaching and learning mathematics can support the re-examination of teacher education programmes and support educational policies (Haser & Star, 2009).

Several scholars have also investigated pre-service teachers' beliefs, which they (pre-service teachers) bring to the teacher education programme from their own early schooling (Maasz & Schlöglmann, 2009; Richardson, 2003) and from their childhood experiences (Stuart & Thurlow, 2000). Such as beliefs about teaching and learning instruction, and subject content (Richardson, 2003). For instance, many pre-service teachers come into the teacher education programme believing that teacher-centred teaching approach facilitates the student's learning process. Whereas some pre-service teachers believe there is less to learn in the teacher training programme compared to when doing practice (Richardson, 2003). However, as pre-service teachers engage in learning how to teach during the programme they change their prior perception to learner-centred approach, placing students at the centre of their learning process (Moos & Ringdal, 2012). With reference to students experiencing MLD, such belief shift might have implications, for instance, on the way pre-service teachers perceive how these students learn with understanding.

According to Haciomeroglu (2013), pre-service teachers' beliefs about teaching and learning mathematics usually align with the beliefs promoted in the teacher education programme. In the programme, pre-service teachers' teaching practices become anchored in the way students learn and how student learning can be maximised (Stuart & Thurlow, 2000). They become conscious of the effects of their

beliefs on their classroom practices (Stuart & Thurlow, 2000). These notions are deeply rooted in pre-service teachers' belief systems, before they ever enter the teacher education programme (Handal, 2003). These arguments seem to indicate that the beliefs that the pre-service teachers bring with to the teacher education were not in line with the ones emphasised in the teacher education only that they were not aware of those beliefs.

2.2.2 Students' mathematical beliefs

Students' beliefs reflect their perception of the learning process (Buehl, 2003) and influence their engagement in the learning of mathematics (Markovits & Forgasz, 2017), impact their interest and enjoyment of the subject (Kloosterman, 2002), influence their mathematics learning (Maasz & Schlöglmann, 2009) and contribute to their academic achievement (Geisler & Rolka, 2021). According to Kloosterman (2002), students' beliefs determine their academic choices and the effort they put into learning mathematics. Early experiences in mathematics (Schommer-Aikins, 2002) are crucial in shaping students' beliefs and learning outcomes; this includes the way teachers teach (Black, Williams, Choudry, Pickard-Smith, & Ryan, 2018). Students' perceptions and beliefs about mathematics depend on what they do and what their teacher does in the classroom (Carter & Norwood, 1997). Students hold beliefs about what mathematics comprises, what it means to know and perform mathematics, and who they are as mathematics students—they relate their beliefs to their mathematics identities (NCTM, 1989; Ndlovu & Ngcobo, 2018). These beliefs are developed during the teaching and learning process, and they influence the students' construction of knowledge and their academic performance (Steele & Ambady, 2006). This indicates that students' prime source of mathematical experience is the classroom, and what occurs in mathematics classrooms influences their beliefs (Relich, 1995).

Students' learning outcomes are strongly linked to their beliefs about learning mathematics; their beliefs are also associated with their academic performance in mathematics (Furinghetti & Pehkonen, 2002; Kislenko et al., 2007; McLeod, 1994; Mensah, Okyere, & Kuranchie, 2013; Schloglmann, 2009). The more positive these beliefs are, the better the outcomes (Mensah et al., 2013). Beliefs about MLD are outlined in the literature as beliefs about mathematics as a subject, oneself in relation to mathematics, the social context, teaching mathematics, and learning mathematics. These beliefs influence students' mathematical learning significantly (De Corte et al., 2002; Schommer-Aikins et al., 2005). Below are the common categories of students' mathematics related beliefs found in the literature:

i. Beliefs about mathematics as a subject refer to the way in which students perceive mathematics (Op't Eynde, De Corte, & Verschaffel, 2002). Some students may find mathematics easy, while most believe that it is difficult,

and some perceive that mathematics is founded on rules and that only bright students can succeed in the subject (Ndlovu & Ngcobo, 2018).

ii. Beliefs about oneself in relation to mathematics describe the way students perceive mathematics, their self-efficacy (confidence, and determination) in solving mathematical problems and their understanding of success or failure in learning mathematics (Carter & Norwood, 1997; Kislenko et al., 2007; Ndlovu & Ngcobo, 2018; Op't Eynde et al., 2002).

iii. Beliefs about social norms in learning mathematics refer to beliefs about teacher and student roles and responsibilities in a mathematics class (Op't Eynde et al., 2002).

iv. Beliefs about teaching mathematics include beliefs about the way in which mathematics should be taught (Carter & Norwood, 1997).

v. Beliefs about learning mathematics explain the ways in which students make sense of learning mathematics. These beliefs impact students' interest and motivation in learning mathematics (Kele & Sharma, 2014; Kislenko et al., 2007).

Many studies have been conducted on beliefs in mathematics education; however, research on beliefs about students experiencing MLD and teachers' beliefs (including pre-service teachers and teacher educators) about MLD is limited. Therefore, this dissertation focuses on the beliefs of teachers and students experiencing MLD.

2.3 Beliefs about mathematics learning difficulties

There are many studies specifically on beliefs about mathematics, and on learning difficulties. However, in the literature, few studies on beliefs about MLD exist, despite learners' different educational needs. In teaching mathematics, teachers employ their experience and knowledge to teach and observe students with and without MLD. These observations can influence beliefs regarding the reasons students experience MLD, students' characteristics, and what strategies may best support their learning in an inclusive mathematics classroom.

Researchers define MLD based on educational contexts and research traditions, such as mathematics education, special education, pedagogy and psychology (Scherer et al., 2016). In mathematics education, MLD has been approached from various theoretical perspectives, such as genetic, cognitive, neuro-scientific and pedagogical (Räsänen et al., 2019). Researchers have discussed disabilities associated with learning mathematics, mathematical difficulties, developmental

dyscalculia, and the effect of these on students' mathematical performance. For instance, MLD refers to barriers that hinder cognitive processes in learning mathematics (Mazzocco, 2007), which negatively affects students' progress in mathematics (Jones, Wilson, & Bhojwani, 1997). Morgan, Farka, and Wu (2009) relate MLD to low mathematical proficiency and low numerical processing skills. Similarly, Karagiannakis, Baccaglini-Frank, and Papadatos (2014) associate MLD with obstacles to, and difficulties in, processing numbers. Moreover, the deficits in mathematical skills can make learning or comprehending arithmetic relationships difficult (Kaufmann & von Aster, 2012).

According to Mtetwa and Garofalo (1989), every mathematics teacher can name different reasons for why students experience MLD. When discussing beliefs about mathematics and MLD, Mtetwa and Garofalo mention healthy beliefs and methods that teachers can employ to address students' unhealthy beliefs (these are non-availing beliefs according to Muis (2004)) about learning mathematics. They state that the underlying causes of students' MLD include their lack of understanding of mathematical concepts, as well as difficulties in employing the correct procedures when solving tasks. Conversely, their study reveals that students' MLD could be attributed to their beliefs about mathematics as a school subject and mathematics tasks; these beliefs influence students' mathematics task performance and learning. They suggest appropriate mathematics instructional activities that engage students. In their conclusion, they emphasise that students' beliefs about the nature of the mathematics tasks play a significant role in their success or failure in completing mathematics tasks.

On the other hand, teachers' beliefs also contribute to their teaching success or failure (Mtetwa & Garofalo, 1989). Beswick (2007/2008) studied teachers' beliefs about appropriate methods for teaching numeracy to students experiencing MLD. The participants were teachers in a professional learning programme that aimed to improve their practical teaching strategies. The goal of the study was to influence teachers' beliefs about inclusive mathematics teaching. The questionnaire results reveal that teachers held different beliefs about students with and without MLD. The findings proved that teachers' beliefs about mathematics, mathematics teaching and students experiencing MLD were difficult to change. The results suggest that professional learning programmes might reduce these belief differences (Beswick, 2007/2008). Furthermore, Soodak and Podell (1994) questioned teachers' thinking regarding their difficulties when teaching students and meeting students' needs. They also investigated teachers' beliefs about the causes of students' difficulties and what they believed to be effective strategies for students experiencing MLD. According to the case studies, teachers' beliefs about MLD were found to be related to the strategies they offered to students. At the same time, some teachers attributed students' MLD to home environments rather than the strategies offered. In another study, Hassan (2014) investigated mathematics teachers' beliefs about teaching students experiencing MLD.

The results indicate that teachers employed various teaching methods, such as story-telling, brainstorming and task analysis, to remediate learning difficulties and account for student diversity. Participants believed that there is no one single teaching method that is appropriate for all students in all situations (Hassan, 2014).

The following sub-chapter elaborate further on the causes of MLD and the characteristics of students experiencing MLD.

2.3.1 Causes of MLD

According to Miles and Forcht (1995), MLD can become a problem starting in primary school. It is difficult for students to learn mathematics if they do not have an adequate subject background (Grossman & Stodolsky, 1995). Consequently, students with a poor background in mathematics often experience difficulty understanding high school mathematics and believe that the course content is advanced (Miles & Forcht, 1995; Rivera & Bryant, 1992). Other factors, such as students' intellectual abilities, mathematics anxiety, personal beliefs, subject content, teachers' practices and the learning environment contribute to MLD. Table 3 outlines the categories of factors from the literature that can hinder students' mathematics learning and that scholars believe to contribute to MLD. One category is Social and includes, for example, biases and peer pressure; another is Academic which includes, for example, students' fear based on negative past experiences and issues with instruction and materials.

Table 3. Factors believed by students and/or teachers to contribute to students' MLD.

Categories	Factors	Literature connection
Academic	Inadequate mathematics learning foundation and teachers' incompetency	Montague, 1992; Siyepu, 2013; Voigts, 1998
	Inappropriate teaching and learning materials and low teaching quality	Siyepu, 2013
	An unconducive learning environment and insufficient teaching materials	Mundia, 2010; Sarsani & Maddini, 2009
	Insufficient teaching time and students' low self-esteem	Montague, 1992
Social	Peer pressure, fear resulting from negative past experiences, mathematics avoidance and the belief that mathematics is a difficult subject	Bekdemir, 2010

2.3.2 Characteristics of students experiencing MLD

Studies have investigated different characteristics of students experiencing MLD that explain both procedural (lack of arithmetic skills and difficulty recalling arithmetic facts) and conceptual (inability to build connections among mathematics concepts) difficulties. Jones et al. (1997) state that students experiencing MLD make inadequate progress in mathematics. They struggle in mathematics and display slower progress in learning (Miles & Forcht, 1995; National Council of Teachers of Mathematics, 2007). Students experiencing MLD usually believe that mathematics is difficult; they are not confident and are uninterested in pursuing scientific subjects at higher education institutions (Zakaria, Chin, & Daud, 2010). They also have difficulties solving word problems (Powell, Fuschs, Fuschs, Cirino, & Fletcher, 2009). Students experiencing MLD work at a slow pace, and have difficulty representing mathematical concepts (Wang, Du, & Liu, 2009) as well as retrieving basic facts (Micallef & Prior, 2004). They tend to forget faster, more easily abandon a task, and have difficulty grasping concepts quickly (Gafoor & Kurukkan, 2015). These deficits contribute to MLD and students' achievements when compared to their peers (Jitendra et al., 2013), which may also influence their beliefs.

2.4 The relationship between teachers' beliefs and their knowledge

In his work, Shulman (1986, 1987) analysed knowledge components that were essential for teaching any particular subject area: subject matter knowledge, knowledge of learners and their characteristics, general pedagogical knowledge, pedagogical content knowledge, knowledge of the educational context, and curriculum knowledge. Many studies have extended Shulman's ideas by identifying content knowledge for teaching (Ball, Thames, & Phelps, 2008) and conceptualising mathematics knowledge for teaching (Selling, Garcia, & Ball, 2016). This dissertation encompasses *beliefs*; however, in Study III, teachers' knowledge of MLD was investigated by employing Ball et al.'s framework. Although Ball et al. do not mention beliefs as part of pedagogical content knowledge (PCK), in this dissertation, I associate knowledge with beliefs since teachers sometimes employ their beliefs as knowledge (Liljedahl, 2008), and they therefore could form part of PCK. Beliefs may interact with the PCK; however, studies in this area are rather scarce (e.g., Bulut, 2021).

How are teachers' beliefs about MLD different from their knowledge? Teachers' beliefs and knowledge of students experiencing MLD (as part of PCK) can influence their teaching decisions and practices. Teachers may teach based on what they know and/or believe about students experiencing MLD and about teaching those students. For instance, teachers should recognise that students have different learning abilities.

They are expected to know what causes some students to experience learning difficulties and what interventions can improve their learning.

This dissertation, however, addresses only one part of one aspect of PCK: teachers' knowledge of students and their characteristics (Ball et al., 2008; Shulman, 1986), with a particular focus on students experiencing MLD. It aims to extend Shulman's and other scholars' ideas regarding what teachers know about students experiencing MLD and the ways in which they can employ that knowledge to teach these students effectively and support their learning and understanding of mathematics. This knowledge can aid in the early detection of difficulties associated with learning mathematics and the promotion of inclusion in mathematics classes; all learners have different educational needs, and learners studying mathematics are no exception. Additionally, teachers' knowledge about students is essential for effective teaching and for detailed intervention to support students' learning needs (Ball et al., 2008). Teachers with this knowledge can support students' achievements according to the students' ability levels. Although teachers' knowledge has been widely researched, this doctoral dissertation highlights teachers' knowledge in identifying students experiencing MLD which is rarely addressed in the field of mathematics education.

3 Research Methodology

3.1 Research context

This research focuses on Namibia, a democratic nation in southern Africa. Namibia shares borders with Angola and Zambia to the north, Botswana to the east, and South Africa to the south, as illustrated in Figure 3. Nearly two-thirds of the entire school population in Namibia is concentrated in rural areas in the six northern regions of Omusati, Oshana, Ohangwena, Oshikoto, Caprivi and Okavango. I conducted the study in Namibia because of my familiarity with its education system, policies and

Figure 3. Location of Namibia in Africa. Source: Ministry of Agriculture, Water and Land Reform, Directorate of Survey and Mapping of Namibia, 2008.

history. Additionally, I have a sound understanding of the cultures prevalent in these regions, which primarily aided the facilitation of the process of obtaining research permission and contacting the schools. Moreover, my role as a Namibian mathematics teacher was sufficient to gain access to the participants and the educational institutions.

3.2 Research design and approach

The phenomenological approach addresses educational experiences and individual perceptions (Selvi, 2008; Sohn et al., 2017) and it allows the participants to explain their unique individualities regarding feelings, experiences, thoughts, understandings and beliefs (Selvi, 2008). Using phenomenological approach in this study allowed exploring and understanding the beliefs about mathematics learning difficulties as it emerged from the participants via in-depth semi-structured interviews instead of quantitative measures.

Various Namibian education stakeholders in relation to MLD were considered while I was planning the research presented in this dissertation. Consideration of stakeholders is of utmost importance for the diversity and variation of research findings regarding the beliefs about MLD. Such diversity may also avoid bias that can easily occur when all participants are from the same educational context and level. Furthermore, I utilised multiple data sources to obtain a holistic overview of the beliefs about MLD in the Namibian context (Creswell, 2009). I employed a qualitative approach, specifically a qualitative survey (Study I) and in-depth individual interviews (Studies II, III and IV) to explore the beliefs about MLD, the barriers that make mathematics difficult to learn and the needs of students experiencing MLD in Namibian schools. Qualitative research produces data by bringing together preconceptions, beliefs, and other personal theories that affect the interaction between the researcher and the research field, in relation to theory and data analyses. It involves interpretations of the voices of the participants in order to make sense of meanings and assumptions they express based on their feelings, experiences, and beliefs (Creswell, 2009; Creswell & Poth, 2018; Selvi, 2008). Thus, I explored beliefs about MLD considering other scholars' arguments on the significance of choosing qualitative research. A summary of the research approach of the studies in this dissertation is presented in Table 4.

Table 4. A summary of the research designs and approach.

Study	Designs of the study	Study approach	Participants	
			Type of participants	Number of participants
I	Survey	Qualitative: Written responses to survey questions	Mathematics teachers representing primary and secondary levels (100 and 131 respectively)	231
II	Phenomenology	Qualitative: Semi-structured interview	Grade 11 students experiencing MLD	27
III	Phenomenology	Qualitative: Semi-structured interview	Grade 12 students who participated in Study II Grade 12 mathematics teachers	23 6
IV	Phenomenology	Qualitative: Semi-structured interview	Senior pre-service high school mathematics teachers (in the final year of their teacher education programme) Mathematics teacher educators	4 3

3.3 Research instruments

The questions in this study investigated the participants' understanding of MLD based on their experiences, observations, practices, the impact of MLD on students' learning and the way in which MLD should be addressed in the classroom. An overview of the research method of the studies in this dissertation is presented in Table 5.

3.3.1 Study I: Survey

A survey containing both closed and open-ended questions used in a previous study (Kelley, Clark, Brown, & Sitzia, 2003) was suitable for collecting data about mathematics teachers' perspectives of MLD because of its flexibility in providing rich data (Zhanga & Zhou, 2016). The instrument consisted of two parts: Part A (background information) and Part B (perspectives of MLD). For Study I, I only analysed Part B due to the space limitation for submission.

3.3.2 Studies II and III: Semi-structured interview

Interviews are a way of collecting qualitative data; they can provide complete information that is not available in written form (Arsenault & Anderson, 2000). They enable participants to discuss their interpretations of the world in which they live as they express their thoughts, views and ideas about an aspect specific to the research. In both Studies II and III, I deemed a semi-structured interview the most appropriate approach for obtaining an in-depth understanding of teachers' and students' beliefs about MLD, as it allowed for participant flexibility. Indeed, this is the approach of phenomenological research.

I developed common interview frameworks about beliefs and MLD that expand on the literature for the current study (Baccaglini-Frank, & Papadastos, 2014; Carnine et a1., 1997; Karagiannakis et al., 2014; Lewis, 2014; Nambira et al., 2009; Wang et al., 2009). The frameworks were designed with the assistance of my supervisors and some experts in the field; the aim was to ensure that the research objectives were met. The frameworks guided the interviews, and they were suited for the study purposes, as shown in Table 3. The design of the questions encouraged participants to share their experiences (Bekdemir, 2010), either in teaching practices, observation or learning mathematics. I arranged the guided interview questions in a manner that promoted comfortability in the discussions (Kingry, Tiedje, & Friedman, 1990).

In Study II, the questions sought to elicit beliefs about MLD (Schommer-Aikins, 2002), the reasons why students experienced MLD (Lewis, 2014), the process of learning mathematics, as well as the methods of improving the teaching and learning of mathematics (Wang et al., 2009). Study III explored the relationship between the teachers' knowledge of the assessment criteria for students experiencing MLD and their students' criteria regarding MLD, based on the framework of Ball and Bass (2002). The focus was whether teachers and their students agreed with one another, which resulted from different perceptions that emerged in Study II.

3.3.3 Study IV: Semi-structured interviews

Study IV was also a qualitative study completed in two phases. I gathered data from participants by means of semi-structured interviews; the participants reflected on their related experiences and observations, as well as the meaning of these experiences (Merriam, 2009), especially at the high school level. These experiences and observations contribute to the beliefs of pre-service teachers (Pajares, 1992; Philipp, 2007).

In Part I, I designed the interview protocol in a way that considered the reviewed literature regarding teachers' beliefs about MLD and learning difficulties, as well as the definition of the term, belief, adopted in this study. The questions were designed

to produce belief statements about students experiencing MLD. In Part II, the interview comprising eight main questions elicited the beliefs of teachers (former pre-service teachers) about MLD based on their experiences and observations in their first year as mathematics teachers.

3.4 Procedure and data collection

3.4.1 Study I: Survey of Namibian mathematics teachers' congress

I collected data during the annual Namibian Mathematics Teachers' Congress in March 2017. Permission to conduct the survey was granted by the congress coordinator. Access to the congress provided a beneficial opportunity for gaining understanding of MLD from the points of view of Namibian teachers across the country. A total of 231 mathematics teachers, representing both primary and secondary schools, voluntarily participated in the survey.

3.4.2 Studies II and III: Interviews at northern Namibian high schools

Through the relevant Namibian authorities, for example, the Chief Executive Officer and Regional Directors, I was granted permission to gather qualitative data via semi-structured interviews in four northern Namibian regions. The selection of the four regions of Omusati, Oshana, Ohangwena and Oshikoto was based on my familiarity with the areas, thereby facilitating access. Additionally, I am a native speaker of Oshiwambo, the native language of the four regions of Namibia, which eased communication.

All the selected high schools (n = 23) that I approached accepted voluntarily to participate in Study II. At each school, mathematics teachers identified five Grade 11 students who were considered by their teachers to experience MLD. Students were selected to create an equal representation of gender in the study. I collected data in October 2018. I informed the participants about the study procedures and the measures in place for maintaining confidentiality and anonymity. All the students who participated in this study were eligible to give their own consent as research participants in terms of the Finnish ethics regulations for conducting academic research studies. Their ages were above 15 years old, and their guardians were informed through their school principals' offices. All the selected students voluntarily agreed to participate in the study and signed the consent forms. To prevent any loss of data, I audio-recorded all the interviews, except for one, at the participant's request. I conducted interviews in the language with which the students

felt comfortable, either English or Oshiwambo, the students' native language. Although the students had been previously observed as experiencing MLD, only some self-identified as experiencing MLD during the interview. Therefore, we narrowed the group in Study II to include only those students who self-identified as experiencing MLD (n = 27).

Interviewing the teachers, based on the criteria employed for identifying students experiencing MLD in Study II, was beneficial because of their differing views. This factor was crucial for conducting Study III. When teachers identify students as having MLD, but students classify themselves as not having MLD, a problem in the field of mathematics education becomes apparent. Therefore, in July 2019, I selected six of the 23 schools to constitute the sample for Study III based on their accessibility. The procedure and research ethics principles in Study II were also followed in Study III. The teachers and their students shared their perceptions of MLD. I was curious about, and interested in, exploring the characteristics of students experiencing MLD.

3.4.3 Study IV: Interviews at a Namibian university

One of the University of Namibia's campuses that offers a teacher training programme participated in Study IV. I made this selection based on convenience for data collection. The head of the Department of Mathematics, Science, Sport and Education at the campus assisted in contacting the mathematics teacher educators in the department and the potential senior pre-service, high school mathematics teachers in their final year of the teacher education programme. Recruitment was not easy due to the pre-service teachers' busy schedules and workload. Hence, the pre-service teachers who were available and willing to participate in the study became the participants. The sample was gender-balanced. I explained the aim and research ethics, including the voluntary agreement to participate in the study and reassurance of anonymity. The interviews were guided by the interview protocol, audio-recorded and transcribed verbatim.

Part I was conducted in October 2018 and focused on beliefs about MLD that were developed during the teacher training programme. The aim was to provide insights into pre-service teachers' beliefs, the type of knowledge and practices that might be emphasised by teacher educators about MLD in teacher education programmes and ways in which teachers could support students experiencing MLD. The pre-service teachers completed the programme and they were appointed as teachers at different state schools. They were contacted to participate in the follow-up interview (Part II) during the second semester of their first year of teaching. The interviews explored the teachers' beliefs about MLD that they had developed in their first year of teaching, reflecting on their classroom practices and experiences.

3.5 Participants

3.5.1 Study I: Participants at the Namibian Mathematics Teachers' Congress

Mathematics teachers from both primary (n = 100) and secondary schools (n = 131) completed the survey. Their experiences and educational backgrounds varied. The majority of these teachers had a teaching qualification (teaching diploma as per Namibian teaching requirements).

3.5.2 Studies II and III: Participants at northern Namibian high schools

The selected Grade 11 students (n = 115), who had been observed by their mathematics teachers as experiencing MLD, participated in the semi-structured interviews. Of the 115 students from 23 schools, 27 self-identified during the interview as experiencing difficulties in learning mathematics. To reiterate, our interest was in students experiencing MLD; thus, the data reported in Study II related to these 27 students (8 males and 19 females). They came from different school environments: urban (n = 14), semi-urban (n = 7) and rural (n = 6) high schools. The students' ages ranged from 16 years to 19 years.

For Study III, 24 students (12 females and 12 males) from six selected schools voluntarily agreed to participate in the follow-up interview in July 2019. Students who self-identified as experiencing MLD as well as those who did not self-identify were included. In the 2018 interviews, seven of these students self-identified as experiencing difficulties, whereas 17 did not. These students studied ordinary mathematics, that is, an extended level in Grade 11 before moving to the core level in Grade 12. Additionally, in Study III, six mathematics teachers (two females and four males), who had selected the Grade 11 students in 2018, voluntarily agreed to be interviewed. Moreover, one additional teacher (a female) agreed to participate in the study as she was the only teacher responsible for teaching Grade 12, although she had not participated in the selection process. The teacher who had been involved in the selection of the students had moved to a different school. All the participating teachers had a bachelor's degree, which qualified them to teach high school mathematics (Grades 11 and 12). Additionally, they all had more than two years of teaching experience, except for the teacher who had not participated in the students' selection process; this teacher had half a year of teaching experience when the interviews were conducted.

3.5.3 Study IV: Participants at a Namibian university

Four senior, pre-service, high school, mathematics teachers and three mathematics teacher educators from one of the Namibian general public university campuses volunteered to participate in Study IV; they constituted the sample of Study IV. The four pre-service teachers (ages 22 to 26) were nearing the completion of their final year in the teacher education programme. They had completed numerous mathematics education courses, such as educational research and teaching methods.

At the same time, the three teacher educators were the only mathematics educators in the department during the time of data collection. They had more than 15 years of previous experience teaching high school mathematics, and they were working at the university during the data collection for this study. Their highest qualifications ranged from a bachelor's degree with honours in mathematics to a doctoral degree in mathematics education. They were responsible for teaching mathematics and mathematics education courses in the programme while simultaneously guiding and monitoring pre-service teachers in their practicums at schools.

3.6 Data analysis

3.6.1 Study I: Analysis of the data from the teachers' survey

A qualitative approach was employed to interpret the data from Part B. I utilised a thematic analysis (Ritchie & Lewis, 2003) to enhance understanding of the data (Braun & Clarke, 2006). After data familiarisation, five keywords were identified and employed in coding. They were disabilities, inability, less ability, lack of interest and struggling. I, furthermore, determined the categories that summarised and represented the data set during the coding process. Three themes were generated: difficulties resulting from learning competency, students' low interest in mathematics and an inadequate foundation in learning mathematics; however, the last theme was excluded from the study due to space limitations.

3.6.2 Studies II and III: Analysis of data from students and teachers' interview

The interviews I conducted in Study II and Study III were transcribed. I replayed the recordings repeatedly to ensure accuracy. The use of a coding framework facilitated the thematic analysis of the transcripts (Corbin & Strauss, 1990). I highlighted significant words and common phrases and noted these based on the research

questions that guided the studies. However, in Study III, I also conducted a theory-driven content analysis to reduce qualitative data through codification and theme development (Creswell, 2009). In Study II, I examined students' interviews to identify their beliefs about MLD. By contrast, Study III explored the relationship between teachers and their students' perceptions concerning MLD, primarily focusing on the teachers' knowledge of students experiencing MLD.

In Study II, my analysis resulted in the following themes: students' access to teachers, students' emotional responses and learner-centred practices (the level of teaching instruction and the pace of teaching). In Study III, the analysis produced five themes: anxiety about mathematics, the lack of class participation, low achievement in school assessments, a slow learning pace and asking low-level questions. Low achievement in school assessment was emphasized with the greater frequency, and I considered it to have the most validity in identifying MLD among students.

3.6.3 Study IV: Analysis of the data from interviews with pre-service teachers and teacher educators

I transcribed all the audio-recorded interviews verbatim and employed an open coding procedure. Employing open coding with qualitative data allowed me to identify all the possible understandings that the participants expressed concerning their observations and experiences as collected in the data set (Corbin & Strauss, 2008); this also comprised their beliefs (Philipp, 2007). This approach allowed me to explore possible connections among participants' beliefs in their belief systems.

Firstly, I read the data repeatedly to familiarise myself with the content. I simultaneously took notes on the commonalities of meanings; furthermore, I noted the understandings of pre-service teachers and teacher educators in the data separately. The interpretations of notes resulted in a list of codes that I employed in the data analysis (Corbin & Strauss, 2008). I discussed the initial findings by employing the codes. The co-author of the study revised the coding process; a second coding was later conducted to refine the code list. A joint discussion of the themes grounded in the data, which answered the research questions, was subsequently held.

Table 5. Overview of the research method.

Phases	Study			
	I	**II**	**III**	**IV**
Conceptual background	Mathematics teachers' beliefs and practice	Beliefs: students' beliefs, beliefs about teaching and learning, MLD, mathematics personalities	Knowledge required for teaching students experiencing MLD, the concept of MLD	Beliefs: beliefs about MLD: teacher educator and pre-service teachers
Participants	Mathematics teachers (*n* = 321)	Grade 11 students (*n* = 27)	Grade 12 students (*n* = 23) Grade 12 mathematics teachers (*n* = 6)	Senior pre-service high school mathematics teachers (*n* = *4*) Mathematics teacher educators (*n* = 3)
Data collection	(Questionnaire) Survey: Google drive and drop-box allotted at a conference venue	Semi-structured interview	Semi-structured interview	Semi-structured interview
Data analysis	Qualitative: thematic analysis (Braun & Clarke, 2006; Ritchie & Lewis, 2003)	Qualitative: thematic analysis (Corbin & Strauss, 1990)	Qualitative: content analysis and theme development (Creswell, 2009)	Qualitative: Open coding (Corbin & Strauss, 2008)

3.7 Research ethics and trustworthiness of the study

This study was conducted in an environment characterised by respect for individuals and respect for knowledge. I approached the study with ethical issues in mind, for example, seeking permission to conduct the study, which was granted by relevant Namibian authorities in the Ministry of Education, Arts and Culture and seeking voluntary participation, as well as obtaining signed consent letters. I presented the participants with information about the purpose and method of data collection. They granted permission for the interviews to be audio-recorded, and I informed them of confidentiality regarding their identities, as well as the research sites. I stored the responses from the survey and audio-recorded interviews securely and employed the data only for this study. In most cases, I referred to interview data as specific

individuals. Therefore, in Studies II and III, the students were referred to with pseudonyms for anonymity, whereas the teachers were referred to as Teacher 1, 2 and so forth. In Study IV, I denoted the participants by their generic titles, such as 'teacher educators' or 'pre-service teachers'.

Moreover, other research ethics that I took into consideration involved recognising the right of participants to withdraw from the study. Additionally, I respected the different experiences of the participants. However, it is recognised that the success and quality of a research project depend on the relationships formed with the participants, the participants' institutions and authorities, as well as agreeing to adhere to a set of guidelines. For instance, trust and ethical standards must be maintained. Also, distractions from schoolwork needed to be avoided; thus, the data collection took place after school hours.

To establish the validity of this study, I employed two methods of data collection (Creswell & Poth, 2018), namely a survey and interviews. The survey and interviews provided detailed descriptions regarding beliefs about MLD, and both data collection methods provided essential knowledge. Furthermore, I collected information from different sources (teachers, teacher educators, pre-service teachers and students). These provided raw data in the form of necessary information concerning beliefs about MLD that complemented each other, thereby broadening the scope of this dissertation. This aspect was also an attempt to achieve a balanced representation of multiple stakeholders in Namibian education by employing thematic analysis and open coding to capture the beliefs about MLD in the data set. Another strategy that increased the credibility and validity of this study was the participants' engagement during the semi-structured interviews, which allowed for an increased understanding and a detailed explanation. Fostering honesty in research was also a strategy for maintaining validity during the entire research process. I employed direct quotations from the data set to ensure the accuracy of the findings and to maintain the participants' voices.

For trustworthiness, I utilised feedback from experts in mathematics education, such as supervisors and fellow researchers in mathematics education, to revise the survey tools, the interview protocols and the data analysis, which increased the repeatability of the study findings. I conducted the data collection and analysis processes with an undivided focus on the research aims. The co-authors of Study II and Study IV acted as peer reviewers in a process referred to as peer debriefing in the literature (Janesick, 2015). The co-authors validated different perspectives by asking critical questions. This strategy also increased the trustworthiness of the study findings and reduced bias based on my knowledge of the Namibian education system. Furthermore, I considered the critique and feedback received from independent peer reviews, supervisors and fellow researchers throughout the

research process. They all provided new insights that enhanced the reliability and generally improved the content of this dissertation, as summarised in Table 6.

Table 6. The trustworthiness of the study.

Study	Trustworthiness
I	Completion of the survey by the participants was conducted concurrently. Experienced researchers in the field identified and refined the categories.
II	We analysed the data and reached an agreement through critical discussions about the accuracy of the coding list and categories.
III	I shared the data and analysis with an expert in the field, and modification was done in terms of codification and theme development.
IV	The second author acted as a peer reviewer, monitored the coding process of the data closely, and asked critical questions to reach accurate coding and precise reporting. There was a careful and closely monitored coding process and critical discussion among the authors.

4 Results of the Empirical Studies

This chapter presents the results of the four original articles.

4.1 Study I: teachers' beliefs about MLD

Hamukwaya, S. T. (2019). K–12 Namibian teachers' beliefs on learning difficulties in mathematics: Reflections on teachers' practice. In U. T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis (Eds.). *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 4652–4659). Utrecht, the Netherlands: Utrecht, the Netherlands: Freudenthal Group & Freudenthal, Institute, Utrecht University and ERME. https://hal.science/hal-02431483

Study I reports the data from a survey completed by 231 Namibian mathematics teachers during an annual national mathematics conference. This study, firstly, examined the teachers' beliefs about students experiencing MLD and, secondly, explored teachers' perceptions of problems affecting MLD. The teachers' experiences varied, which may have increased the uniqueness of the feelings, perceptions and emotions they expressed. The participants reflected on their practices and experiences, and the results revealed a wide range of beliefs. From a general perspective, teachers understood the term MLD within a deficit framework, believing the students' difficulties emanated from a lack of competence.

Additionally, teachers appeared to have beliefs that do not support providing better learning experiences for the students experiencing MLD. This may contribute to inadequate foundation in the subject that can cause MLD as indicated in the literature review. My interpretation here is that, if teachers do not act accordingly and try to support students by focusing on gaps in their learning, this may limit students' learning and understanding and may contribute to MLD. They identified potential causes of MLD, including students' lack of interest in learning mathematics, low student participation and an inadequate foundation in the subject. These aspects could affect the students' learning opportunities negatively and lead to poor performance in a specific topic area of mathematics as a subject. Findings indicate that these personal beliefs could influence the teaching practices related to students experiencing MLD negatively.

4.2 Study II: High school students' mathematics difficulties

Hamukwaya, S. T. & Ruttenberg-Rozen, R. (2023). Believing in your own abilities: What Namibian high school students experiencing mathematical difficulties can teach us. *Canadian Journal of Science, Mathematics, and Technology Education*, 1–19. https://doi.org/10.1007/s42330-023-00260-7

It is essential to understand the experiences and beliefs of learners who experience difficulties in mathematics classes; this can be achieved through listening to their narratives. Study II reports findings from semi-structured interviews with 27 Grade 11 students who were reported to experience MLD by their mathematics teachers. Study II aimed to investigate students' beliefs and their experiences in the Namibian mathematics education system. To achieve this, the study explored the beliefs concerning teaching and learning mathematics and the barriers that hindered the learning ability of students experiencing MLD. The study revealed that negative experiences of learning mathematics shaped students' beliefs.

Although students were identified by teachers as having MLD, the students themselves believed they had the potential and ability to learn mathematics and felt comfortable with the subject. Though they acknowledged that they were not successful in mathematics, they did not believe it was because of MLD. Students perceived that their surroundings were inherently unfair and did not contribute to their success in mathematics. For instance, the Namibian schooling system was viewed as limiting the opportunity for students to learn mathematics, and this was believed to have contributed to their MLD. Students mentioned teacher absenteeism, which they believed occurred more often than in other grades and thus affected their learning opportunities negatively. The students were approaching the end of high school; they considered access to their teachers a necessity.

Teaching practices, such as the fast pace of teaching and a lack of learning support, were other systemic aspects believed to contribute to MLD. Students suggested different teaching practices that would support their mathematics learning and understanding. They wanted a slower teaching pace and increased depth of learning and teaching that built on prior knowledge to support their mathematical understanding. Students believed that the depth of learning and understanding of mathematical ideas would be essential to overcome MLD. They also believed that, if they had the opportunity to learn mathematics with sufficient academic support, they would not experience difficulties.

Moreover, some of the students indicated that emotions presented barriers to learning mathematics. These results demonstrate the complicated relationship between the beliefs in one's own ability to do something and responses when barriers are presented. Thus, the students' beliefs were often tied to emotional dispositions

and intense feelings, and they believed that these prevented their learning and achievement. Students believed they should ask teachers for help, but they felt shy or recalled negative experiences that prevented them from doing so. Furthermore, students believed they could achieve adequately in mathematics but felt panicked and disempowered when the pace was rapid. This illustrates the way in which emotional reactions impacted students' responses to barriers.

4.3 Study III: Perspectives of teachers and students on learning difficulties

Hamukwaya, S. T. (2022). A comparison of the perceived difficulties in learning high school mathematics from the perspective of the teachers and the students. *International Journal for Mathematics Teaching and Learning*, *23*(1), 46–63. https://doi.org/10.4256/ijmtl.v23i1.317

Study III contributes to understanding teachers' knowledge of students and their characteristics, as well as the students' knowledge of their learning. It was based on the Ball et al. (2002) MKT framework of teachers' knowledge of students experiencing MLD and the way in which their knowledge is expressed in relation to self-identification. Six Namibian high school mathematics teachers and 23 of their Grade 12 students were the participants of the study. Through semi-structured interviews, teachers shared their criteria for identifying students experiencing MLD, and the students voiced their thoughts about their MLD. The findings revealed participants' views on school achievement, class participation, students' pace, level of asking questions and mathematics anxiety.

A large number of teachers and students voiced their belief that low achievement occurred because students had difficulties learning mathematics. For instance, teachers expressed the conviction that students with poor mathematics backgrounds could not solve basic mathematics questions, leading to low achievement. This suggests that teachers identified students experiencing MLD based on the aspect of under-performance. Additionally, most of the participating teachers suggested that students experiencing MLD were slow to solve questions. In contrast, this view was not common among students. Furthermore, many students mentioned anxiety concerning MLD, whereas few teachers mentioned this perspective. Other factors that were perceived as barriers to students' learning were a new learning environment and anxiety due to the fear of failing. The findings suggest that anxiety could influence the pace at which students solve mathematics problems and students' participation, which, in turn, might contribute to MLD. Primary factors believed by teachers to affect the pace of students were the students' doubts about their answers and their difficulty with time management.

Teachers also viewed participation as more critical in determining students' difficulties in learning, which contrasted with students, who typically held the opposite view. Students who did not consider participation vital, indicated that they could get assistance from their peers. Students' reasons for not participating were influenced either by their negative learning experiences or personal preferences. Conversely, some of the students agreed that they were not active participants in the class. Despite their difficulties, students voiced that they were curious and eager to learn mathematics; however, they needed learning support.

Furthermore, teachers indicated that some students experiencing MLD asked questions, but these questions were often below their grade level, contrary to what the teachers expected. However, the fact that students were asking questions reveals that they were interested in learning mathematics. As there is a lack of supportive evidence on this aspect, classroom observation would be appropriate to strengthen these findings.

4.4 Study IV: Teacher educators' and pre-service teachers' beliefs about MLD

Hamukwaya, S. T. & Haser, Ç. (2021). "It does not mean that they cannot do mathematics": Beliefs about mathematics learning difficulties. *International Electronic Journal Mathematics Education*, 16(1), em9569. https://doi.org/10.29333/iejme/9569

Study IV documents the beliefs of teacher educators (n = 3) and pre-service teachers (n = 4) regarding MLD. I selected participants from one Namibian university campus and explored their experiences and the meaning of these experiences through semi-structured interviews, which were conducted in two parts. Part I investigated pre-service teachers' beliefs about MLD among high school students that they had developed at the end of the teacher education programme and the views and practices that might be emphasised in the programme. Part II explored possible changes in teachers' beliefs during their first year of teaching.

The most critical factors believed to cause MLD at the high school level were a lack of fundamental knowledge, personal factors and systemic factors in the school system. Teacher educators and pre-service teachers characterised students experiencing MLD in terms of beliefs and attitudes. They suggested individual instructional support and mixed ability grouping as means of supporting high school students experiencing MLD. Teacher educators expressed beliefs about teachers' knowledge, beliefs and practices concerning MLD. They mentioned individual support, while pre-service teachers addressed pair and group work to support students experiencing MLD. Results of Part I illustrate that pre-service teachers had developed beliefs about MLD that were somewhat in line with the

ideas and practices emphasised in the education programme. Pre-service teachers addressed the negative consequences of students' existing knowledge in terms of MLD. They observed previous teachers' influence on students' MLD, although they did not reflect on the way that their knowledge and practices might influence students experiencing MLD or might cause MLD. The teacher educators also discussed the effects of studying in different school environments on students' difficulties, whereas pre-service teachers discussed MLD in relation to the curriculum and teachers' workload.

Pre-service teachers elaborated on MLD in their first year of teaching, although their beliefs did not seem to have changed substantially. They did not comment on the way in which teachers' knowledge and practices might influence students' experiencing MLD. They addressed procedural difficulties and elaborated on the challenges of teaching students experiencing MLD and the experiences of, and possible consequences for, students experiencing MLD. They also realised that their strategies to deal with students experiencing MLD might not work for all students because of the curriculum-related difficulties. The findings illustrate that it was difficult for them, as new teachers, to implement the curriculum and include students with and without MLD, especially when the class size was large and the teaching load was heavy. The study shows that teachers starting their career did not seem to consider individual differences while planning their teaching. In both parts of the study (Part I and Part II), the importance of practices concerning MLD was emphasised. Table 7 summarises the main findings of the four studies. Based on the collected data of this study and the literature, these findings can have implication on teaching and learning, teacher education and mathematics education as explained in Table 7 (e.g., Ndlovu & Ngcobo, 2018; Bendixen, 2002; Haser & Star, 2009; Leatham, 2006).

Table 7. Summary of the empirical studies regarding beliefs about MLD.

Study	Main findings
I	(i) Teachers framed students' difficulties as emanating from a lack of competence. This findings is important because it may indicate a need of a belief change among mathematics teachers. Framing students experiencing MLD in such a way may result in teachers not supporting and accommodating these students in the teaching and learning and this might be the reason for MLD.
II	(i) Students believed that some mathematical experiences such as a slower teaching pace and teaching that builds upon their prior knowledge may support their mathematics learning. This finding is important and should be investigated further because such learning experiences may be essential for students experiencing MLD as they may improve their mathematics learning and reduce their MLD. (ii) Students believed that they had the potential and ability to learn mathematics. This findings indicated an availing belief expressed by students experiencing MLD which may positively influence their learning of mathematics. If students held such belief, it is possible that they will be more eager to learn mathematics when they have the opportunities.
III	(i) Teachers illustrated knowledge of students experiencing MLD and believed addressing individual students' needs improved their mathematics learning. These beliefs may be essential to support students experiencing MLD in the mathematics classroom and improve their mathematics understanding. (ii) Students self-identified as potential learners of mathematics. This students' belief may also be important for teachers' pedagogical knowledge, especially to understand individual students needs and how to support their learning which may reduce MLD.
IV	(i) Pre-service teachers developed some availing mathematic-related beliefs about MLD as a result of the practices emphasised in the education programme, for instance, grouping students experiencing MLD in mixed learning ability. This finding may indicate the influence of teacher education in general regarding the nature of mathematics and teaching and learning in relation to MLD. Then, if pre-service teachers have these availing beliefs, they should be supported in their efforts to teach students experiencing MLD so that their practices will help students.

5 General Discussion

The belief about MLD is an essential concept in mathematics education that can be analysed from both teachers' and students' perspectives. For this reason, it becomes imperative to understand the fundamental concept of the belief about MLD with the key questions of what these beliefs are, what causes these beliefs, how these beliefs can be changed to promote mathematics learning, and why students experience MLD. The interest for carrying out this study was influenced by my teaching experience as a Namibian mathematics teacher. Working in this role, I observed that teachers did not seem to believe that they were part of the system that reinforced MLD among students. This may reinforce poor learners' low academic performance in mathematics. Based on the findings, for the students experiencing MLD to learn mathematics with understanding and perform better, they need teachers to guide, assist and support them. In addition, I noted that many students experienced MLD at higher grade levels.

Specifically, using a phenomenological approach, I wanted to understand teachers' (pre-service, in-service, and teacher educators) beliefs and students' beliefs about MLD—why these beliefs appear, and why they experience certain practices and adopt these beliefs—as, according to scholars, beliefs impact teaching and learning interaction (Steele & Ambady, 2006; Markovits & Forgasz, 2017). I also wanted to understand the beliefs about MLD within the Namibian context from a scientific perspective in order to draw conclusions related to policy, teaching, and learning; and to further change the beliefs and, ultimately, the ways in which Namibian teachers teach and students learn so that MLD may be reduced. At the same time, my aim was to suggest ways to decrease non-availing beliefs about MLD by increasing availing beliefs among teachers and students.

Therefore, this chapter discusses the overall findings of the dissertation, which are related to teachers (and teaching), students experiencing MLD, and ways to support their learning process. It is based on four aims that constitute the basis of this study as presented in Section 1.3.

5.1 Beliefs about MLD

In this chapter, first, I addressed the (i) systemic nature of beliefs about MLD from the four empirical studies, (ii) beliefs about practices which may contribute to MLD, (iii) beliefs about practice that would support students experiencing MLD, and (iv) beliefs about MLD that had developed by the end of the teacher education program; focused on the (beliefs) implications concerning MLD that were revealed from the studies. In my discussion, I also suggest action research for students, teachers, and teacher education as regards practice, policy, and theory. In doing so, I provided a broad picture of what occurred in some Namibian schools in terms of beliefs about MLD based on the findings of this dissertation. Second, I looked at the contributions of the study to this field of research. Finally, I suggested further research based on the findings and limitations of the study.

5.1.1 The systemic nature of beliefs about MLD

In the four studies that I conducted, the participants expressed their beliefs about MLD based on their own experiences, knowledge, and practices. In this section, I discuss different participants' beliefs regarding MLD and the ways in which these can be compared. The common aspects that are believed to lead and/or contribute to MLD were: (i) the lack of mathematical understanding, mathematical knowledge, and arithmetic skills, (ii) students' beliefs regarding the nature of mathematics and teachers' beliefs regarding students who experience MLD, (iii) students' habits and emotional dispositions, and (iv) pedagogical factors. First, I will discuss the findings in relation to the first aim of this study, exploring beliefs about MLD.

Similar to the findings of other researchers (Mazzocco, 2007; Wang et al., 2009), in this study, MLD was regarded by pre-service and in-service teachers and teacher educators as an obstacle that affected students' academic performance negatively and delayed the process of learning mathematics. Pre-service teachers indicated that when students who lacked essential mathematical knowledge moved to higher grade levels, their learning of new concepts was challenged, and they were likely to develop MLD. In other words, students' prerequisite knowledge may be incomplete, which may make it difficult for them to understand the new mathematics concepts.

Teachers addressed procedural difficulties regarding tasks that students who experience MLD could, or could not do, in the mathematics class. They believed that students who experience MLD lacked arithmetic skills and had difficulty recalling arithmetic facts. Additionally, teachers and teacher educators regarded these students as being unable to perform at the expected base level in particular learning areas or at certain levels, which consequently, may result in poor performance. They tended to believe that these students had low learning abilities or that their skills were insufficient. This may suggest that, in general, students who experience MLD are

believed to have a poor understanding of procedural mathematics. On the other hand, teachers' expressions such as, '*students are unable to…'* carry negative connotations that could be considered to potentially influence their professional decisions and actions. Such belief may not effectively include students who experience MLD in the teaching and learning process. As stated in the literature, this belief may influence teachers' instructional decisions and the outcomes for students (Philipp, 2007, Schommer-Aikins et al., 2005). There is a need, therefore, for challenging and discussing Namibian mathematics teachers' beliefs about students who experience MLD in teacher education and professional development programmes.

Another factor believed by educators as contributing to the students' MLD, especially at the beginning of their high school careers, was students' beliefs about the nature of the subject, namely, that mathematics is difficult. Teacher educators and pre-service teachers indicated that this belief might affect students' behaviour towards mathematics teachers or the subject itself negatively, as students might choose not to focus on understanding the concepts in the class. Such a belief has the potential to jeopardise students' future success in mathematics and may have a negative impact on their chances to achieve a career in the field of science. Several studies cited this belief in the literature review (e.g., Siyepu, 2013; Mundia, 2010; Bekdemir, 2010); stating that it could have been formed based on the students' early achievements in mathematics or by familial, academic success, or lack thereof, in mathematics. Surprisingly, none of the students shared such a belief in this study. However, there is a possibility that they might not be aware of such a belief even if they have it.

The study has revealed that students who experience MLD have some basic emotions in relation to learning mathematics. Students' emotional reactions and experiences were based on, firstly, their interpretations of the learning situations; for instance, some students believed that they were placed in an underperforming school/class. Secondly, their personal mathematics experience. I undertook this study expecting that negative prior experiences of the students experiencing mathematics difficulties would cause them to disassociate from mathematics (Mazzocco et al., 2012; Ndlovu & Ngcobo, 2018). However, students in this study believed in their ability to learn and understand mathematics despite experiencing mathematics difficulties. Thirdly, students also believed that the emotional states arose from situations, such as a new learning environment or low achievement. To understand the nature of the emotional experiences of students with MLD who wanted to increase their understanding of mathematics, it is necessary to analyse classroom practices and focus on the relationship between certain emotional dispositions and practices.

The literature indicates that students with a positive emotional disposition towards mathematics also tend to experience positive emotions (Hannula, 2015;

2014). In comparison, the current study revealed that students who expressed negative emotions believed that they had the potential to understand and improve their mathematics learning. The students' negative emotional dispositions did not seem to be associated with the belief that they were not capable of doing mathematics. Rather, they tended to have availing beliefs that could help them in learning mathematics. Students may have associated their negative emotions with issues outside of their control and therefore, if these issues were eliminated, students believed that they would probably not experience MLD.

With a different perspective, the students in this study felt comfortable learning mathematics as a school subject. Nevertheless, they still feared taking tests or asking teachers questions when they did not understand properly. According to Geisler and Rolka (2021) these emotions limited their learning. Study II revealed that students' beliefs were tied to emotions in the self-identification of their mathematics personalities as well as to personal emotional responses. By expressing who they were in relation to learning and understanding mathematics, students tended to relate negative emotions to failure. The students indicated that they felt shy, panicky, or embarrassed when they sensed a need to ask for learning support or when they did not understand the subject matter. They reported that these emotions were influenced by their earlier experiences in learning mathematics. This indicates how students' past mathematics experiences could shape their emotions and beliefs as indicated in the literature (e.g., Black et al., 2018, Schommer-Aikins et al., 2005). It seems students' negative emotional disposition towards mathematics and their beliefs about mathematics affected their learning experiences in different ways. In order to reduce students' negative emotions, it is vitally important for teachers to consider carefully the diversity of students' learning needs in the classroom (Di Martino & Zan, 2011). This might be a challenge, particularly if the classroom is overpopulated and the majority of students are experiencing MLD. My interpretation could be that, if students recognise that they are not sufficiently knowledgeable in mathematics, they can look for ways to improve the situation, for instance, by asking questions in order not to have MLD, and by maintaining availing beliefs and emotions.

According to Hannula (2004), a negative affect, such as mathematics anxiety, undermines the students' academic outcomes, and students with such emotional reactions may fail to develop their full mathematical potential. Hannula advocates that, where possible, the formation of mathematics anxiety such as frustration and fear – which is an expression of anxiety (Ardi et al., 2019; Zhang, Zhao, & Kong, 2019). should be avoided to reduce negative experiences. The participants in the current study acknowledged anxiety (which is part of emotion) as another factor that could contribute to MLD. For example, students felt anxious when meeting new teachers and peers. Based on the students' beliefs (in Study II) and teacher educators' beleifs, anxiety was also considered an obstacle to learning when students had

difficulties in learning mathematics. When students who experienced MLD had to take a mathematics examination, they were scared and confused. Participants believed that anxiety could influence the pace at which students solved mathematical problems and, therefore, lead them to perform poorly.

Students expressed some attitude statements, such as an interest in improving their understanding of mathematics, as none of them voiced a lack of interest in learning mathematics or a lack of commitment. They also did not connect their lack of practice or participation to MLD. This finding is in contrast with studies that revealed that students with MLD found mathematics not interesting (e.g., Eshun-Famiyeh, 2005; Mundia, 2010; Sarsani & Maddini, 2009). However, teachers and teacher educators shared different observations and beliefs about students who experience MLD. Some observed that students seemed disinterested in learning mathematics while others had different observations and concluded that students were interested. With such observation, it is worth noting that mathematics teachers should include interesting activities that capture and build on learners' interest in mathematics during the teaching and learning process.

It can be said that participants' beliefs about MLD were linked to factors such as teaching and learning environments, and the pedagogy teachers employed. The students, pre-service teachers, and teacher educators believed that MLD emanated from insufficient learning opportunities and support. Although students indicated availing beliefs (such as interest in learning and understanding mathematics), they believed that they had not acquired an adequate mathematical foundation and thus, were experiencing MLD. The educators also related MLD to a lack of sufficient mathematics knowledge and skills which, they believed, to be the result of inadequate mathematics knowledge acquisition in lower grade levels. However, the most important finding could be that students believed they would understand mathematics despite their difficulty in learning the subject and their emotional responses in the classroom. The discussion above addressed the first aim of this study, and next, I will discuss the findings in relation to the second aim, exploring beliefs about the practices which may contribute to MLD.

5.1.2 Beliefs about pedagogical factors that could contribute to MLD

Dkeidek, Mamlok-Naaman and Hofstein (2011) argue that a proper learning foundation plays a significant role in the development of the ability of students to ask questions. Hence, teachers in the present study believed that a poor foundation in learning mathematics could be the reason why students experience MLD and were observed to ask questions below their level of learning. Moreover, in the present study, students experiencing MLD were also seen as slow in attempting to solve

fundamental questions, similar to the findings of Wang et al. (2009). Although students' questions were believed (by teachers) to be below their grade level, in my interpretation, asking questions may have enhanced students' understanding and demonstrated their interest in the subject.

Some teacher educators regarded memorisation as meaningless because most students often forgot the content learned. They considered it as a teaching method which does not build on previous knowledge, experience, and understanding; and which may limit learning, as students may not see the connection between what is taught and what is learned in school mathematics. Teacher educators believed that the strategies to solve mathematical problems were not well explained to students. They observed that the emphasis in schools was on students to follow a set of procedures. The educators believed that this is not an effective teaching practice and it would not lead to students' learning. They believed that teachers should focus more on urging the students to understand the procedures and explain why following procedures is helpful. This implies that there is a greater possibility for the students of the teachers who emphasize memorization to learn mathematics without understanding. Thus, opportunities for students to understand mathematical concepts are deemed essential for overcoming MLD in students. Nevertheless, evidence to support teacher educators' beliefs is needed to build arguments for this teaching practice.

Some students appeared to ignore the importance of class participation in the mathematics lesson and based on teachers' beliefs, low classroom participation was an indicator of MLD. Such attitudes seemed to make teachers believe that the students were experiencing MLD. According to the teachers, students' participation in the mathematics class demonstrates their competence and strengthens their mathematical identity. A possible conclusion is then that difficulty in learning mathematics might affect a student's level of classroom participation and consequently, the learning that the student achieves.

Teacher educators observed a challenge in the lack of sufficient subject knowledge in mathematics among first-year mathematics pre-service teachers who had studied mathematics at a core level. In comparison, pre-service teachers who had studied mathematics at a higher or extended level were observed to be more competent in mathematics skills. It is likely that the lack of adequate mathematics skills among pre-service teachers could result in ineffective teaching, which may contribute to MLD. This also points to the fact that Namibian high school students should be encouraged to take higher-level mathematics if they are considering becoming mathematics teachers in the future. This will give them a strong mathematics foundation to understand the concepts covered in the teacher-training program. Future studies should explore the two curriculum contents (core and extended) and how this affects teaching and learning processes. Such studies would

provide suggestions for policy makers for necessary adjustments in the policies. Accordingly, studying a higher-level mathematics could be a prerequisite to apply for entry into mathematics teacher education programmes. However, considering the lack of teachers' subject knowledge mentioned in this study, it is equally important to provide continued support for teachers' development in subject content and in teaching practices.

Moreover, the study identified some practical factors that could potentially limit students' opportunities to learn mathematics, and may, therefore, contribute to MLD. It uncovered a number of systemic barriers for Namibian teachers and students. The participants believed that MLD is often connected to factors at school (e.g., unqualified teachers) and other systemic factors and/or aspects of the Namibian schooling system and practices. One of the systemic implications revealed in the study was that the wrong placement of mathematics teachers in some Namibian schools resulted in a weakened essential mathematics knowledge base for students, which may have contributed to MLD. The teacher educators' common belief was that some primary mathematics teachers were assigned to teach mathematics at higher-grade levels (which they were less familiar with) or vice versa. These teachers usually might not have the necessary knowledge and skills to teach mathematics at those levels. Teacher educators believed that those teachers were not competent to teach mathematics concepts effectively. This may result in students not gaining adequate mathematics knowledge, which could negatively affect their learning of mathematics and, subsequently, contribute to MLD. Thus, it is critical for the school management to ensure that teachers do teach the subjects that they are qualified to teach and that they are placed at the grade level of their specialization. This might also address the wider educational policy issue regarding teacher shortage and teacher placement in Namibian schools, which should be taken into consideration.

The findings indicated other systemic implications, such as teachers' workload and curriculum demands, which also hindered students' abilities to learn mathematics sufficiently. The pre-service teachers and students believed that the density of the high school mathematics curriculum makes it difficult to support students experiencing MLD. They believed that there exists a lack of balance between the pace of the curriculum and the teacher's workload, which does not facilitate accommodating students experiencing MLD. These beliefs are a result of individual experiences (Bendixen, 2002) and struggles, and have several interrelated points to address in terms of policy. In summary, curriculum load accompanied by the lack of sufficient mathematics teachers indicates a possibility for teachers not providing effective learning experiences for students with MLD.

The findings further display a challenge to teaching and learning in overcrowded classrooms in Namibian schools and the possible consequences for students experiencing MLD. This conforms with existing studies by Ndjangala, Abah, and

Mashebe (2021) and Zimba, Mufune, Ndjangala, Abah, and Mashebe (2021), which concur with the present study that pre-service teachers believed such challenge made it difficult to provide students with adequate individual learning support. Specifically, when first-year teachers encountered this challenge in practice – when they wanted to provide a series of mathematics activities and individual learner support with effective feedback, their beliefs and practices could have potentially changed. Although they expressed the desire for diverse approaches to addressing MLD, their beliefs about MLD did not, in fact, seem to change considerably during their first year of teaching.

MLD could also be related to broader policy issues beyond mathematics teaching and learning, which mathematics teachers cannot influence. I referred, for instance to English, which is the medium of teaching instruction in Namibian schools. Mathematics and other subjects are taught in English, which is not the mother tongue of most of the students. The lack of sufficient language skills was believed by the first-year teachers to hamper students' mathematics learning, eventually leading to MLD. They believed that some students experienced MLD because they did not understand the teachers' explanations or questions. Students should then improve their language proficiency as they progress with their schooling, which would help them to understand instructions for other school subjects, including mathematics.

In the Namibian national curriculum, teaching the entire curriculum is emphasised, and the overall time allocation for teaching each subject is determined by the Ministry (Ministry of Education, 2010). The findings of the present study indicated that some essential aspects of mathematics remain untaught due to the absenteeism of teachers. Grade 11 students perceived teachers' absenteeism as a cause of the lack of support and restricted access to proper instruction. Literature shows that teacher absenteeism can result in an incomplete syllabus, which may negatively affect students' learning experience and opportunities to learn mathematics (Roby, 2013) and in turn, MLD among students may increase. Students in this study indicated that they wanted to learn mathematics and gain consistent access to their teachers. Although Hipondoka (2017) reported that the absenteeism of teachers in Namibia could have been the result of their participation in professional development activities, their lack of commitment or their inadequate awareness of how to utilise their own leave appropriately may thus contribute to MLD. The absenteeism of teachers teaching Grade 11 classes may not be similar to that in other grade levels and may influence the teaching processes negatively and, thus, contribute to MLD. If this is the case, teacher absenteeism requires further exploration to determine possible relationships between students' narratives in relation to MLD and their access to teachers. This summarised the discussion of the findings in relation to the second aim of this study. The following discussion

addresses the third aim, exploring beliefs about practice which would support students expereincing MLD.

5.1.3 Beliefs about practices that would support students experiencing MLD

Pre-service teachers believed it is a challenge to effectively teach students experiencing MLD when they considered the amount of mathematics content that should be covered and the number of students per class. Especially taking their beliefs that stated students who experience MLD could acquire a better understanding of mathematics through in-depth teaching, slow-paced instruction, and practise, as well as personalised learning support into account. Attending to the needs of every student who experiences MLD may pose a challenge because of the diversity of the students and the makeup of the classroom population. Thus, pre-service teachers believed that building on students' prior knowledge in ways that would link new mathematics knowledge with their existing knowledge is essential for learning mathematics as it may improve students mathematical understanding. When students do not build on prior knowledge, there is a danger that learning could become rote (Lazarowitz & Lieb, 2006); which may contribute to MLD. However, teachers must be aware of the ways in which students' diversity affects the class in order to optimise teaching opportunities (Roos, 2017).

According to Tomlinson (1999) and Zakaria et al. (2010) the individual students may need different teaching methods. This study also illustrates various beliefs that address diversity in teaching mathematics to meet the student's needs and to create opportunities for learning. For instance, teachers in studies III and IV seemed eager to diversify their teaching methods to help their students with different learning needs. However, the teacher educators believed that teachers should understand first *why* students experienced MLD. Such understanding may aid teachers in identifying a suitable teaching method that improves teaching practices for their students and creates an inclusive mathematics classroom. One practice was the pace of teaching instruction. Whereby students believed that they were not effectively learning mathematics because of the fast pace of teaching. The teacher educators expressed this belief when they talked about providing learning support to students. They believed if learning support is offered at a reduced pace to students who experience MLD, it would increase their learning through understanding. This indicates that students may develop MLD if the teaching pace is not adjusted to their learning pace, which would allow them to follow the lessons. These findings could be addressed in teacher education and professional development programmes.

Moreover, all groups of participants believed that additional learning support, along with integrating different teaching strategies with a series of practical

activities, could assist students who experience MLD, as well as those who promote their understanding of mathematics. Teachers believed this approach could only be successful when students were also eager to learn mathematics and if teachers were willing to give them support. Significantly, the students believed in their potential and ability to learn mathematics with sufficient, individualised learning support. They believed that it was necessary for their teachers to provide essential, individual learning support with positive reinforcement. Individual support combined with pair and group work, was advocated by students and teacher educators as beneficial to students who experience MLD since it employed different teaching strategies and provided students with many activities to practice. For instance, the first-year teachers concurred that mixing students by ability enabled those experiencing MLD to enhance their understanding of mathematics. However, the overload of teachers' responsibilities and large class sizes caused difficulties in implementing this strategy successfully.

One significant finding was that students mentioned that they were eager to learn mathematics. They believed that they could study high school mathematics despite the obstacles in their learning and achievement in mathematics that they previously experienced. They are of the view that if they were given an opportunity to study mathematics adequately, they could have mastered the mathematical concepts. The beginning teachers also expressed that some students who experience MLD have shown interest and a desire to understand mathematics. These availing beliefs may serve to strengthen students' perceptions of themselves as capable mathematics students. Hence, students suggested that an increased depth of learning and understanding of mathematics would be essential regardless of schooling and achievements, but only if they had the opportunity to study mathematics adequately.

Additionally, pre-service teachers realised that students who experience MLD needed to practise, concentrate, participate, and put an effort into learning mathematics. The students and pre-service teachers alike underscored the revision of mathematical concepts to enhance students' understanding and reduce their MLD. However, beginning teachers emphasised the use of the students' mother tongue when re-explaining concepts. They observed that such practice helps students to learn better with a greater understanding of the mathematics concepts. In general, there were availing beliefs and a positive degree of enthusiasm from both students and teachers to improve mathematics learning, and I suggest a need to sustain and improve those beliefs and perceptions. In the above discussion, I have summarised the findings in relation to the third aim of this study. The next discussion is about exploring beliefs about MLD that had developed by the end of the teacher education programme.

5.1.4 Beliefs about MLD developed by the end of the teacher education programme

Although the beginning teachers addressed the importance of understanding the nature of MLD, they seemingly disregarded individual differences while planning their lessons. This result indicates that teacher education programmes should emphasise multiple approaches to address different forms of MLD such as highlighting flexible planning tasks to accommodate students who experience MLD in the classroom. Therefore, teacher education programmes should provide pre-service teachers with opportunities to reflect on their knowledge, beliefs, and practices, as well as the possible consequences of their own lack of essential mathematical and pedagogical knowledge when considering MLD. Moreover, first-year teachers believed that collaboration among mathematics teachers and language teachers, with parental involvement, supported their practices, especially when they had trouble explaining mathematical concepts to students. They acknowledged that such practices would help students who experienced MLD to improve their learning and understanding of mathematics. Furthermore, first-year teachers considered these practices a practical aid to students who experience MLD. Simultaneously, they believed they can identify whether a student's difficulties were linked to language barriers or mathematics as a subject.

Pre-service teachers' beliefs regarding MLD in terms of their own mathematics education courses during their studies differed from their beliefs regarding the subject as teachers when it came to students who experience MLD. Teachers who were just beginning their careers explained that their experiences in dealing with MLD had not always been successful. It is possible that they were still in the process of developing their beliefs, including the belief that students who experience MLD learn differently. Moreover, teachers, pre-service teachers, and students underlined the need for effective mathematics teaching practices in the Namibian high school system to support students who experience MLD. For instance, many of the student participants in Study II commented that they would like to receive in-depth teaching that would expand their prior learning and abilities and lead to new understanding and meaningful learning.

It seemed that teacher training programme resulted in beliefs that connected MLD to mathematics teachers' heavy teaching loads, overcrowded classrooms, and insufficient teaching and learning support, plus other factors specific to the Namibian context. Pre-service teachers expressed their own practices or those of other teachers. They did not seem to consider individual differences in learning styles while planning their teaching. This finding suggests that teacher education programmes should emphasise employing multiple approaches when dealing with different types of MLD in classes and, particularly, being flexible in the planning of tasks to

accommodate students who experience MLD. This summarises the discussion of the findings in relation to the last aim of this study (fourth aim).

In summary, with this dissertation, I aimed to explore beliefs about MLD from the Namibian perspective. Students' difficulties; experienced and those observed, are a compelling topic in mathematics education. Using a phenomenological approach, teachers' and students' beliefs regarding MLD and teachers' knowledge about students experiencing MLD were discussed. This study confirmed that the teachers' practice of identifying students who experience MLD is a highly complex and professional task (Ben-Peretz, 2011). It is connected to teachers' competencies, knowledge of their students and their own beliefs. The educators had non-availing beliefs (about students experiencing MLD); for example, they believed that the students were unable to solve basic mathematics problems, which could be considered an obstacle in teaching these students. Yet, they indicated that they wanted to help their students. Similarly, students stated that they wanted to expand their understanding of mathematics as they learn it. The participants believed that opportunities to understand mathematics were essential to mitigate MLD. Teachers and teacher educators indicated that learning difficulties were not unique to students experiencing MLD but that all individuals have some learning difficulties. Although some of the beliefs expressed by the participants were similar, their beliefs generally addressed different aspects of beliefs associated with MLD.

5.2 Contribution of the study

The study contributes to filling a gap in the research literature, as it provides insights into a rarely researched topic and country in the field of mathematics education. Although the study extends the current limited knowledge regarding beliefs about MLD, its contribution is notable because it highlights an otherwise silent aspect, namely beliefs about MLD. It is reasonable to believe that this study can serve as a guide for future research on optimising learning opportunities for students as it contributes to the understanding of the systemic features of the Namibian education system regarding the ways in which different stakeholders view barriers to the teaching and learning of students who experience MLD.

Throughout this research study, I realised that those students who experience MLD did not believe that they were incapable of learning mathematics; rather, they needed the opportunity to reach their highest potential. Students believed that the opportunity to understand mathematics was essential to reduce MLD. Although the beliefs shared by participants were based on a phased-out curriculum, the vital message is that understanding beliefs regarding MLD and the means of changing those beliefs are important. Notably, students, teachers and teacher educators believe that teachers and educators should be responsible for limiting learning difficulties in

mathematics, and the participants believed that opportunities to understand mathematics were essential for both teachers and students in order to mitigate MLD.

Although some teachers' beliefs revealed non-availing beliefs and negative connotations, the students' potential for mathematics learning was encouraging, even though negative emotional dispositions impeded their learning progress, for instance, fear of taking mathematics tests. However, how to change teachers' negative perceptions of students who experience MLD still remains an open question and could be another direction for future research. Perhaps, beliefs regarding MLD could be considered when allocating teaching assignments, teaching support, and professional development. However, more comparative, and predictive work would be required before any recommendations could be made.

5.3 Implication of the findings and recommendations to stakeholders in Namibian education

Based on the study's findings, some practical implications in relation to practice, policy, and theory arose from a teaching and organisational perspective. This study uncovered a widely held belief that teachers are teaching at different grade levels than they are qualified for. If this is the case, then teachers teaching mathematics at a grade level that they are not qualified for may be the reason for MLD. Based on this finding, the implications could be assigning teachers to teach subjects they qualified for so that grade 11 mathematics lessons are not taught by teachers who are not qualified to teach at that grade level. Another reason found in this study for MLD might be that mathematics teachers were absent in the classrooms as participants believed. Based on this finding, students believed that they had limited learning opportunities. In this case, the implication may be that grade 11 mathematics lessons (and possibly the mathematics lessons until this grade level) should not be without a teacher. From these implications, I recommend that the grade 11 mathematics teachers may be placed at the grade level of their qualifications and specialisation and schools should pay attention to teacher absenteeism. These all might contribute to the implication that the wider educational policies may address the issues regarding teachers' recruitment and placement adequately if this may be the reason for MLD.

Another implication revealed by teachers and preservice teachers from the study was the teachers' workload and curriculum demand which does not facilitate accommodating students experiencing MLD. They believed that the grade 11 mathematics curriculum and the teachers' teaching responsibilities might also be the reason for MLD. These implications illustrate a need for Namibian stakeholders in education and national education policies to consider such implications when

designing the curriculum that there should be a balance between the teachers' workload and the content of the grade 11 mathematics curriculum.

The study also found that participants believed that teachers did not consider individual differences in their teaching, but they wanted to provide learning support to individual students experiencing MLD. If this is the case, teachers might not provide learning support to individual students, and students may experience MLD. Teaching and learning in overcrowded classrooms may be a reason for this practical implication, as it is revealed in the study by teachers, teacher educators, and pre-service teachers. From these findings, I develop an implication that there may be a need for change in teacher education to provide tools that may assist teachers to support learning mathematics in an overcrowded classroom. Additionally, school authorities may consider the issue of individual students' learning needs to accommodate students who experience MLD in the mathematics classroom. However, these implications should be considered within the limitations that are explained in detail in the next section.

In a nutshell, let me remind readers that the aim or the basis of this study was to explore beliefs about MLD. As the literature indicated in section 2.1, beliefs are individual constructs, based on experience and thinking (e.g., Hannula, 2014; Kaasila, 2007). Hence, the findings presented in this dissertation are beliefs about MLD from the teachers, preservice teachers, teacher educators, and students' perspectives. So, there is a need for readers to be slightly more careful when interpreting the implications of the findings of this study.

5.4 Limitations and future research

The study aimed to explore beliefs about MLD; however, it was limited to the Namibian context, which, consequently, limits the generalisability of the results of this study. As far as the development of a theory is concerned, several limitations have emerged and must be considered when interpreting the results. First, this dissertation is not based on direct evidence from classroom practice; classroom observations could be an essential factor in strengthening the findings. Second, the studies presented in this dissertation explicitly focused on the beliefs about MLD from the Namibian perspective. Applying the results to other geographical and cultural contexts should be undertaken with caution. Third, even though Studies II and III indicated some crucial strengths, such as the large-scale nature of the data, they did not represent all Namibian educational regions. Thus, generalisations cannot be made about MLD based on these findings, even within the Namibian context. Fourth, Study IV's findings are limited to a small number of participants and the findings could have been different if the sample had covered all the state university campuses in Namibia. Fifth, the survey employed in Study I was utilised to collect

self-reported data, and it was not piloted. Nevertheless, the data set out of this questionnaire was adequate to answer the research questions I set for this study.

Belief in one's ability is not always enough to overcome barriers, especially as far as the results of Study II indicated; thus, designers of future studies to assist students who experience difficulties in learning mathematics could benefit from recognising the relationship between emotions and beliefs. Hence, this can be a limitation if research and/or practice focus only on beliefs but ignore emotions.

Meanwhile, the collection of qualitative, self-reported data from Studies II and III was useful, as the data suggested that students had positive beliefs regarding their ability to learn mathematics. However, in the future, collecting supporting evidence about students' beliefs, emotions, and participation that may be revealed in the classroom, and students' achievement by means of direct observations and carefully designed tasks could improve the understanding of MLD. Furthermore, as no teaching practice was observed for this study, future research could benefit from investigating teachers' beliefs about MLD in relation to classroom practices. The literature also acknowledges these methodological limitations and advocates that the investigation of teachers' mathematical beliefs should focus on their instructional practices, verbally and observationally (Thompson, 1992). This knowledge could also be essential for professional development, and it could influence educational policy makers and educators when addressing learning difficulties in mathematics classes.

The areas listed above are indeed essential in informing education practice and conduct in Namibia. Due to the rather wide scope of the approach and the study topic, the resulting design considerations remain at a general level; however, the findings show promising research directions for future work. Based on the findings, discussions, and limitations of this dissertation, further research should be conducted to address both this topic and the context more thoroughly.

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