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ABSTRACT

Mathematics is a comprehensive, even esthetical experience, affecting a person intellectually, emotionally and physically. The purpose of this study is to determine and examine the dimensions of technology-enhanced mathematics learning.

The three learning domains cognitive, psychomotor and affective, ranging from uncomplicated to more complex learning outcomes, as defined by Bloom, have been used a great deal in mathematics pedagogy (Krathwohl, Bloom, & Masia, 1964). This study goes deeper and also examines motivation theory and learning theories when applying technology to the teaching of mathematics.

To get a broad picture of the impact of these dimensions on mathematics learning via technology, research was conducted in an array of contexts, including South Africa, Mozambique, Germany and Finland. The cross-cultural and cross-countries approach was chosen to ensure wider generalizability of the research. The study involved an action design research (ADR) approach of creating and evaluating artifacts; (i) a novel pedagogical INBECOM model for mathematics learning advocating both behavioristic and constructivist perspectives, and (ii) a newly designed and created story-based UFractions mobile game for learning of fractions incorporating tangible manipulatives. In particular, the affective domain of participants in the study was being studied throughout a ten-year research process from 2009 to 2019.

The INBECOM pedagogical model was tested by organizing a fraction course for 21 grade 10 students. The development and evaluation of the pedagogical INBECOM model gives a concrete example of how two learning approaches, constructivism and behaviourism, can be combined in teaching fractions. Furthermore, the results of the qualitative evaluation confirm the view that successful instructional practices have features that are supported by both constructivism and behaviorism.

The UFractions mobile game was evaluated with 305 grade 8 students and 12 teachers. Empirical tests indicate that combining concrete manipulatives and mobile phones is a meaningful way to learn the abstract concept of fractions, increasing active student participation. On the basis of the collected data, I initiated a taxonomy for the variety of play motivations in the UFractions game. The dynamics between game motivations and disturbance factors (DF) was analysed. Each motivation relates to a set of DFs typically affecting the player motivation negatively. By becoming aware of these relations, we are able to design more motivating educational games and give guidelines for game developers, users and educators.

To explore the affective learning experiences of the three groups of research participants, the qualitative data was derived from the interviews with researchers, teachers and students, as well as from learning diaries, feelings blogs, observations (311 documents) and quantitized (Saldaña, 2009). All the data was explored from the affective perspective, by labelling the feelings the participants experienced according to the affective levels of the Krathwohl et al. (1964) framework. I concluded that affective learning at all five levels was recognized among the three groups of participants. However, the results show that affective learning mostly took place at the receiving level, indicating that the participants received more than they responded, valued, organized or internalized. There was also a significant effect of research participants pertaining to receive; students' affective learning occurred more at the receiving level than that of the teachers; and teachers' affective learning emerged more at the value level.

Moreover, I define a dimension taxonomy of learning to be used as a framework in the design and implementation of technology-enhanced mathematics teaching and learning including the following three dimensions: (i) Domains of learning, (ii) Orientation of learning, and (iii) Motivation of learning. More precisely, the five domains of learning are cognitive, psychomotor, affective, interpersonal, and intrapersonal. Considering orientation of learning, combining behaviorism and constructivism, would lead to more motivating and meaningful teaching and learning strategies. Furthermore, the level of technology integration, the level of students' cognitive process, and the level of teachers' knowledge, are intertwined. Motivational factors are an essential part of learning, and it is important to acknowledge connections between motivations and disturbances, when using technology.

KEYWORDS: Learning technology, Motivation, Affective learning, Learning theories, Tutoring systems, Serious games, Mathematics pedagogy, Global South, Design Science research, Cultural context

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TIIVISTELMÄ

Matematiikka on moniulotteinen kokemus vaikuttaen henkilöön älyllisesti ja tunnetasolla samalla kytkeytyen myös fyysiseen ulottuvuuteen. Tämä tutkimus määrittää ja tarkastelee teknologia-avusteisen matematiikan oppimisen dimensioita.

Bloomin määrittämät kolme oppimisen osa-aluetta, kognitiivinen, psykomotorinen ja affektiivinen, jotka etenevät yksinkertaisista monimutkaisempiin oppimisen tasoihin, ovat olleet laajasti käytössä matematiikan pedagogiikassa (Krathwohl, Bloom & Masia, 1964). Tämä tutkimus laajentaa käsitystä oppimisesta tutkimalla motivaatio- ja oppimisteorioita sekä niiden käytännön soveltamista teknologia-avusteisessa matematiikan opetuksessa.

Laajan ymmärryksen saavuttamiseksi siitä, miten nämä tekijät vaikuttavat teknologia-avusteiseen matematiikan oppimiseen, tutkimusta toteutettiin monissa eri ympäristöissä, mukaan lukien Etelä-Afrikka, Mosambik, Saksa ja Suomi. Tutkimuksessa huomioitiin kulttuuriset ja kansainväliset näkökulmat tulosten laajemman yleistettävyyden varmistamiseksi. Tutkimus hyödynsi suunnittelutoimintatutkimuksen (Action Design Research, ADR) menetelmää artefaktien luomiseksi ja evaluoimiseksi: (i) uudenlaista behavioristisia ja konstruktivistisia näkökulmia yhdistävää pedagogista INBECOM-mallia matematiikan oppimiseen, ja (ii) käsinkosketeltavia matematiikan apuvälineitä hyödyntävää UFractions-mobiilipeliä murtolukujen oppimiseen. Erityisesti osallistujien affektiivista oppimista tutkittiin kymmenen vuoden tutkimusprosessin aikana vuosina 2009–2019.

INBECOM-pedagogista mallia testattiin järjestämällä murtolukukurssi kansanopiston 10-luokalle, jolla oli 21 oppilasta. Pedagogisen INBECOM-mallin kehitys ja arviointi antavat konkreettisen esimerkin siitä, miten kahden oppimisteorian, konstruktivismin ja behaviorismin, voi yhdistää murtolukujen opetuksessa. Lisäksi laadullisen arvioinnin tulokset vahvistavat käsitystä siitä, että menestyksellisillä opetusmenetelmillä on piirteitä, jotka hyödyntävät sekä konstruktivistisia että behavioristisia periaatteita.

UFractions-mobiilipeli arvioitiin 305 8-luokan opiskelijan ja 12 opettajan avulla. Empiiriset testit osoittavat, että konkreettisten apuvälineiden ja matkapuhelimien yhdistäminen on mielekäs tapa oppia abstrakti murtoluvun käsite ja edistää opiskelijoiden aktiivista osallistumista. Kerätyn datan perusteella kehitettiin taksonomia UFractions-pelin pelimotivaatioista. Pelimotivaatioiden ja häiriötekijöiden (Disturbance Factors, DF) välistä dynamiikkaa analysoitiin. Jokainen motivaatio liittyy tiettyihin häiriötekijöihin, jotka yleensä vaikuttavat pelaajan motivaatioon negatiivi-

sesti. Näiden suhteiden tiedostaminen auttaa suunnittelemaan motivoivampia opetuspelejä ja antaa suuntaviivoja pelikehittäjille, käyttäjille ja opettajille.

Affektiivisen oppimisen kokemusten tutkimiseksi tutkimukseen osallistuneiden kolmen ryhmän dataa tarkasteltiin laadullisen tutkimuksen keinoin; tutkijoiden, opettajien ja opiskelijoiden haastattelut, oppimispäiväkirjat, tunneblogi sekä havainnot (311 asiakirjaa) kvantifioitiin (Saldaña, 2009). Kaikki data analysoitiin affektiivisesta näkökulmasta merkitsemällä osallistujien kokemat tunteet Krathwohlin ym. (1964) viitekehyksen affektiivisten tasojen mukaisesti. Tutkimus osoitti, että affektiivista oppimista tunnistettiin kolmen osallistujaryhmän keskuudessa kaikilla viidellä tasolla. Tulokset osoittavat kuitenkin, että affektiivinen oppiminen tapahtui pääasiassa vastaanottotasolla, mikä viittaa siihen, että osallistujat vastaanottivat enemmän kuin he vastasivat, arvostivat, järjestivät tai sisäistivät. Myös osallistujaryhmien affektiivista oppimista koskevat tulokset vaihtelivat merkittävästi: opiskelijoiden affektiivinen oppiminen tapahtui enemmän matalammalla vastaanottotasolla kuin opettajien, ja opettajien affektiivinen oppiminen ilmeni enemmän korkeamman, arvotason oppimisena.

Lisäksi tutkimuksessa määritellään oppimisen ulottuvuuksien taksonomia, jota käytetään teknologia-avusteisen matematiikan opetuksen ja oppimisen suunnittelussa ja toteutuksessa. Tähän kuuluu seuraavat kolme ulottuvuutta: (i) Oppimisen osa-alueet, (ii) Oppimisen orientaatio ja (iii) Oppimisen motivaatio. Tarkemmin sanottuna viisi oppimisen osa-aluetta ovat kognitiivinen, psykomotorinen, affektiivinen, interpersonaalinen ja intrapersonaalinen. Yhdistämällä behavioristisia ja konstruktivistisia elementtejä saaadaan innostavia ja merkityksellisiä opetus- ja oppimisstrategioita. Motivaatiotekijät ovat olennainen osa oppimista, ja teknologiaa käytettäessä on tärkeää tunnistaa yhteydet motivaation ja erilaisten häiriötekijöiden välillä. Lisäksi teknologian integraation taso, opiskelijoiden kognitiivinen prosessi ja opettajien tietotaso ovat kietoutuneet toisiinsa.

ASIASANAT: Oppimisteknologia, Motivaatio, Affektiivinen oppiminen, Oppimisteoriat, Älykkäät oppimisympäristöt, Hyötypelit, Matematiikan pedagogiikka, Globaali etelä, Suunnittelutoimintatutkimus, Kulttuurinen konteksti

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Abbreviations

ADR	An Action Design Research
AH	Adaptive Hypermedia System
AI	Artificial intelligence
AR	Augmented Reality
CAS	A Computer Algebra System
CMS	Course Management Systems
CK	Content Knowledge
DGE	Dynamic Geometry Environment
DF	Disturbance factor
DSR	Design Science Research
ICT	Information and Communication Technology
IGS	Interactive Geometry Software
INBECOM	A pedagogical model Integrating Behaviorism and Constructivism in Mathematics
ITS	Intelligent Tutoring System
LMS	Learning Management Systems
OER	Open Educational Resources
PCK	Pedagogical Content Knowledge
PISA	Programme for International Student Assessment
PK	Pedagogical Knowledge
SAMR Model	Substitution, Augmentation, Modification, and Redefinition model
SITES	Second Information Technology in Education Study
STEM education	Science, Technology, Engineering and Mathematics education
TCK	Technological Content Knowledge
TPACK	Technological pedagogical content knowledge
TK	Technology knowledge
TPK	Technological Pedagogical Knowledge
VLE	Virtual Learning Environments

List of Included Publications

This thesis is based on the following original publications. The publication reprints of the included publications are presented in Part II of the thesis. My contributions for each of the papers listed below are detailed in section 1.3.

I **Turtiainen, E.**, Blignaut, S., Els, C., Laine, T.H. & Sutinen, E. Story-based UFractions Mobile Game in South Africa: Contextualization Process and Multidimensional Playing Experiences. 2009. Proceedings of the Second Workshop of Story Telling and Educational Games (STEG 2009).

II Laine, T.H. and **Nygren, E.** and Sutinen, E. and Islas Sedano, C. and Joy, M.S. & Blignaut, S. Ubiquitous Mathematics from South Africa to Finland: Does Reverse Transfer Work? 2011. In: Ubiquitous Learning: Strategies for Pedagogy, Course Design and Technology. Chen, I. and Kidd, T. (eds.), Information Age Publishing, Charlotte, NC, pp. 249-282.

III **Nygren, E.**, Sutinen, E., Blignaut, A.S., Laine, T.H. & Els, C.J. The Ufractions Mobile Manipulative Game: Opportunities for South African Grade 8 Learners. 2012. International Journal of Ubiquitous Computing (IJUC), 2(1): pp. 1-18.

IV **Nygren, E.**, Sutinen, E., Blignaut, A. S., Laine, T. H., & Els, C. J. Motivations for Play in the UFractions Mobile Game in Three Countries. 2012. International Journal of Mobile and Blended Learning (IJMBL), 4(2), 30-48.

V **Nygren, E**., Sutinen, E. & Laine T. H. Dynamics between disturbances and motivators in educational mobile games. 2018. International Journal of Interactive Mobile Technologies (iJIM), 12(3), pp.120-141.

VI **Nygren, E.**, Blignaut, A. S., Leendertz, V. & Sutinen E. Quantitizing Affective Data as Project Evaluation on the Use of a Mathematics Mobile Game and Intelligent Tutoring System. 2019. Informatics in Education, 18(2), pp. 375-402.

The permission of the publishers has been obtained for the use of the articles as partial publications of the dissertation.

1 Introduction

1.1 Background and motivation for the research

Learning of Mathematics is a complicated endeavor, and integrating technologies to the challenge raises new perspectives. In order to derive the dimensions of learning Mathematics, I completed the research within an array of context: South Africa, Mozambique, Germany, and Finland, employing *an action design research* (ADR) approach to create and evaluate two artifacts; (i) compiled a *pedagogical model* for mathematics learning which advocated an innovative behaviorist-constructivist perspective while designing and (ii) using a *mobile game for learning* of fractions.

The mere use of modern technology in teaching does not guarantee good learning outcomes and teaching effectiveness, but it is essential to think about how to use technology and what skills to learn. When teachers choose a tool to be used in teaching—or a designer develops a tool to be used—they have to think about the underlying learning principle; how do people learn? Developing useful learning tools and materials, and using them in a meaningful way requires an understanding of the principles underlying how people learn. Different kinds of learning strategies emphasize different sides of knowledge, reality, values, and humanity. Before the 1960s, the most prevalent of learning theories was behaviorism, whereby human is a mechanical machine that responds to stimuli, and emotions, motivation, or other states of mind play no role in learning. The more recent significant learning theories are cognitivism and constructivism. Central to these approaches is the learner's role in learning, understanding, and thinking. The learning context also has a significant role in both the use and development of educational technology.

Mathematics is born out of people's needs to solve practical problems. According to Polya (1962), it is easy to imitate solutions to problems that resemble each other. He suggests that school math should contain more problems that require creativity and ingenuity. Furthermore, he describes the process of problem-solving as follows: "Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable" (Polya, 1962, p. ix).

Learning mathematics is a multi-dimensional experience. Bloom classified learning according to three domains: cognitive, psychomotor, and affective, ranging from uncomplicated to more complex learning outcomes (Krathwohl et al., 1964).

We understand these three dimensions through learning objectives; learning mathematics, which involves not only acquiring cognitive knowledge, but also attaining psychomotor skills and growth in the affective domain, for example, creation of attitudes, appreciations, and relationships.

Bloom's cognitive domain has become a significant model for assessing learning outcomes, as well as for planning teaching and learning activities (Reigeluth, 1999). However, also other two domains should be considered in teaching and learning, as they are equally important (Adkins, 2004; Bolin, Khramtsova, & Saarnio, 2005; Griffith & Nguyen, 2006). Aspects of the affective domain are often neglected (Grootenboer & Marshman, 2016). Moreover, some teachers claim that the affective domain is beyond the scope of their teaching, and they feel that each student copes with their feelings independently (Duncan-Hewitt, Leise, & Hall, 2005). They also claim that it is too time-consuming to consider the affective domain when planning their classes (Griffith & Nguyen, 2006). For that reason, when assessing the results, cognitive objectives are emphasized instead of affective experiences (Bolin et al., 2005). We should also discuss the importance of the affective aspects of learning, although the outcome of learning is cognitive of nature (Duncan-Hewitt et al., 2005). The development of affective skills, from the simplest to the most complex, improves learning across all three domains (Duncan-Hewitt et al., 2005). The higher levels of the cognitive domain are strenuous to reach if teachers do not introduce the supplementary skills of the affective domain in teaching (Griffith & Nguyen, 2006).

However, Bloom's taxonomy and its revisions has been criticized for many reasons; it is said to be too abstract and not empirically validated focusing too much on individual students rather than social learning. Taxonomy is based on the idea that learning progresses from an easier level to another, although in reality learning is not quite so straightforward. Furthermore, motivation has not been taken into account in the theory, even though it is an essential part of learning (Kompa, 2017; Tutkun, Güzel, Köroğlu, & Ilhan, 2012).

Technology can change students' and teachers' attitudes positively towards Mathematics (Alessi & Trollip, 2001; Lin, 2008) and, for example, towards problem-solving (Rahman, Ghazali, & Ismail, 2003). Sulakshana (2005) points out that when technology is used effectively, the learning environment becomes flexible and education shifts away from teacher-led teaching to be more student centered. According to Mayer (2019), computer software provides instant personal feedback allowing students to move forward at their own pace. They can go back to easier problems or move on to more difficult according to their own skill level without getting frustrated or bored.

Technological devices and digital games have become part of young people's lives. Computers have been part of learning since childhood for many students (Pivec, 2007). Computer games could be a useful help in learning and enhance learning in many areas of mathematics (Alessi & Trollip, 2001; Kiili, Koskinen, Lindstedt, & Ninaus, 2018; Mayer, 2019; Rieber, 2001). The games designed for learning purposes or serious games has increased significantly over the past 15 years (Wouters & van Oostendorp, 2017). Most of the present educational games are edutainment, such as game-like-drills and practice activities that mainly address the extrinsic motivational side of games and are used to achieve lower level learning goals, for example, drilling multiplication tables or arithmetic. The possibilities that modern Information and Communication Technology (ICT) offers are not used to attract intrinsic motivation and, thus, gain deeper learning. The role of the teachers is a crucial factor; equally, the selection of appropriate games in achieving good learning results with serious games (Iten & Petko, 2016). However, teaching professionals do not have much experience in designing learning games (Ma, Williams, Prejean, & Richard, 2007; Pivec, 2007).

While the use of learning games has increased tremendously, the amount of research related to them has also increased. In particular, the effectiveness of games has been studied widely from a learning perspective, focusing mostly on the cognitive dimension of learning (Wouters & van Oostendorp, 2017). More research concerning educational games are needed and questions like: Under which conditions is a given game an excellent and useful learning tool? Which features of the games support learning and student excitement? (Boyle, Connolly, Hainey, & Boyle, 2012; Boyle et al., 2016; Pivec, 2007).

Interest in the impact of learning games on the student's affective domain has grown steadily in recent years (Vankúš, 2021). Vankuš's (2021) carried out a systematic literature review and applied a PRISMA statement on 57 journal articles finding out that the majority of the papers (84%) reported positive effects of game-based learning on students' affective domain. The most articles in the review reported that games improve positive attitudes and beliefs towards mathematics, and increase motivation and engagement. The impact of beliefs and attitudes on mathematics learning is as such a broad field of research, and the first studies related to attitude were published more than 60 years ago (Martino & Zan, 2015). The relationship between computer games and attitudes has been studied to some extent (White & McCoy, 2019; Yesilyurt, Dogan, & Ilhan, 2019), but above all research related to affective learning has consisted of motivational research (Boyle et al., 2016; Higgins, Huscroft-D'Angelo, & Crawford, 2019). Various motivations for playing games have been studied a lot, but more detailed and systematic research is needed (Boyle et al., 2012). Moreover, using qualitative research looking into players' experiences in playing games and distinguishing motives and subjective experience as separate components (Boyle et al., 2016). Vankuš (2021) found that some of the studies on affective learning and game play produced mixed results most probably because of inappropriate research instruments, mistakes with the game design, or due to poorly selected or designed research groups, and suggested that this area should be further explored using carefully designed and conducted studies.

In addition to computer games, the materials and tools available for learning and instruction of mathematics provided by the Internet have also increased significantly. Intelligent tutoring systems (ITSs), computer software designed to emulate a human tutor's behavior and guidance, have been developed and researched since the 1970s (Nwana, 1990). Many studies have documented that ITSs are powerful learning tools for mathematics teaching and learning, like UZWEBMAT teaching the unit of probability (Özyurt, Özyurt, Baki, & Güvenb, 2013) and supplementary application-based tutorials in the multivariable calculus (Verner, Aroshas, & Berman, 2008). Few empirical studies have addressed what kind of students the ITSs are most suitable, like for Steenbergen-Hu and Cooper (2013), who reported a positive impact of ITS for students who are motivated and can self-regulate learning but not necessarily too low achievers in K–12 students' mathematical learning and instruction. A significant feature of it is the accessibility of instructional resources such as hints, multimedia examples, tutorial dialogues, and other tools the student can use during problem-solving (Beal, Arroyo, Cohen, Woolf, & Beal, 2010). These tools help the student learn to solve increasingly challenging problems in a particular area (Beal *et al*., 2010). I chose an intelligent tutoring system, ActiveMath, to be part of this study. The ActiveMath system guides the student in self-regulated learning, can adapt to individual level of knowledge, personal interests, and goals. ActiveMath system is configurable with pedagogical strategies and various content. The system utilizes several AI-techniques achieving an adaptive course generation and student modeling. Furthermore, the learners can do interactive exercises and get immediate feedback as ActiveMath has a unique mechanism that provides different feedback strategies and modifying the runtime behavior of an exercise (Melis & Siekmann, 2004a; Narciss et al., 2014).

1.2 Research design and research questions

In the study, I follow a constructivist epistemology, by which learners construct their knowledge within their realities and contexts. In order to support the learning process, the construction of knowledge needs to be interpreted. For testing and building theory, an action design research (ADR) approach was used in an interpretivist way (Sein, Henfridsson, Purao, Rossi, & Lindgren, 2011). The methodology combines design science (Hevner & Chatterjee, 2010), which is often used to develop artifacts, and action research where the researcher is involved in improving the current processes. Design science research is a pragmatic, problem-solving paradigm that suits for studying complex issues in many different environments. shows a multi-environment extension of the design science method used in this study. The progress of this study is one instance of how the elaborated action design research process model by Mullarkey and Hevner can be used (2019). I will describe more the research methodology and action design science research method in Chapter 3.

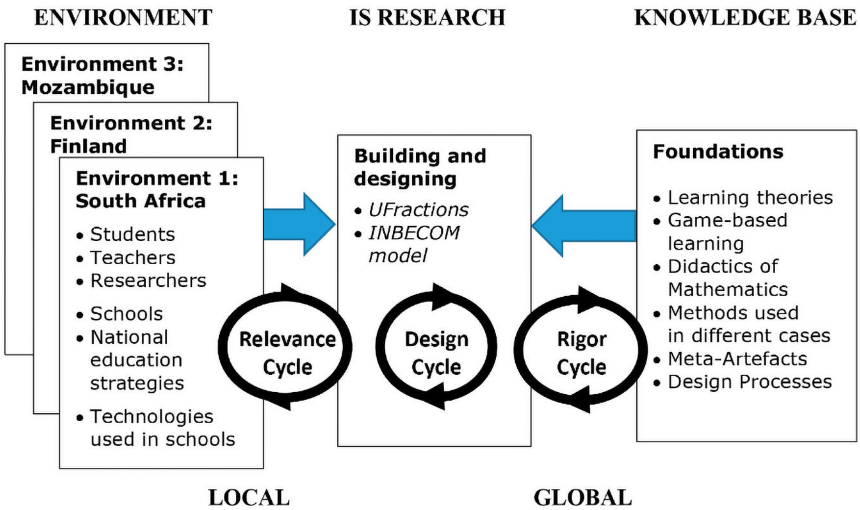

Figure 1: Multi-environment extension of the design science method used in this study (Hevner, 2007).

This research comprised of seven underpinning research questions, which were addressed in six articles (Listed on page iii). The seven research questions and their objectives that emerged from the environments shown in Figure 1 were:

Q1 What are the dimensions of learning mathematics with technology-enhanced learning?

By answering this research question I aim at defining a dimension taxonomy of learning that can be used as a framework in the design and implementation of technology-enhanced mathematics teaching and learning. This taxonomy is largely based on previous research, and for its development I make an extensive literature review in Chapter 2. At the same time, the reader of the dissertation will become familiar with this topic. The goal is also to determine how follow-up research I did in different contexts confirm or infirm the taxonomy.

Q2 How can artifacts for mathematics learning be developed?

The aim of this research question is to identify the features of the artifacts for learning mathematics that meet the requirements of research context, and to describe the process of building artifacts. Two artifacts were created and evaluated during this research; (i) a *pedagogical INBECOM model* for mathematics learning advocating both behavioristic and constructivist perspectives, and (ii) a story-based *UFractions mobile game for learning* of fractions incorporating tangible manipulatives. Chapter 6 elaborates on the development of the INBECOM (Integrating Behaviourism and Constructivism) pedagogical model, the underlying idea of this study. I will describe the evaluation of the UFractions game in subsection 4.2.1.

Q3 What are the considerations to be taken into account during the design of a mobile game for learning fractions in diverse contexts?

The third research question aims to examine how different cultures influence game design. Exceptionally, in this study, I designed the game in a developing country and then evaluated it in two other contexts, including a developed country. In general, the development of games is just the opposite. This process is called a reverse transfer and it facilitated me to analyze the contextual factors of technology for learning. The research contexts are described in Subsection 4.1.

In addition, the aim of this reseach question is to investigate how factors that interfere gaming (disturbance factors, DFs) and motivational factors could be taken into account in the design of games.

Q4 What strategies do we need in order to compile a mobile game for learning for diverse contexts?

The math learning game UFractions is tested in three different contexts, and these tests are evaluated to see what factors need to be considered in different contexts. The aim is to derive practical guidelines compiling a mobile game for game users and educators from the interplay of disturbance factors (DF) and motivations.

Q5 How can artifacts, such as games for learning, address the different dimensions of learning mathematics?

The aim of this research question is to find out how different dimensions of mathematics learning can be taken into account when choosing suitable technologies and their uses for mathematics learning. An analysis of specific example artifacts is used to see how they map onto dimension of learning mathematics, for example development of a taxonomy for the variety of play motivations in the UFractions. Based on this analysis, my aim is to give instructions on how to analyse an artefact, in order to establish what dimensions it is mapped onto. In Chapter 2, the dimensions of learning mathematics are examined from the perspectives of different theories, and in Subsection 2.5, the following three dimensions are selected for more specific examination:1) Motivation theory, 2) Learning theories, 3) Bloom’s taxonomy: domains of learning.

Q6 What is the relationship between motivation and disturbances during learning mathematics with games for learning?

Sometimes the game is played even if there are a lot of factors that hinder it, and sometimes even the finest game goes unplayed in class. The aim of this research question is to find out how the game motivations and disturbance factors (DF) are related. By becoming aware of these relations, we can design more motivating educational games and give guidelines for game developers, users, and educators.

Q7 How does an approach of creating interventions and artifacts based on the taxonomy for dimensions of learning mathematics trigger affective learning?

The aim of the last research question is to examine how technology-enhanced learning in particular triggers affective learning, looking in particular at the different dimensions of mathematics learning. To answer this question, I am extensively analyzing the entire research process in terms of affective learning.

1.3 My contribution

My PhD path started with a focus on generating an instructional design theory for mathematics teaching and learning using educational technology tools as a support mechanism. In my research, I delved into the cognitive, psychomotor, and affective dimensions of learning, along with exploring motivation and learning theories in the context of integrating technology into mathematics education. I took a cross-cultural perspective to ensure the findings had broader relevance.

The two artifacts created during the adventure were (i) *a pedagogical model INBECOM* advocating innovative behaviorist-constructivist perspective towards mathematics teaching and learning, and (ii) *a story-based context-aware mobile game UFractions* using mathematical manipulatives.

The pedagogical model named INBECOM (Integrating Behaviorism and Constructivism in Mathematics) utilizes games and tutoring systems to foster productive and meaningful learning of mathematical concepts. Fractions were chosen as a mathematical concept under examination as it is known throughout mathematics communities as one of the most challenging concepts (Nabors, 2003).

In a mobile game UFractions, the player solves the fraction problems associated with the story of two leopards living in Savannah. The combination of the two different learning tools provided us with new opportunities not only to motivate the pupils but also to create new connections between a fictitious story, fraction theory, and physical objects.

Based on the data I gathered, I devised *a taxonomy to categorize motivational factors* within the UFractions game and analyzed the relationship between these motivations and Disturbance Factors (DF). This insight can inform the development of more engaging educational games and offer valuable guidance to game developers, users, and educators.

Moreover, I formulated *a dimension-based taxonomy to facilitate the teaching of fractions with technology*, which includes domains of learning, learning orientation, and learning motivation.

My research was conducted in accordance with the principles of co-design in the group I lead. Research design was made together with professor Sutinen and professor Blignaut, who were my supervisors. In addition, Professor Teemu H. Laine attended actively to the research design especially for Paper II, where he is the main contributor. I was the main contributor to all the manuscripts except for article II. My role in each article is described in Appendix 1 in more detail.

Limitations of the study relate to the following aspects. I incorporated only two technological tools in the research, the *UFractions* mobile game, and the *ActiveMath* intelligent tutoring system. Although the study took place over a multitude of years from 2007-2019, it made use of a snapshot evaluation; a longitudinal analysis could have provided broader results. Development and evaluation of the UFractions game and ActiveMath content were resource-intensive so far that the INBECOM model was only tested once in one mathematics course.

This study provides a deep understanding of the use of educational technology in mathematics teaching and learning in different contexts. Cross-cultural and cross-countries approach was chosen to ensure wider generalisability of the research. I explored both design a mobile game and bringing it to different environments after the development. Both motivations and distractions were looked at when using a mobile game as a learning tool. I incorporated the new UFractions mobile game into a new pedagogical model with both constructivist and behavioristic elements. This study did not so much examine the effectiveness of technological tools in the traditional sense but focused on what affordances technology brought to the learning situation and process. The results of this research can be utilized by both teachers and curriculum planners, as well as instructional designers.

1.4 My journey

The methodology of this research includes many qualitative parts, and the results are partly interpreted based on my own experience as a teacher. Since this study base itself on a constructivist epistemology, it is inevitable and necessary to make interpretations and guidelines that are subjective and, hence, based on my background and knowledge base.

I have always been interested in almost everything. After high school, it was tough to choose a career place from several opportunities. I chose mathematics because I thought that mathematics gives explanations to the structures of the sophisticated world. After completing my Master's thesis on Möbius transformations in R^3 (Turtiainen, 2002), I immersed myself in the world of equations for some years and completed the licentiate thesis on Modified Clifford Analysis and Dirac Operator on Manifolds (Turtiainen, 2004). During my years as a researcher in mathematics, I got an opportunity to imagine and feel the beautiful world of mathematics. Professor Ilpo Laine described the complex analysis equations so well that they started to live their own life in my mind. Professor Laine, for example, described a part of an equation as a ripple of a wave. Professor Sirkka-Liisa Eriksson and Professor Heinz Leutwiler familiarized me with the higher dimensional analysis and applications using geometric algebras. They taught me how to handle equations in Clifford's geometric algebra that is the smallest algebra extension of the n-dimensional Euclidean space inheriting its algebraic, geometric, and metric properties (Eriksson-Bique & Leutwiler, 2001).

However, after doing mathematics research for some years, life brought me as a teacher in a folk high school. Folk high school and its unique community pedagogy ("folk high school pedagogy") have inspired me a lot. Community pedagogy has its roots in the Danish folk high school movement of the early 20th century, originating in the educational ideas of Grundtvig (Kulich, 1964), who emphasized the "living word" and the importance of local people's experiences. Grundvig wanted to provide education to local people, and believed that the "living word" gives youth inspiration and kindles them to action—it is something more than the "dead word" that is academic knowledge of the Latin schools.

While working for over ten years as a mathematics teacher, mostly to 16-17 years old students and adult immigrants in the Evangelical Folk High School of Kitee, I noticed that mathematics seems to be tedious and challenging to learn for most if not all students. Specifically, abstract concepts cause problems for students. The experimental dimension of the learning process is fundamental because doing mathematics is being able to solve mathematical problems (Polya, 2004). A fundamental problem of learning mathematics is that mathematics is done inductively or empirically while it is taught deductively, theoretically (Polya, 1962).

During my teaching career, I have taught several students with learning disabilities and limited language skills. It has aroused my interest in different ways and different dimensions of learning. I have also questioned the meaning of school mathematics; what kind of mathematics do students need to cope with life or to succeed in life? Does it make sense to teach many things if most of them are learned only superficially? Is there enough time for creative thinking in mathematics lessons?

In my first years as a teacher, I was shocked about the negative feelings my students had towards mathematics. I asked them to draw pictures of mathematics lessons they had experienced during their nine years of primary education. The result was a variety of drawings with dark colors, gallows, and skulls, as well as pictures that describe headache and constriction. Since I have always liked mathematics, it was a surprise to me that so many felt it distressing. I started to think about how senseless it is to teach without taking into account the feelings of the students, and how the negative feelings affect the learning results.

Professor Johannes Cronje from South Africa came to visit my school in 2007 and presented his idea of a new structure to the mathematics lessons consisting of a behavioristic part and a constructivist part (Cronje, 2006). Prof. Cronje suggested to me that I could try the model, for example, with mathematical games. I became enthusiastic because I wanted to find new ways to teach mathematics in a meaningful and motivating way. I felt that this model might be a solution to change students' attitudes toward mathematics. At the same time, I wanted to explore how people learn mathematics by using technology. During my teaching years, I had noticed that students needed learner-centered actions to obtain motivation and to be able to search for knowledge by themselves, but also a teacher-centered approach to obtain necessary skills. Doing the research alongside work took many years, but it also enabled me to learn a great deal during this interesting journey.

1.5 Structure of the thesis

This dissertation examines the field of educational technology, especially ICT-supported learning of mathematics. The thesis contains seven chapters providing a comprehensive description of the different stages of the study. The present introduction explains the background and demand for this research, describes the research problem, paradigm, and aim of the research. In the introduction, I briefly discuss the story of my research and the progress of my journey. This study took place in an exceptional environment, including diverse countries and cultures. Therefore, Chapter 2 illustrates the background and problem space in detail. Chapter 3 describes the research design and used methodologies. Chapter 4 elaborates on the research narrative.

The thesis follows the idea that every project produces different knowledge (Drechsler & Hevner, 2018). It is important to describe as broadly as possible the contribution of this study without omitting solution design knowledge. Accordingly, Chapter 5 summarizes the results reported in the publications included in this thesis, and Chapter 6 describes the aspects of the artifact evaluation process in the several cases in different environments, such as the different versions of the INBECOM model and factors influencing the change of the model during the years. In Chapter 7, I give a summary of the answers to the research questions, reflect on the explanations and restrictions of the results, including remarks on methodology and content, and present recommendations to future research.

2 Background

This chapter covers a description of the problem space, including the main learning theories, learning issues, historical foundations of the study of learning, as well as an illustration of the technological tools used in mathematics learning. This dissertation creates a pedagogical model that uses both behaviorism and constructivism as background theories of learning. Selection of the technological tools for learning fractions based on how and what kind of learning takes place according to different learning theories. In Chapter 6, I describe in more detail the development of the model and what kind of tools were chosen for the experiments, as well as what learning theories underlie which tool. The perspectives of different learning theories as well as theory of math games were also taken into account in the development of the mobile game UFractions, that is represented in Section 4.2.1.

2.1 The main learning paradigms

In this section, the primary learning paradigms, *behaviorism, cognitivism, and constructivism,* are defined and discussed. There also exist other learning theories, but they are not addressed here because they fall outside the scope of this study. These three learning theories were also the three most influential theories of learning in the 20th century (Ertmer & Newby, 2013). They are presented here in chronological order, although it should be noted that they all appear in classrooms and computer-assisted teaching today. Primary sources for presenting the theories are “Learning theories an educational perspective sixth edition” (Schunk, 2012), “Theory of teaching, from constructivism to realism” (Puolimatka, 2003) and ”Constructivism: A psychological theory of learning” (Fosnot & Perry, 1996).

2.1.1 Behaviorism

During the past hundred years, the reigning learning paradigm has changed many times. The most dominant learning theory of the 20th century has been **behaviorism**, whose early representants, Thorndike (1874-1949), Pavlov (1849-1936), Watson (1878–1958) and Skinner (1904-1990), based their theories, for the most part, on animal research (Tynjälä, 2000). According to the behaviorist concept of learning,

learning is a reaction to stimulus coming from outside, and emotions, motivation, or other mental functions do not affect the learning process (Puolimatka, 2003; Tynjälä, 2000). Behaviorism is a conditioning theory based on the view that learning means preferably an increase in knowledge, and learning is developed through interactions with the environment (Schunk, 2012). The students' progress is assessed by measuring observable results, i.e. behaviors in the given task (Fosnot & Perry, 1996).

In behaviorist instruction, the learning material divided into separate components and then connected to the learning goals, and the right performance of the learner is rewarded (Schunk, 2012; Tynjälä, 2000). One base of learning is a step-by-step progressive practice, which builds the student's knowledge and motivation gradually (Puolimatka, 2003). Behaviorism recognizes that learners do not progress at the same pace, and individualizing instruction would improve effectiveness (Schunk, 2012). The beginning of the 20th century brought the discovery that automated machines could enhance teaching effectiveness (Skinner, 1958). Programmed instruction, referring to instructional materials, developed following operant conditioning principles of learning, would make it possible to begin instruction at learners' present performance levels and allow them to progress at their rate step-by-step (Schunk, 2012).

The design of the first computer-based learning programs was based on the behaviorist idea that the learning material is presented in small parts (Tynjälä, 2000). Educational games, so-called drill-and-practice-games, relate to behaviourist learning (Alessi & Trollip, 2001). Puolimatka (2003) suggests that behaviorist games can secure, for example, students with learning disabilities acquiring necessary skills and knowledge. However, educational software based on behaviorism is often dull, frustrating, difficult to apply to new situations (Alessi & Trollip, 2001), and it produces only a lower degree of learning (Ma et al., 2007).

2.1.2 Cognitivism

The behaviorist model of learning is insufficient to describe complicated processes of learning and does not explain the cognitive change occurring in understanding new concepts (Fosnot & Perry, 1996). Therefore, the cognitive trend began to develop in the 1950s. *The cognitive view* on learning suggests that intangible factors can also affect learning, for example, motivation, attitudes, and memory. According to cognitivism, the learner's actions and inner processes are essential for learning, and learning means a change in a person's knowledge structures (Puolimatka, 2003). People are not "programmed animals" that only respond to environmental stimuli; learning requires the active participation of the learner, and actions are a consequence of thinking (Ertmer & Newby, 1993; Fosnot & Perry, 1996). The cognitive approach focuses on thinking processes by supporting the learners to choose the appropriate learning strategy, and emotions and beliefs are also believed to have an impact on the learning (Ertmer & Newby, 1993).

Moving from a behavioral orientation to a cognitive orientation has affected the instructional design to a great extent. Hence, instead of learning through the material provided by the system, the student interacts with the system (Cooper, 1993; Ertmer & Newby, 1993). Computers can act as “Mindtools” for interpreting and organizing students’ personal knowledge (Jonassen, 2000). Instructional design made according to both behavioristic and cognitivist views uses feedback, as well as learner and task analysis, but for different reasons. Behaviorists use feedback for changing learner behavior, but cognitivism use feedback to guide and support mental connections (Ertmer & Newby, 1993). Accordingly, behaviorists aim to assess the learners' knowledge level, whereas cognitivism searches for indications of existing mental structures (Ertmer & Newby, 1993). Mental-model building can be supported, for example, by concept mapping tools that allow students to visually illustrate relationships, and mind map applications have been used to extend student's cognitive capacities (Martínez, Pérez, Suero, & Pardo, 2013).

2.1.3 Constructivism

Constructivism is a present trend of cognitivism, where the central part of learning is the learner’s own role, understanding of knowledge and thinking. Piaget (1896–1980) and Vygotsky (1896-1934) reknowed psychologists whose theories are the basis of constructivism (Puolimatka, 2003). According to constructivism, information cannot be transferred to the learner as such, but the learner is an active constructor of knowledge. In other words, learners form knowledge structures from their earlier knowledge and experiences (Schunk, 2012; Tynjälä, 2000). When existing structures are challenged, the learner generates possibilities and contradictions, enabling the development of novel, increasingly meaningful mental structures. Gradual structural shifts in learners' perspectives are constructed when they ponder meanings (Fosnot & Perry, 1996).

Constructivism has changed the role of the teacher from an information provider to an instructor of learning processes. The teacher’s task is to support the student’s active attempt to build ideas and models (Ertmer & Newby, 1993; Puolimatka, 2003) and design the learning environment to socially and physically support the development of skills (Rauste-von Wright, von Wright, & Soini, 2003). There are many learning models based on constructivism, such as inquiry-based and problem based learning (Puolimatka, 2003; Rauste-von Wright et al., 2003).

Constructivism uses the concept of disequilibrium, facilitating learning. Errors, such as wrong answers in mathematics, are not avoided or even minimized because they are regarded as a result of learners' conceptions and compelling learning experiences. Learners are exposed to challenging, open-ended investigations in meaningful contexts (Fosnot & Perry, 1996). Constructivism has effected curricula involving students actively in learning, providing experiences that providing experiences that get students to change their beliefs and reorganize their knowledge structures, and integrating studying a big topic from multiple perspectives (Schunk, 2012). Constructivism also takes into account the conditions of learning since knowledge is created together with others, and the environment contributes to an individual's learning (Cobb, 1994). In learning communities, the group transforms their individual, subjective beliefs into inter-subjective, and finally objective mutually agreed construct (Kilpatrick, Jones, & Barrett, 2003). A widely used way in educational sciences is to divide constructivism into individual-centered individual constructivism and social constructivism, these differ in the extent to which the activity is seen as an individual, and to which extent as a social or cultural phenomenon. In individual constructivism, a person is thought to construct knowledge with the help of internal categories or interpretive frameworks, while in social constructivism, knowledge is constructed together and is context-bound (Puolimatka, 2003).

The aim of the constructivist instructional designer is assisting learners to actively explore complex topics that will force them into thinking in a given content area as experts (Schunk, 2012). Learners are encouraged to develop a deeper understanding of the content being learned (Alessi & Trollip, 2001). Information from many sources is essential, and it is presented in different ways (Ertmer & Newby, 1993; Schunk, 2012). Educational technology has used constructivism as a design base, for example, in simulations, virtual realities, and so-called open-learning environments, designed to arouse learners' thinking (Alessi & Trollip, 2001). The problem of the constructivist teaching theory is often a tendency to apply constructivist models one-sidedly and the thought that some specific constructivist model can solve all problems connected to teaching (Puolimatka, 2003).

2.1.4 Comparison of learning paradigms

Learning strategies are often competing for options, and the supporters of different trends criticize other approaches. Especially the behaviorist-constructivist debate has been ongoing for years (Alessi & Trollip, 2001; Ertmer & Newby, 1993). One approach to the debate is that there does not exist a superb paradigm in all instruction, but different strategies based on different theories appear to be necessary and chosen according to the learners' level of knowledge and learning goal (Ertmer & Newby, 1993). Ertmer and Newby (1993) suggest that instructional designers must intelligently choose the appropriate methods based on the learners' present competence level and the type of learning task (Figure 2).

It has been common to plot these two approaches at opposite ends of a straight line (Cronje, 2006). However, criticizing this two-ended continuum results in the realization, that if that were the case, behaviorism would mean the same as the unconstructive learning theory (Puolimatka, 2003). The behaviorist[1]- constructivist division is not a problem if these two theories are seen as possibilities that can complement each other. According to Cronje (2006), learning events can contain both behavioristic and constructivist elements. He suggests that these two approaches are situated at right angles to one another and presented as rectangle lines to form a plottable area. Dividing this rectangle into four sections gives different views on learning and teaching (Figure 3). The Immersion Quadrant is low in both elements of behaviorism and constructivism. An outside entity does not determine the learning process, and learning takes place through crisis management. In the Injection Quadrant, the approach of teaching is direct instruction of public pre-produced information. In Construction Quadrant learners construct new artifacts. Finally, in Integration Quadrant, “integration is the combination of instruction and construction in-appropriate conditions" (Cronje, 2006, p. 9), and the instructional designer selects elements from both the behavioristic and constructivist domains to achieve the desired outcome. This integrative viewpoint of learning theories would lead to more realistic and meaningful learning and teaching strategies. This dissertation originated from this idea of combining the two learning theories. In particular, the development of the pedagogical INBECOM model described in Chapter 6 accurately describes how learning theories were viewed.

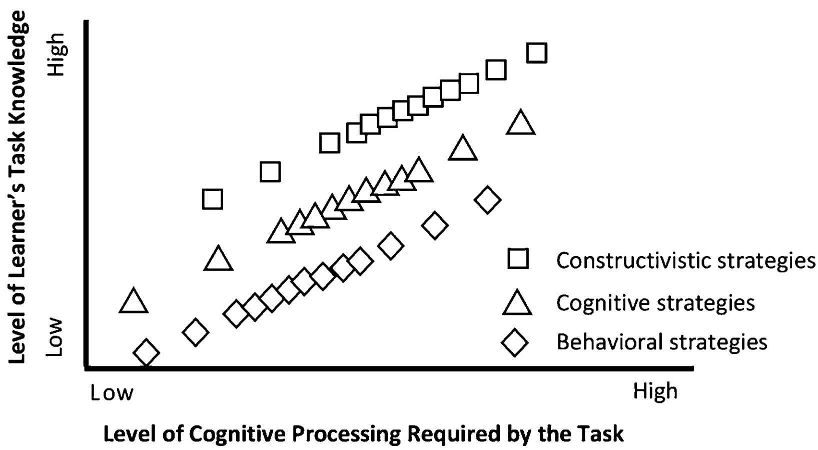

Figure 2: Comparison of the associated instructional strategies based on the learner's level of knowledge and the level of cognitive processing (Ertmer & Newby, 1993, p. 69).

[1] Cronje uses the term *objectivist* instead of behaviorism following Jonassen (1991).

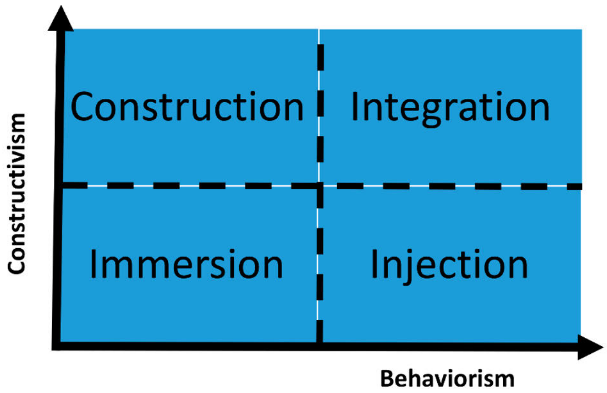

Figure 3: Four-quadrant model (Cronje, 2006).

2.2 Dimensions of learning

As described in the introduction chapter, in addition to combining two learning theories, motivation theory and Bloom's taxonomy classifying learning according to three domains, were chosen as the design framework for this dissertation and the dimensions of mathematics learning when applying technology. This section examines motivation theories and describes Bloom’s three domains of learning in more detail: cognitive, psychomotor, and affective.

2.2.1 Motivation

Motivation is a psychological feature driving people to action (Cofer & Appley, 1967). The fundamental issue in psychology has been seeking answers to the question, "Why do people do x?" (Chmiel, 2000). Psychology researchers have investigated and explained the psychological features of motivation extensively and created different theories trying to explain the motivations fundamental to human behavior. Theories on motivation are divided into *process theories* and *content theories*. Process (or cognitive) theories describe and analyze how human behavior is maintained and directed in the self-directed human cognitive processes. Content theories deal with the individual's needs and essential factors, which are generating, sustaining, or stopping different activities (Borkowski, 2005; Miner, 2005). Motivation can be divided into "intrinsic" and "extrinsic" motivation. Intrinsic motivation refers to doing something because of inherent enjoyment, and extrinsic motivation refers to doing something because of external rewards such as good grades (Deci & Ryan, 2010). As a part of this study, I instantiated a taxonomy for a variety of play motivations in the UFractions game, and studied how the different features of the game are related to these motivations. Chapter 5 and answer to research question Q5 discusses motivators in more detail, and Article IV presents motivation theories more broadly.

2.2.2 Bloom's taxonomy

In 1956, Benjamin Bloom, within the behaviorist paradigm, classified learning according to three domains: cognitive, psychomotor, and affective, ranging from uncomplicated to more complex learning outcomes (Krathwohl et al., 1964). Bloom developed the cognitive model to focus on the acquisition and development of learners' intellectual skills encompassing knowledge, comprehension, application, analysis, synthesis, and evaluation (Bloom, 1956). The cognitive model has become the standard for educators to identify and classify learning objectives, as well as to develop teaching and learning activities (Reigeluth, 1999). Many other theories, after that, have proposed new taxonomies for different types of learning in the cognitive domain (Ausubel, 1968; Gagne, 1985; Merrill, 1983). Additionally, Bloom's taxonomy was revised by Anderson, Krathwohl, and Bloom (2001) as well as synthesized by Reigeluth (1999) to be more suitable for the information age, see Figure 4.

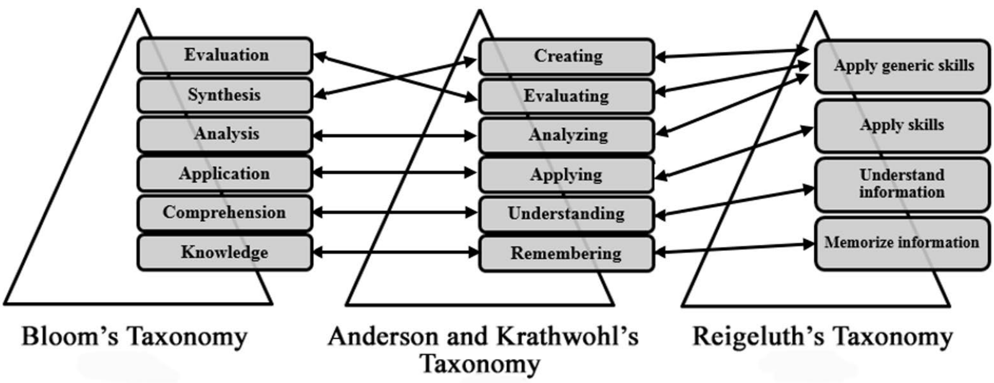

Figure 4: Bloom's taxonomy and its subsequent revisions by Reigeluth (1999) and Anderson et al. (2001).

Reigeluth (1999) categorized the fundamental psychological theories (behaviorist, cognitive, constructivist) within each level of learning. The knowledge level began to *memorize information,* which is the type of learning behaviorists have addressed. The stage *understands information* is similar to Bloom's corresponding stage *understanding*, in which the students understand the relationship among the elements of knowledge and organize the acquired knowledge into existing structures. To *apply skills* is similar to Bloom's *application* level, requiring more than just memorization and understanding (Reigeluth, 1999). Cognitivist theories have explained remarkably learning at this stage (Reigeluth, 1999). *Apply generic skills* encompasses Bloom's *analysis, synthesis, and evaluation.* At this level, learning becomes more complex, including higher-order thinking skills, learning skills, and metacognitive skills. Related to this learning stage, the cognitive approach emphasizes meaningfulness and helping learners to organize knowledge, and constructivism, moreover, emphasizes forming new relationships with knowledge, new representations of knowledge, and making interpretations of the real world (Alessi & Trollip, 2001; Duncan-Hewitt et al., 2005; Even & Tirosh, 2002).

The cognitive domain was first developed extensively, and the other two domains—affective and the psychomotor—were expanded by Anderson *et al.* (2001). Although the cognitive domain receives the most emphasis (Adkins, 2004; Bolin et al., 2005; Griffith & Nguyen, 2006), as it focuses on the acquisition of knowledge and skills, all three domains should be considered during teaching and learning (Griffith & Nguyen, 2006). The highest levels in the cognitive domain are difficult to achieve if teachers do not also develop complementary skills in the affective domain (Griffith & Nguyen, 2006). The emotional side of learning is often neglected because it is complex and students are thought to cope with it on their own (Duncan-Hewitt et al., 2005). Moreover, educators do not have the time to consider the affective domain during their teaching preparation (Griffith & Nguyen, 2006). Also, more emphasis is placed on the objective assessment of knowledge rather than on the affective aspects of learning (Bolin et al., 2005).

Figure 5 outlines the five levels of the affective domain of learning (Krathwohl et al., 1964). The advancement in psychological theories has inspired to explore the affective domain in mathematics education. The first studies of the affective dimension of mathematics focused primarily on attitudes towards mathematics (Ignacio, Nieto, & Barona, 2006). The positive attitudes towards mathematics tend to change to more negative when students grow older and move to secondary school (McLeod, 1992). Attitudes can change in both directions rapidly (Hannula, 2002; Ruffell, Mason, & Allen, 1998). In the last decades, research has addressed the beliefs and emotional reactions of mathematics learners (Hannula, 2002; Ignacio et al., 2006; McLeod, 1992). Beliefs and attitudes are generally stable, but emotions may change quickly and vary in the level of intensity, the time that they take to develop and in the degree they effect on cognitive learning (McLeod, 1992). Emotional reactions play a significant role in mathematics learning, and emotions are a direct link to motivation (Hannula, 2006).

Four axes relating to beliefs can be established: mathematics, oneself, mathematics teaching, and the social context in which mathematics is learned (McLeod, 1992). Notably, the student's self-concept as a mathematics learner is one of the basic descriptors of the affective domain in mathematics (McLeod, 1992). Ignacio et al. (Ignacio et al., 2006) observed that the students' beliefs about themselves as mathematics learners are related neither to gender nor to the year of secondary education that they are studying. Students who are doubting their abilities give quickly up in the face of difficulties, and their negative beliefs weaken their performance (Chapman, 1988). On the other hand, confidence correlates positively with achievement in mathematics (Reyes, 1984). McLeod et al. (McLeod, Metzger, & Craviotto, 1989) observed emotional reactions of solving problems to be the same for expert and novice problem solvers. However, experts can control their feelings better than novices; they stay flexible and try various strategies.

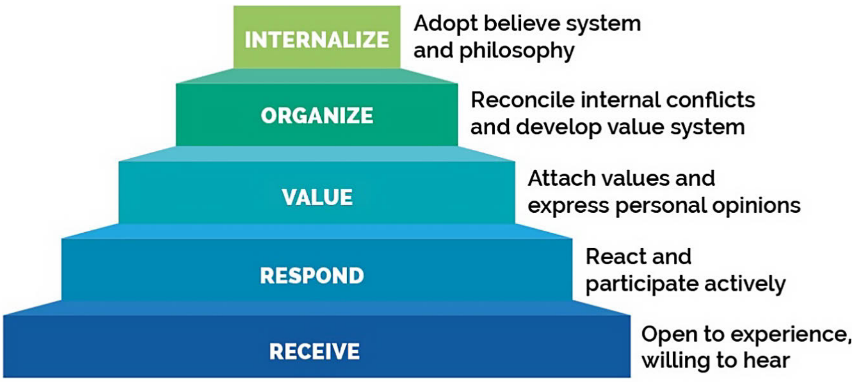

Figure 5: Five stages of Krathwohl's affective domain (Krathwohl et al., 1964).

2.3 Technology, pedagogy, and mathematics

Indeed, the use of technology in mathematics teaching brings variety to the lessons by providing consistency and context to mathematics, giving learners opportunities to experiment and test mathematical understanding and new ways to visualize concepts, explore ideas and discover relationships (Bray & Tangney, 2017; Olive et al., 2009). The use of technology encourages teachers to use problem-solving and cooperative strategies. Mathematics learning can be made more interesting, challenging, and practical by introducing realistic data into the classroom. This section addresses the relations between technology, pedagogy, and mathematics.

A vital issue to address when using technological tools is the question of *whether they support and promote learning*. Drijvers et al. (2016) have carried out an extensive literature review on the subject and concluded that the use of technology as a tool in teaching improves learning outcomes, but with small average effect sizes. They indicate that the benefit of using technology in mathematics education does not appear to be very strong when looking at experimental results. Furthermore, most review studies cover experimental and quantitative studies and do not differentiate educational level, the technology used, and how technology is integrated into the teaching (Drijvers et al., 2016).

Another crucial factor is *how the tools are used in the classrooms*. The role of the teacher is significant, according to many studies, when integrating technology into learning mathematics (Drijvers et al., 2016). Essential factors for using technology effectively are identified to be mathematical knowledge, pedagogical skills, pedagogical content knowledge, curriculum knowledge, and beliefs (Drijvers et al., 2016). One framework for teacher knowledge for technology integration is called technological pedagogical content knowledge (TPACK, or technology, pedagogy, and content knowledge) shown in Figure 6 (Koehler & Mishra, 2009).

According to the TPACK model, teacher's knowledge consists of:

- Content knowledge (CK) about the subject matter to be learned or taught
- Pedagogical knowledge (PK) about the processes and practices or methods of teaching and learning
- Technology knowledge (TK) about applying information technology productively at work and in everyday lives, and to continually adapt to changes in information technology.

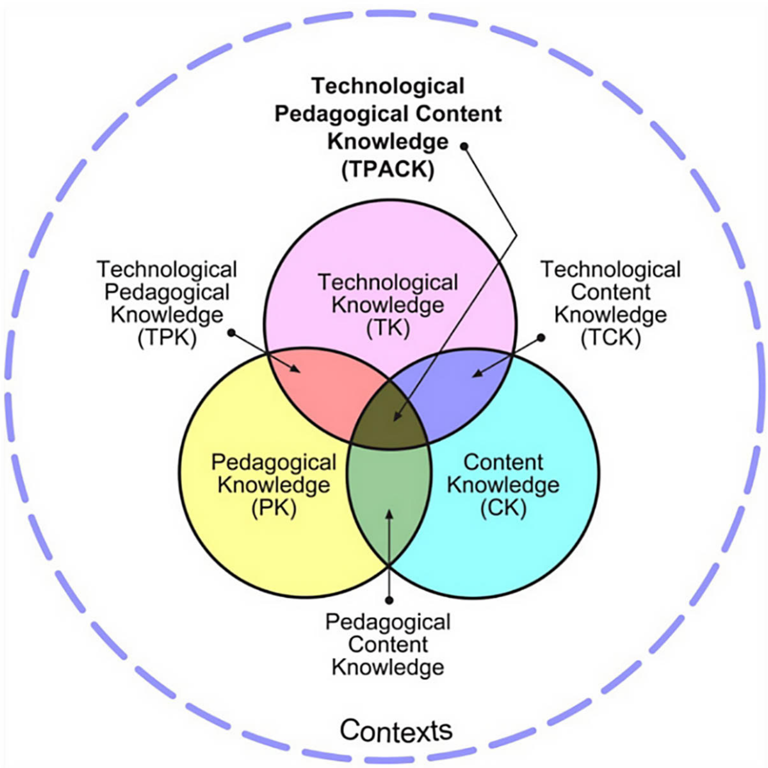

Figure 6: The TPACK framework and its knowledge (Koehler & Mishra, 2009).

When these areas of knowledge overlap, the following teacher expertise emerge:
- Pedagogical Content Knowledge (PCK) about the core business of teaching, learning, curriculum, assessment, and reporting
- Technological Content Knowledge (TCK) of how technology and content influence and constrain one another.
- Technological Pedagogical Knowledge (TPK) of how teaching and learning can change when particular technologies are used in particular ways.

Koehler and Mishra (2009) state: "Underlying significant and profoundly skilled teaching with technology, TPACK is different from knowledge of all three concepts individually. Instead, TPACK is the basis of effective teaching with technology. It requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones." (p. 66)

Bray and Tangney (2017) developed a classification based on the pedagogical approach used by completing a systematic analysis of 139 studies of technology interventions in mathematics education. In the classification, they also considered the level of integration of technology using the top three levels of the Substitution, Augmentation, Modification, and Redefinition (SAMR) model, where:
- *Substitution* describes situations in which the technology acts as a direct tool substitute, without functional or conceptual change, such as measuring and drawing using a graphics program.
- *Augmentation* refers to the situations where the technology is used as a substitute for an existing tool, but with some functional improvement regarding the facilitation of the task, for example, using online materials with links to practical exercises
- At *the modification* level, technology allows a significant task redesign or modifies the solving strategies of the user.
- At *redefinition* level, technology allows for the creation of new tasks that could not be posted without the use of the technology (Bray & Tangney, 2017; Hamilton, Rosenberg, & Akcaoglu, 2016; Puentedura, 2010).

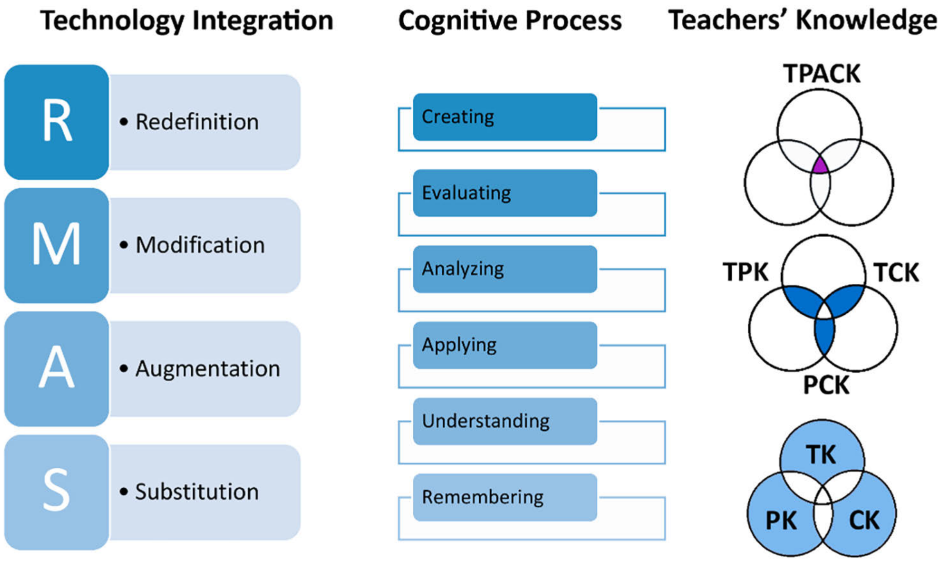

Figure 7: Level of technology integration, cognitive process, and teachers' knowledge, as well as their relationship. They were modified from (Puentedura, 2014).

According to Bray and Tangney (2017), the dominant pedagogical approach in recent empirical research on the use of technology in mathematics education is constructivist in philosophy. Thirty-seven percent of the detailed studies are clear constructivist and thirty-four percent, social constructivist. Furthermore, the usage of technology is at the augmentation level. Although the study was related to published technology interventions, and the reality in classrooms is likely to be quite different, it suggests that these results could explain why teachers are not utilizing the potential benefits of technology in mathematics education fully. Teachers might see technology-enhanced learning often only as an enrichment to the existing classroom practices and do not exploit a new kind of learning extensively.

TPACK models the connection to Bloom's learning model can be used to analyze the use of technology in mathematics teaching regarding learning goals. Figure 7 shows the levels of these models and their relationships modified from work by Puentedura (2014). The lowest levels of technology integration, according to the SAMR model, correspond to the lower levels of cognitive learning, and the upper levels to each other. Similarly, the higher the level of learning or the level of technology integration, the higher the level of required teacher's knowledge is.

2.4 Technological tools used for mathematics education

Technology development has been fast and will undoubtedly continue at the same pace. Future careers require new types of skills, from comprehensive problem-solving to teamwork skills. Implications of technology-rich environments influence the nature of mathematics education and the concepts, and skills students are expected to learn in class (Drijvers et al., 2016). STEM education integrates science, technology, engineering, and mathematics knowledge and skills aiming at finding solutions to complex problems and improving real-life situations (Sen, Ay, & Kiray, 2018).

The teaching of Mathematics varies a lot depending on the grade and the content, for example, university students learning differential equations or primary school pupils learning geometrical shapes. In both cases, technology can be used to enhance learning in hundreds of ways; only the imagination of teachers and designers of technological tools set boundaries in the use of technology. ICT is a tool to perform calculations, draw graphs, and more generally to using data and information sources; organizing and investigating; analyzing and automating processes; models and modelling; and help solve problems. *"Students can develop deeper understanding of mathematics with the appropriate use of technology. Technology can help support investigation by students in every area of mathematics and allow them to focus on decision making, reflection, reasoning, and problem solving. The existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it."* (National Council of Teachers of Mathematics, 2000)

One crucial question is how technology could help in doing mathematics and solving problems experimentally, inductively, even in chaos? According to Polya (1962), knowledge of mathematics consists of information and know-how. In mathematics, know-how is much more important than mere possession of information. Know-how in mathematics means “the ability to solve problems—not merely routine problems but problems requiring some degree of independence, judgment, originality, creativity” (Polya, 1962, p. xi). Therefore, technology should be used not only to process information but also to develop students’ ability to reason and think creatively.

This chapter introduces the essential technological tools used in the teaching of mathematics and then discusses issues related to the use of technology in teaching.

2.4.1 Computer environments for studying school geometry

Computer environments or Interactive Geometry Software (IGS) for studying school geometry can be divided into three groups: supposers, dynamic geometry environments (DGEs), and Logo-based programs. All these computer environments enable students to create and then manipulate geometric constructions, primarily in plane geometry.

With the help of computer environments, teachers can make geometry livelier than with paper and pencil. Geometric supposers allow the user to choose a shape, for example, triangle or rectangle, and make a euclidian construction with the help of chosen shapes (Yerushalmy & Houde, 1986). Supposers are not very complicated and do not have many features to teach more in-depth geometric knowledge. Instead, according to Olive (2000), dynamic geometry environments can completely transform the teaching and learning of mathematics at the secondary level. In DGE, points and lines are primitive objects, and students add various elements, such as circles and polygons quickly. They can move the entire construction conveniently and learn the qualitative "generic" properties of a configuration (Monaghan, Trouche, & Borwein, 2016). DGEs provide a way to teach mathematics based on constructivism because *"Dynamic geometry turns mathematics into a laboratory science rather than the game of mental gymnastics, dominated by computation and symbolic manipulation, that it has become in many of our secondary schools. As a laboratory science, mathematics becomes an investigation of interesting phenomena, and the role of the mathematics student becomes that of the scientist: observing, recording, manipulating, predicting, conjecturing and testing, and developing theory as explanations for the phenomena"* (Olive, 2000, p. 17). Examples of geometry software are the Geometer's Sketchpad[2], Capri geometry[3], and Geogebra[4]. A new generation of geometry programs includes touch screens (e.g., Geometric Constructor, Sketch-Pad Explorer), allowing a variety of simultaneous finger actions (Bairral, Arzarello, & Assis, 2017).

2.4.2 Computer algebra systems

A Computer Algebra System (CAS) is an interactive software used to manipulate mathematical formulae (Balcheff & Kaput, 1996; Ginsburg, Groose, Taylor, & Vernescu, 1997). The main difference between a CAS and a traditional calculator is its ability to deal with equations symbolically rather than numerically. Computer Algebra Systems provide a flexible tool for mathematicians and learners. Using CAS, functions can be visualized and manipulated in many ways, and students can explore realistic and complicated problems in algebra, calculus, and linear algebra. They can be used, for example, to simplify rational functions, factor polynomials,

2 www.dynamicgeometry.com/
3 www.cabri.com/
4 www.geogebra.org

find solutions to a system of equation, symbolically integrate and differentiate arbitrary equations (Ginsburg et al., 1997).

The joint CAS used to manipulate algebraic expressions, and graphing of functions are Maple[5], Mathematica[6], MatLab[7], MathCAD, and Derive. The capabilities and features of these systems vary significantly from one system to another. These systems can also provide a programming language for the users to define their procedures (Ginsburg et al., 1997).

2.4.3 Serious games

"*A serious game* is a digital game created to entertain and to achieve at least one additional goal (e.g., learning or health)" (Dörner, Göbel, Effelsberg, & Wiemeyer, 2016, p. 3).

"*Educational games* denote a subgroup of serious games, tackling the formal educational sector from elementary schools to higher education, vocational training, and collaborative workplace training. Whereas *learning games* address primarily informal learning, educational games focus on formal learning in dedicated educational institutions." (Dörner et al., 2016, p. 9)

With the help of serious games, learning can be more productive and meaningful (Alessi & Trollip, 2001; Klawe, 1999; Rieber, 2001; Sedighian, 1997). In some cases, the games encourage learners to study things that they might not otherwise choose to study at all, providing an extrinsic motivation to players who do not have an intrinsic motivation (Dörner et al., 2016). Learners often spend more time playing educational games than they would if some other studying methodologies were being used (Alessi & Trollip, 2001). Recent technological developments, such as smartphones, and other mobile devices, novel interaction devices increase the chances that the next generation of digital games will strengthen entertainment, challenge, motivation, excitement, and interest (Dörner et al., 2016).

Csikszentmihalyi (1991) has introduced the concept of *flow* that describes a person's immersion in experience and is closely related to games. In a flow experience, people completely involve in the determined task losing track of time. The best educational games give flow experiences to students (Rieber, 2001). The other constructs similar to flow but to emphasize different aspects of the subjective experience have been proposed, such as enjoyment, and arousal (Boyle et al., 2012). Games are believed, above all, to enhance intrinsic motivation.

5 http://www.maplesoft.com/
6 http://www.wolfram.com/mathematica/
7 http://www.mathworks.com/products/matlab/

There exist many types of educational games, for example, adventure, role-play, business, and logic games. Games are often task-based, and rules define the course of the game. Games usually involve some form of competition. Many games combine these elements and contain the challenge one has to overcome or succeed to reach a goal. The player has to start at an easy level and proceed to the highest level to win the game. Fantasy is inherent in many games, and the degree of fantasy ranges from really realistic games to virtual fantasy worlds (Alessi & Trollip, 2001). The serious game player can practice problem-solving strategies in a simulated hypothetical world (Dörner et al., 2016). Different types of games offer different characteristics that have to be taken into account when using the games as a learning tool. The size of the target group, students' technical abilities, availability, and license policy affect the choice of an appropriate game (Pivec, 2007).

Historically, games are an integral part of mathematics since different games requiring mathematical reasoning have been played in different cultures for centuries. However, we often use educational games for the achievement of fundamental mathematical skills, such as automaticity in operation skills (Fokides, 2018). For example, short “mini-games" focuses on a particular topic. They are easy to use, can be played at flexible times, and repeatedly. In order to achieve higher levels of learning, these games need to be more complex and based on experiential learning, exploration, and experimentation (Fokides, 2018; Kebritchi, Hirumi, & Bai, 2010). Video games that not explicitly made for learning, such as Minecraft[8], offer opportunities for exploration in a three-dimensional space, and Miegakure[9] even in a four-dimensional world. Creation of links between abstract mathematical concepts and real-life situations can be made using role-playing games combined with well-designed activities (Ahmad, Shafie, & Latif, 2010; Shaffer, 2006).

Garris Ahlers and Driskell (2002a) presented the model of game-based learning displaying how and when learning occurs when the student is playing an educational game. Mattheiss, Kickmeier-Rust, Steiner, and Albert (2009) developed it further by combining it with the cognitive model of motivation to learn (Heckhausen & Heckhausen, 2006) (Figure 8). According to the model, game-based learning consists of conditions, activities, and outcomes. Conditions include characteristics of the instructional content, the player, and the game. A typical feature of an educational game is that the instructional content is mixed with the game characteristics producing a game cycle of user judgments, user behavior, and system feedback. By repeating the game cycle, the player adopts knowledge while reacting emotionally or cognitively to the events of the game. In other words, the game cycle produces specific cognitive and affective outcomes. Each one of these three parts can be used to

8 https://www.minecraft.net/
9 https://miegakure.com/

enhance motivation. The player's specific expectations regarding activities and outcomes influence the motivation.

What are the essential features of the games? What makes a game good? Several studies identified motivations for playing computer games. Table 1 presents a comparison of seven taxonomies of motivations. On the one hand, these taxonomies overlap, but on the other hand, they differ notably. Most of these studies have concentrated generally on video games, multi-player games, and online games, instead of serious games. At all events, it is clear that each taxonomy is pertinent in its context. The reason for inequalities between the taxonomies is partly due to different contexts and types of the games, and partly due to different definitions of motivations. For example, teamwork, social interaction, and socializing all sound similar, but the definition is slightly different.

Designing educational games is challenging, as they should be both motivating and educational. Boyle et al. (2016) conducted an extensive research review of 143 papers from 2009 to 2014 considering impacts and outcomes of playing digital and serious games, two of which in mathematics. They conclude that research has progressed in understanding how specific game components engage players and support learning. Game features that increase engagement and enjoyment are, for example, animation, graphics, fun, rules, and goals, as well as rewards such as earning points, finding rare game items and fast loading times (Huang, Johnson, & Han, 2013; King, Delfabbro, & Griffiths, 2011). According to Alessi and Trollip (2001), educational games must have worthwhile learning objectives, they must be fun, and the goal of the game must reinforce the learning goals. Malone's (1981) empirical research has proved that an intrinsically motivating environment should offer challenges, involve fantasy, and arouse curiosity. Learners are challenged by reasonable, personally meaningful goals, uncertain outcomes, and randomness. Fantasy contains intrinsic and extrinsic fantasies, cognition, and emotions. Players' curiosity can be intrigued by, for example, audio and visual effects, well-formulated knowledge structures, and informative feedback (Malone & Lepper, 1987). Narrative games offer the player a possibility to influence the development of a story and make tasks meaningful (Dörner et al., 2016).

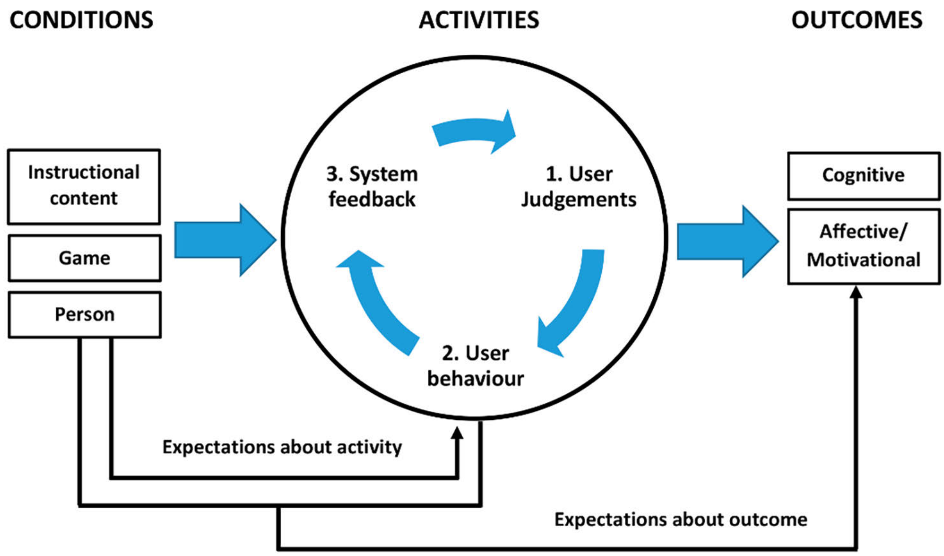

Figure 8: An advanced model of motivation for educational games (Mattheiss et al., 2009)

On the one hand, stories enable quicker comprehension and better remembrance (Dörner et al., 2016). On the other hand, a narrative may distract learners from the learning material because the working memory is loaded of story events withholding from cognitive activities that yield learning (Wouters & Van Oostendorp, 2013). Adaptation and personalization in educational games are essential. For the game to be both practical and exciting, it should fit as tightly as possible to the characteristics of the player (Dörner et al., 2016). Boyle *et al.* (2016) note that extensive systematic research should be carried out mapping game features to engagement and learning. Furthermore, they found out that games are still mainly used to support lower-level learning, which is a disappointment considering the speculation about games' potential to be an engaging and active new method for supporting 21st-century skills.

Game-based learning is a complex learning environment. The use of instructional support in educational games can improve learning (Wouters & Van Oostendorp, 2013). Considering the design and use of digital educational games in mathematics, an embedded instructional module, adjusted to the knowledge requirements of the player and closely linked to the goals of the game, is crucial from the mathematics learning point of view (Klawe, 1999). Notably, how an educational game is used in lessons affects the effectiveness of the game (Klawe, 1999). Garris et al. (2002a) predicate that an essential part of a game-based learning process is debriefing that enables linking the virtual world and the real world after playing.

Table 1: A comparison of seven studies related to motivations for play.

	Malone and Lepper (1987) **Computer games for learning**	**Garris, Ahlers, and Driskell (2002b)** **Computer games for learning**	**Sweetser and Wyeth (2005)** **Real-time strategies games**	**Yee (2006)** **MMORPGs**	**Ryan, Rigby & Przybyiski (2006)** **Computer games**	**Fu, Su & Yu (2009)** **E-learning games**	**De Grove, Cauberghe & Van Looy (2014)** **Digital Games**
Advancement				x			
Agency							x
Autonomy					x		
Believability							x
Challenge	x	x	x			6 elements	
Clear goals			x			4 elements	
Competition	x			x			
Competence					x		
Concentration			x			6 elements	
Control	x	x	x			7 elements	
Cooperation	x						
Curiosity	x						
Customization				x			
Discovery				x			
Escapism				x			x
Fantasy	x	x					
Feedback			x			5 elements	
Habit							x
Immersion			x	4 elements		7 elements	
Involvement							x
Knowledge improvement						7 elements	
Mechanics				x			
Moral self-reaction							x
Mystery		x					
Pastime							x
Player skills			x				
Performance							x
Recognition	x						
Relatedness					x		
Relationship				x			
Role-Playing				x			
Rules/goals		x					
Sensory stimuli		x					
Social interaction			x			6 elements	
Sociability							x
Socializing				x			
Status							x
Teamwork				x			

Games developed for entertainment and learning address different outcomes (Boyle et al., 2016). A typical weakness of educational games is that they do not provide an appropriate balance between playing and learning activities or between challenge and ability. Other disadvantages are aligning the game with the national curricula and the lack of proficient instructional models based on pedagogical standards and didactical methods (Dörner et al., 2016).

2.4.4 Drills

We use drills to provide practice to the learner. Drills can be components within games, tutorials, or web-based materials. According to Alessi and Trollip (2001), most drills follow the typical structure presented in Figure 9. The cycle "select item–question and response–judge response–feedback" is repeated many times. This basic structure forms the basis of many variations for many educational games, which are often drills. Examples of drills are *MathDrill*[10], an interactive site for Math Problem and *Math U See*[11]

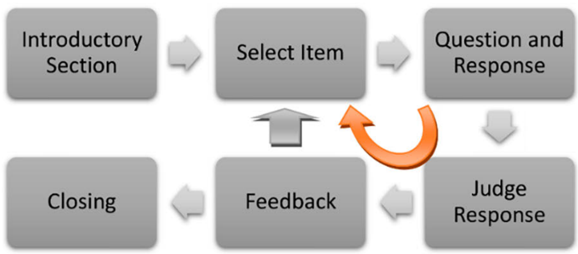

Figure 9: Typical structure of a drill (Alessi & Trollip, 2001)

Computer-based drills fall in the category of behavioristic methodology (Bray & Tangney, 2017), and they have been criticized extensively. Many educational theorists claim that drills do not capitalize on the power of the computer and that learners can accomplish drills fast like workbooks or flashcards without "proper thinking," providing a very narrow level of expressivity (Alessi & Trollip, 2001; Bray & Tangney, 2017). However, *"drills, in combination with tutorials and other methodologies, provide practice and are useful for learning information in which fluency is required, such as basic math skills…"* (Alessi & Trollip, 2001, p. 181).

[10] http://www.mathdrill.com/
[11] https://mathusee.com/e-learning/drills/

2.4.5 Web-based resources

The explosive growth of the internet has brought a significant number of materials and tools available for learning and instruction. Web-based resources are an essential part of today's mathematics lessons, and their quantity—and also quality —is increasing continuously. For mathematics, many internet sites contain, for example, tutorials, games, interactive problems, quizzes, and printable worksheets. Digital libraries in the context of education can be called digital repositories that use learning objects to organize their content (Borba et al., 2017). Examples of learning resources in online repositories are MERLOT[12] (Multimedia Educational Resources for Learning and Online Teaching) and Khan Academy[13].

The web also offers platforms, Virtual Learning Environments (VLE), for distance learning and a new type of communication, such as Open Source[14] and Moodle[15]. VLEs also referred to as Learning Management Systems (LMS) or Course Management Systems (CMS). “VLEs/LMSs/CMSs are web-based systems which are usually password-protected and allow people to make a range of digitized materials and online activities available to students”(McAvinia, 2016, p. 1).

Many types of research evidence the advantages of using web-based material, for example, Lin’s (2008) study on the efficacy of web-based workshops in elementary school. According to Lin, especially the visual representation and 3D shapes enhance student understanding and interest in mathematics. Not everything on the internet is lovely, and educators should not turn to it as the only approach. “The ever-changing Web landscape, with sites appearing, disappearing, and changing daily, makes it difficult for educators to depend on it for essential information” (Alessi & Trollip, 2001, pp. 397-398).

One solution to these challenges is to develop systems with an ability to adapt their behavior to the goals, tasks, interests, and needs of individual users (Brusilovsky & Maybury, 2002). Adaptive hypermedia (AH) and Adaptive Web systems build a model of the individual user and apply it for adaptation to that user (Brusilovsky, 1998; Brusilovsky & Maybury, 2002). Examples of Adaptive Web systems for mathematics learning are SHARP Online for solving math problems (Gil, Rodríguez, García-Peñalvo, & López, 2008), and a hypermedia tool, Hipatia for self-regulated learning, developing specific math skills, and promoting practical problem solving (Cueli, González-Castro, Krawec, Núñez, & González-Pienda, 2016). Another is an interactive Platform for Learning Calculus called PIAC intended for overcoming mathematical difficulties in calculus (Andrade-Aréchiga, López, & López-Morteo, 2012).

12 https://www.merlot.org/
13 https://www.khanacademy.org
14 www.opensource.org
15 www.moodle.org

When thinking about the internet and learning, the development of internet technologies must be taken into account. Web 1.0 made content available online for viewing, and there was no direct communication between the reader and the writer or publisher of the content. Web 2.0 considered a dynamic web where the users can read, write, and collaborate to a certain extent. Social networking platforms such as Myspace[16], Twitter[17] and Facebook[18] evolved, and the boundaries between authors and the users became slim (Rego, Moreira, Morales, & Garcia, 2010). Web 2.0 enabled a collaborative way of learning where knowledge can be socially constructed and provided new ways, such as Google Docs, wikis, and blogs, of creating and sharing knowledge (Abdelmalak, 2015). Building and sustaining an online learning community proved to be crucial and necessary to the learning process (Abdelmalak, 2015). Web 3.0 is called the semantic web or the intelligent web, and it is the transformed version of Web 2.0 with technologies and functionalities such as intelligent collaborative filtering, cloud computing, big data, linked data, openness, machine learning, 3D visualization and smart mobility (Hussain, 2012). Web 3.0 technologies will transform the learning with the help of web-based resources to the learning in virtual learning environments where students benefit from more advanced learning personalization, learner support, assessment, and record keeping (Kurilovas, Kubilinskiene, & Dagiene, 2014; Morris, 2011).

2.4.6 Tutorials and intelligent tutoring systems

Tutorials present information or model skills and guide the learner through the first use of information or skills (Alessi & Trollip, 2001). Figure 10 shows the structure of a typical tutorial: According to Alessi and Trollip (2001), a typical tutorial starts with an introductory section informing the learner of the purpose and nature of the program. After that, information is presented and elaborated, and the learner must answer a question or questions. The tutorial program judges the response to assess comprehension or skill and gives some feedback to the learner. The red arrow in **Figure 10** shows that the cycle continues until the program is terminated by either a learner or the program. The closing point might incorporate summary and closing remarks, but usually without assessment of the learning.

The simplest type of a program sequence in tutorials is linear, where the program progresses from one topic or concept to the next by presenting information and asking questions. All users have to go through all the material in a predefined order, and the tutorial does not adapt to individual learners (Alessi & Trollip, 2001). Branching

[16] https://myspace.com/
[17] https://twitter.com/
[18] https://www.facebook.com/

tutorials have a more complicated type of a program sequence because learners using these can affect the sequence by their performance and choice. Nowadays, the internet contains many mathematics tutorials, for instance, Visual Math Learning[19] and Finite mathematics and Applied calculus Online Tutorials[20] .

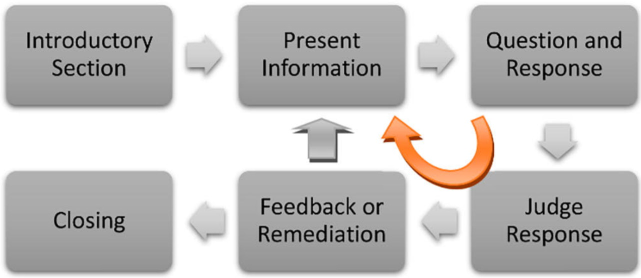

Figure 10: The general structure and sequence of a tutorial program (Alessi & Trollip, 2001).

Tutorials using artificial intelligence (AI) are more advanced learning tools. Intelligent Tutoring Systems (ITS) "are computer programs designed to incorporate techniques from the AI community in order to provide tutors who know what they teach, whom they teach, and how to teach it". Adaptive Hypermedia Systems (AH) provide instruction in skills and better suited for the instruction of concepts. In contrast, ITSs generally assist in the use of these concepts to solve problems (Nwana, 1990; Phobun & Vicheanpanya, 2010, p. 252). ITS usually contains four modules: (i) the Expert Module including the content of the particular learning domain; (ii) the Tutoring Module guiding instructional interactions with the students (iii) the Student Module, which is a dynamic representation of the students' current state of knowledge; and (iv) the User Interface controlling interaction between the student and the system (Ramesh, Rao, & Ramanathan, 2015).

A significant feature of ITSs is the accessibility of instructional resources such as hints, multimedia examples, tutorial dialogues, and other tools the student can use during problem-solving (Beal et al., 2010). These tools help the student learns to solve increasingly challenging problems in a particular area (Beal *et al*., 2010). ITSs can be implemented in various ways, for example building upon the LMS Moodle (Ramesh et al., 2015) or developing a separate web-based, adaptive learning environment such as ActiveMath (Melis & Siekmann, 2004b).

[19] http://www.visualmathlearning.com/
[20] http://www.zweigmedia.com/RealWorld/tutindex.html

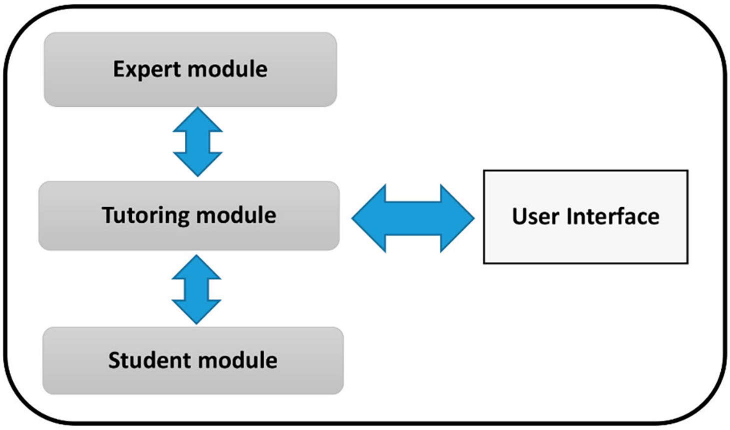

Figure 11: The components of an intelligent tutoring system (Ramesh et al., 2015).

ITSs have been discovered to support learning well and mainly to be an excellent supplement to contact teaching (Lowe, Mestel, & Wiliams, 2016; Weeraratne & Chin, 2018). In mathematics, they have, for example, proved to enhance meaningful learning of arithmetic and algebra knowledge (Sabo, Atkinson, Barrus, Joseph, & Perez, 2013), and improve performance in state standardized tests, as well as enhanced engagement (Arroyo et al., 2014). On the whole, the advantages of ITSs are their ability to contextually follow a student's performance and adjust the teaching approach to a student's learning needs (Woolf, 2010). The future development of ITSs progresses towards generating highly individualized, pedagogically valid, and readily available learning material (Woolf, 2010).

2.4.7 Manipulatives

One of the first supporters of concreteness was Pestalozzi (1746-1827), who reasoned that student learns best through physical activity, using their senses. From this point of view, Froebel (1782- 1852) started to create different concrete manipulatives to help to learn in kindergarten. Montessori (1870-1952) familiarized himself with the ideas of Pestalozzi and Froebel and developed her philosophy of education, which emphasizes the child's activity and where the learning environment and equipment play a significant role. According to Montessori, the purpose of manipulatives is to help development of reasoning and abstract thinking, and foster creativeness (Montessori, 1972). Some decades later, psychologist Piaget's (1896-1980) research on the development of children's intelligence reported the necessity for actual manipulation of objects when learning formal, abstract mathematical concepts (Piaget, 1969). The use of manipulatives is related to psychomotor skills in Bloom's system.

One example of concrete manipulatives is a unique set of tiles for arithmetics learning known as Cuisenaire rods exploiting the possibility of different spatial arrangements to exemplify mathematical principles like number composition and fractions (Cuisenaire, 1968).

The idea that young children learn best through interacting with concrete objects has caused the frequent use of mathematics manipulatives (Gürbüz, 2010; Pires et al., 2019; Sherman & Bisanz, 2009; Uttal, Scudder, & DeLoache, 1997). The Finnish National Core Curriculum for Comprehensive Schools of 2004 emphasized the importance of concreteness as a link between the student's experiences, thinking structures, and the abstract structure of mathematics (2004). The most recent Finnish National Core Curriculum for Comprehensive Schools (2014) does not explain in detail the use of manipulatives. However, instead, concreteness and activities are the central elements regarding mathematics teaching and learning. During grades 3 to 6, the curriculum states that mathematics is studied in learning environments where concretizing and manipulatives are essential. Furthermore, manipulatives must be readily available.

Research has demonstrated that the long-term use of concrete manipulatives improves students' achievements in mathematics (Clements, 2000). According to Kamii et al. (2001), manipulatives can be used to encourage students' thinking and conclusions while solving mathematical problems. Sowell (1989) notes that manipulatives improve attitudes towards mathematics if teachers know how to use them. Uttal et al. (1997) suggest that the use of concrete manipulatives can be useful, but the use of concrete objects does not guarantee an understanding of the concept. Students may not perceive the relation between manipulatives and principles of mathematics unless these relations are specifically highlighted, and teachers must take into account students' conceptions of what the manipulatives represent. When concrete manipulatives are used, pedagogical planning is crucial for their effectiveness (Clements, 2000; Kamii et al., 2001).

Digital manipulatives are computationally enhanced versions of physical objects. They offer many opportunities for interaction and serious play (Rieber, 2001). The idea of digital manipulatives was born in the 1970s when Papert and his students at Massachusetts Institute of Technology (MIT) Artificial Intelligence laboratory, started to think about how to combine concrete manipulatives and computers with helping mathematical learning (Papert, 1980). Papert designed the Logo programming language to move a line drawing turtle on the floor. Logo programming became popular in the schools in the 1980s, although in the school version, the concrete turtle had moved from floor to screen. Papert noticed that Logo programming was both fun and an advantageous learning methodology. By actively doing children learned to speak the mathematics language and use mathematical concepts, such as geometric shapes, degrees, speed, processes, and procedures.

The researchers of the MIT laboratory continued research on digital manipulatives (McNerney, 2004). In the 1990s, Logo programming was applied to digitally enhanced versions of Lego bricks. Both programming languages and environments have become comfortable to use, including many features and allowing students to easily explore topics in product design and prototyping (Danahy et al., 2014). Logo programming follows a constructionist approach to learning and supports the formation of the link between students' actions and symbolic representations (Bray & Tangney, 2017), and it is also suitable for learning non-Euclidean geometries (Hoyles & Lagrange, 2010).

Virtual manipulatives are interactive visual models typically on the computer screen used often in mathematics teaching. Moyer-Packenham and Westenskow (2013) survey of virtual manipulatives found 32 studies showing a moderate effect of virtual manipulatives on math performance compared to typical instruction in the general education classroom. The latest advances in cellular technology, cellphone cameras, GPS, and web development virtual manipulatives further by using augmented reality (AR). AR geometry applications can be used to improve students' understanding of mathematics, especially in 3D space (Cahyono, Firdaus, Budiman, & Wati, 2018). Nonetheless, as Dillenbourg (2016) suggests, the swift progress of technologies is blurring the line between the digital and the physical, making the distinction less straightforward.

2.5 What is the research gap?

This chapter introduced learning theories, the various technological tools used in the teaching of mathematics, and the relationship between technology, pedagogy, and mathematics. Based on the literature review, more information is needed on how to utilize the potential of technology in teaching and learning mathematics. Technology integration into classrooms at a deeper level requires more knowledge of selecting appropriate learning approaches and tools. Substantial financial and human capital has been invested in developing and introducing technological tools into classrooms and developing teacher's competencies in technology integration to enhance mathematics education. Regardless of the investment, the impact on the reality of school practices has been limited (Bano, Zowghi, Kearney, Schuck, & Aubusson, 2018; Hoyles & Lagrange, 2010). TPACK levels of teachers seem to be only moderate, and they lack effective strategies for integration of ICT (Niess et al., 2009; Njiku, Mutarutinya, & Maniraho, 2021).

Research on the use of digital technologies in the classroom has not concentrated sufficiently on context, which would be very important for students to construct the meaning of what they are learning (Hoyles & Lagrange, 2010). This is in line with the results of systematic literature review by Bano et al. (2018). They analyzed 49 studies (60 papers) published during 2003 – 2016, focusing on investigating mathematics or science learning and teaching with mobile apps and technologies in secondary school education. They discovered that none of these empirical studies have been replicated in different contexts to increase the reliability and generalizability of the results. This is actually surprising, considering that one of the most important advantages of mobile learning is both the context awareness of the technology and the facility of re-contextualization of learning (Schroeder, 2013).

Bloom's taxonomy is a functional tool for planning mathematics teaching and evaluating learning. However, it oversimplifies the learning process. Motivation is essential for learning, but more exploration is needed in relation to the use of technology in teaching mathematics.

Learning theories are a crucial part of teacher’s pedagogical knowledge. During the last ten years, educational technology tools have been reported to be based mainly on behaviorism, or their design is not based on pedagogical theories at all (Cheung & Hew, 2009; Goodwin & Highfield, 2013; Zydney & Warner, 2016). However, Bano et al. (2018) found that the most recent studies reported mobile applications were mainly based on three learning orientations; Collaboration, Inquiry-based Learning (IBL) and Realistic Learning. They request if this is a sign of current change in approaches of software designers, and suggest further research on which learning theories are applied in mathematics classes in schools today, and on what learning orientation the design of present applications is based. Several researchers suggest combining different learning approaches (Cronje, 2006; Sidney, 2015; Weegar & Pacis, 2012), but there are only a few concrete examples of how behaviorism an constructivism can be taken into account in pedagogical models simultaneously.

Based on this literature review and the above mentioned research gaps, the following three dimensions of *learning mathematics with technology-enhanced learning* were chosen as the research framework for this thesis (Figure 12): 1) Motivation theory, 2) Learning theories, 3) Bloom’s taxonomy: domains of learning.

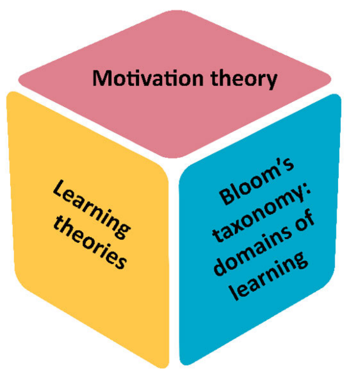

Figure 12: The dimensions of learning mathematics with technology-enhanced learning.

Looking at these three dimensions brings more insight into how teachers can move up from lower to higher levels of teaching with technology, and what factors should be considered when choosing a suitable tool for learning situations. These classifications provide one way to categorize and interpret new developments in the field, and challenge designers and researchers to reflect the critical question of which learning theory could be applicable in designing computer-enhanced teaching in mathematics.

The goal is also to determine how the follow-up research I did in different contexts confirms or infirms the taxonomy. Replicating empirical studies in different environments is an important part of research, and brings more information about the connections between context and teaching mathematics using information technology.

3 Research Design

The focus of this research is the design and development of artifacts for mathematics learning, and their evaluation in relation to different dimensions of mathematics learning. I have explored how technology can be used to teach fractions to 8th graders in different contexts, including South Africa, Mozambique, Germany, and Finland. This research comprised of seven underpinning research questions presented in the subsection 1.2. and addressed in seven articles (listed on page iii and enclosed in the thesis as appendices).

This chapter introduces the research methodology. I used an action design science (ADR) approach of creating and evaluating artifacts; compiled a *pedagogical model* for mathematics learning, which advocated an innovative behaviorist-constructivist perspective. The model included a *mobile game for learning* fractions designed during the study. Hence, the chapter demonstrates the ADR process through its iterative steps.

3.1 Action design research

This thesis follows *an action design research* approach for testing and building theory (Sein et al., 2011). This methodology combines *action research* (Cohen, Manion, & Morrison, 2018) and *design science* (Hevner & Chatterjee, 2010). The broad scope of case-study research alternatives makes it an adaptable research approach for information systems (Cavaye, 1996). I have used a case-study in the interpretivist tradition, for testing and building theory, with multiple case-study designs, using mixed methods. This section provides a precise description of the methodologies used. The overall research approach relates to an exploratory and conceptual interplay that is deriving from a literature review and a series of longitudinal empirical studies in different contexts. The integrated work focuses on the emerging phenomena in the learning of mathematics through technology-enhanced learning. The research uses a discursive and narrative approach in order to highlight and explain the complexity of the process.

Action research is a methodology for researchers, especially for teachers to improve teaching practices, and more generally, to understand and generate knowledge on educational practices and their complexity (Cohen et al., 2018). Action research is a systematic collaborative approach and presupposing that all people involved in the issue investigated should be included in the process of inquiry (Stringer, 2013). This research engaged in total 311 participants. I was the principal researcher working both as a teacher and a researcher. Also, the study involved 12 teachers and 12 researchers, as well as nearly 300 students.

Action research proposes that generalized solutions do not fit all contexts or groups to which they are applied, they have to be adapted and modified to find an appropriate solution for particular dynamics in a local situation (Stringer, 2013). To understand and create optimal generalizable results, I have used three different contexts in cases under examination (South Africa, Mozambique, and Finland).

Design science is a research paradigm aiming at creating innovative and purposeful artifacts through a problem-solving process in a specified problem domain, in a real-life context (Hevner & Chatterjee, 2010; Hevner, March, Park, & Ram, 2004). When building and applying an artifact, having an explicit knowledge and understanding of the design problem, as well as its solution, are acquired. The artifact can be, for example, models (abstractions and representations), methods (algorithms and practices), or instantiations (implemented and prototype systems) (Hevner et al., 2004). In this thesis, I analyze and examine the process of the designing and implementing of two artifacts: i) a new pedagogical INBECOM model for mathematics, and ii) the UFractions mobile game. The name INBECOM stands for Integrating Behaviorism and Constructivism in Mathematics, and the model utilizes narrative games and tutoring systems to foster productive and meaningful learning of mathematical concepts.

Unification of action research and design science is called *action design research (ADR)* (Sein et al., 2011). The progress of this study is one instance of how the elaborated action design research process model of Mullarkey and Hevner could be used (2019); as illustrated in Figure 13. Research according to the principles of action design research could be described employing three separate parts; environment, design science research, and knowledge base. In this research, three different environments manifest in three different cases. Each environment consists of the application domain, containing students, teachers, researchers as well as schools, national education strategies, local curricula, and technologies used in schools. The knowledge base consists of research foundations, learning theories, methods used in different cases, available technologies, and articles presenting results. Three cycles represent different actions; the relevance cycle, design cycle, and rigor cycle (Hevner, 2007). The relevance cycle is local in its nature and bridges the contextual environment with the process of creating artifacts, including producing requirements and field testing. The design cycle is in the center of the research, including several design iterations of artifacts, with appropriate evaluations. The rigor cycle is global and provides scientific information to support the design cycles and supplement the knowledge base with new research results. It is noteworthy that design science research makes research contributions to both the application environment and the knowledge base, providing solutions to practical problems that emerged from the problem space (Hevner et al., 2004). Chapter 5 highlights the different knowledge types the research has produced and discusses solution design knowledge in more detail.

Figure 14 shows the overall progress of the study that can be divided into six stages. The design of two artifacts included four main iterations shown in Figure 15. These iterations were done in different contexts that are described in detail in the following chapter. Figure 13 illustrates the elaborated action design research process model defining precisely the course of the study through five levels; problem formulation and planning, artifact creation, evaluation, reflection, and learning. Every iteration included the above five levels. The eight ADR principles (Mullarkey & Hevner, 2019; Sein et al., 2011) guided the research point of entry in different environments and iterations. Being either problem, objective, development, or observation centered (Mullarkey & Hevner, 2019). The eight ADR principles further guided the ADR intervention cycles

1) *Practice-Inspired Research* principle. Field problems were viewed as knowledge-creation opportunities, and the problems inspired the research activities.
2) *Theory-Ingrained Artefact* principle. The design of artifacts, UFractions, and INBECOM model was based on the theories presented in Chapter 2.
3) *Reciprocal Shaping* principle. There were inseparable influences of the two artifacts and the research contexts.
4) *Mutually Influential Roles* mutual learning among the different project participants (students, teachers, and researchers) was necessary.
5) *Authentic and Concurrent Evaluation* principle. All decisions about designing, shaping, and reshaping the artifacts were interwoven with continuous evaluation.
6) *Guided Emergence* principle. The design process of artifacts reflected the preliminary plan created by the researchers and participants' needs and perspectives.
7) *Generalized Outcomes* principle. The solutions to the problems were generalized.
8) *Abstraction* principle. Every ADR intervention cycle introduced an artifact at the appropriate level of abstraction for the stage of project activity and goals.

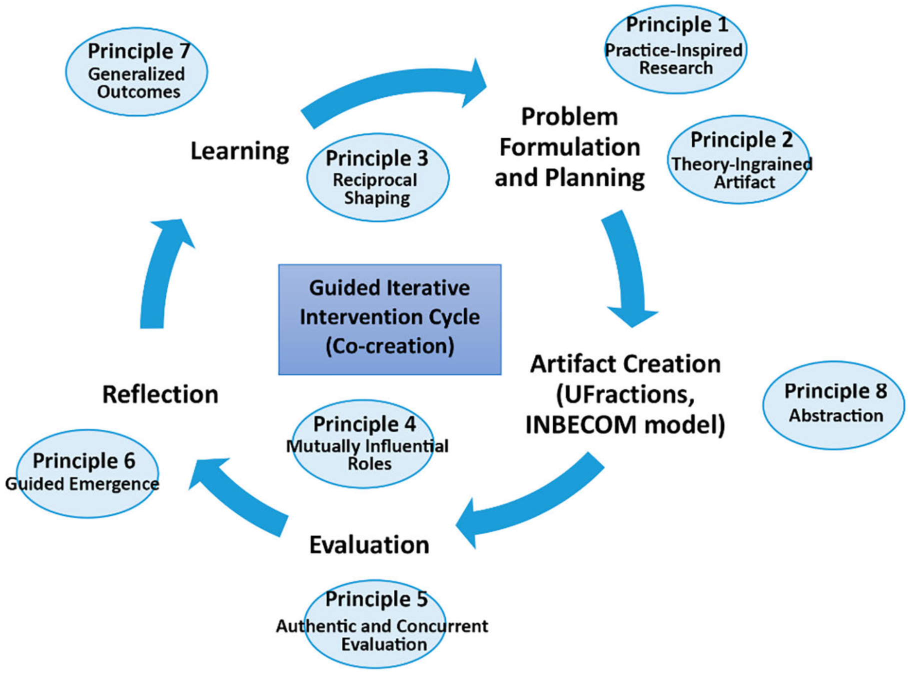

Figure 13: The elaborated action design research cycle (Mullarkey & Hevner, 2019, p. 8).

3.2 Phases of the action design science research framework

I completed the research in six phases (Figure 14)

1. idea and planning of an educational model,
2. development, experiments, and evaluation of the UFractions mobile game,
3. content and translations, experiments and evaluation of the ActiveMath intelligent tutoring system
4. organizing fraction course using the INBECOM model and
5. theory generating of the INBECOM model
6. theory generating of dynamics between disturbances and motivators, and quantitizing useful data as project evaluation.

The research was from 2009-2019.

Throughout the process, I wrote a learning diary that was also a research instrument. I learned considerably and developed as a teacher and researcher during these years. At stages 5 and 6 (Figure 14), I gathered all the experiences and knowledge I gained during the process and formed a new theory. In this rigor cycle, I combined the research activities with the existing knowledge base.

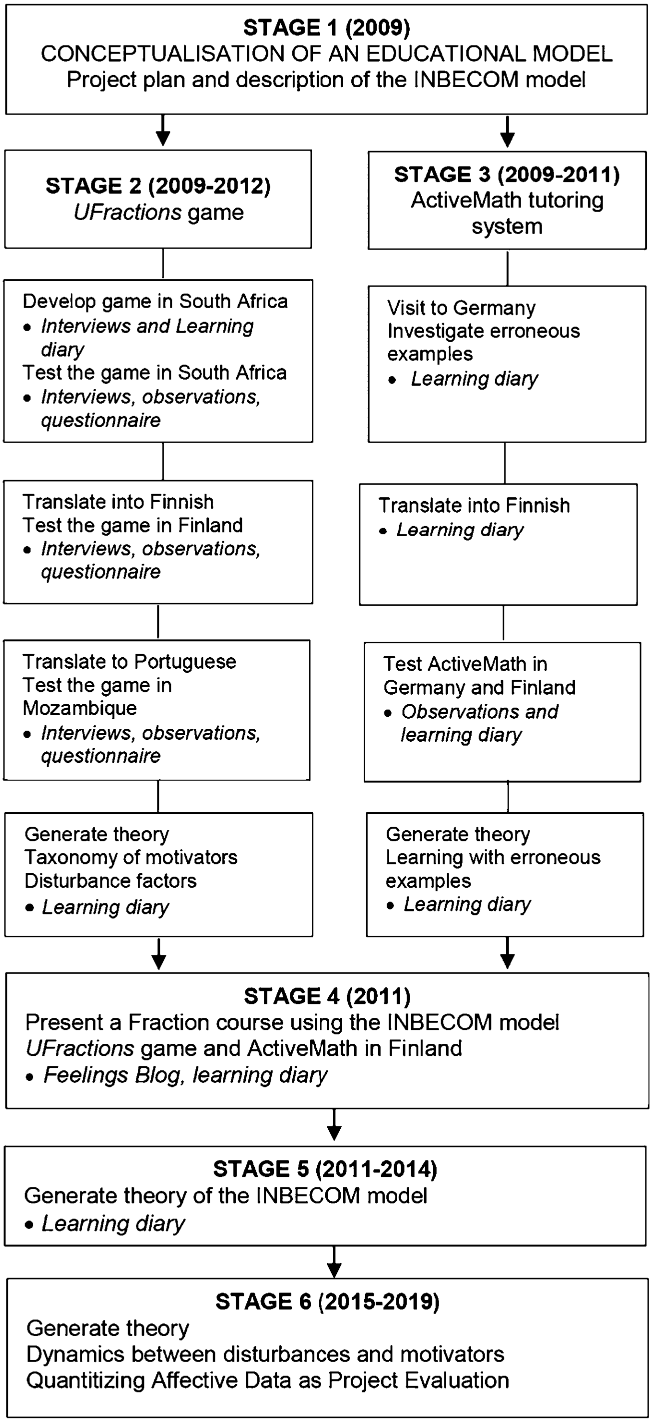

Figure 14: Six stages of the design of the study.

The two artifacts created were decentralized in various research settings, which are the environments in the multi-environment extension of Hevner's model (Hevner, 2007). The design cycles with building and evaluation activities took me to design the UFractions mobile game in South Africa and test it in different contexts in Mozambique, Finland, and South Africa, as well as deploying an intelligent tutoring system in Germany and Finland, and teaching Finnish students a fraction course with the INBECOM model. In Chapter 4, I will describe more the different contexts and the progress of the research. One element of this dissertation is the story of how the model has evolved over the years. In Chapter 6, I will elaborate on the pedagogical model development and design thinking that developed during the journey.

As the study was conducted according to the action design research principle (Hevner & Chatterjee, 2010; Hevner, 2007; Sein et al., 2011), the design of artifacts included four iterations where the design was done through iterations of building and evaluating activities. Figure 15 shows the four main iterations of the research, contexts of iterations, and publications relations to iterations. The whole process included learning more about the problem domain and possible solutions, and evaluation was done at every phase of the process (Goldkuhl, Ågerfalk, & Sjöström, 2017). The research point of entry varied in iterations; being problem, objective, development, or observation centered (Mullarkey & Hevner, 2019). Each iteration produced information utilized in the later iterations. Multiple instruments were used for evaluation, as shown in Figure 14.

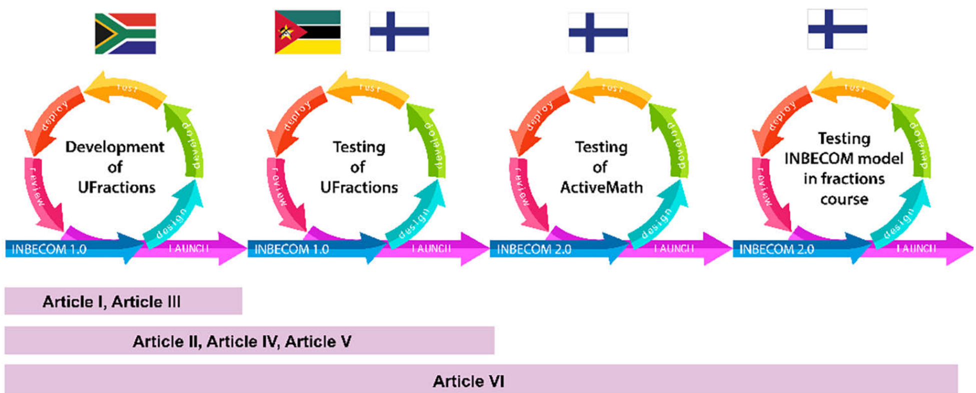

Figure 15: Four iterations of the research, contexts of iterations and publications relations to iterations.

3.3 Research methodology

Next, I present the methodology related to each research question. Table 2 lists the research approaches used in each of the articles and research questions they answer. In Chapter 4 you can find more information about the contexts and the progress of the research. Each of the attached individual papers describes their specific environment and problem space, as well as a methodological approach in detail. The whole research was divided into six different phases shown in Figure 14. At each level, different research instruments were used, which are listed in the figure. In this study, the researcher used research instruments as advised by Cohen et al. (2005).

Table 2: Research approaches used in this study.

RQ	Research Approach	Thesis/ Paper
Q1	• Literature review complemented by the results of empirical studies	Chapter 2
Q2	• A multi-method approach comprising qualitative and quantitative research strategies • The Exploratory Software Development (ESD)	I, III
Q3	• Contextualized design approach • Multi-method approach where the results are primarily derived from quantitative questionnaire data, and qualitative comments from the participants support the findings	I, II, III, V
Q4	• A multi-method approach where the dominant method was a qualitative case study, quantitative methods were used to extend the qualitative approach. • Disturbance factors; see Q6	III, V
Q5	• A multi-method approach, comprising of both qualitative and quantitative strategies was employed	I, IV
Q6	• Multi-method approach, where the focus was especially on the analysis of qualitative data sets • A specific technology integration evaluation tool	V
Q7	• Quantitized project evaluation according to the affective framework of Krathwohl.	VI

Since the research used multi-method approach, different sampling methods were applied to obtain an adequate representative sample. Convenience sample was used when selecting countries, and testing the INBECOM model for an entire course. Due to limited resources, it was necessary to choose countries where either the main researcher (Finland) or supervisors (South Africa and Mozambique) lived, as well as the provinces close to the researchers' locations. Considering INBECOM model qualitative evaluations the researcher taught and observed her own class in the school where she worked. When both qualitative and quantitative data were needed for the analysis of the UFRactions game, a larger sample size was chosen to represent all

8th graders appropriately. For that, purposive sampling procedures were adopted. The schools selected for the evaluations varied in economical background, sizes, owners (private/government) and delineation of geographic area (urban/rural).

Q1 ***What are the dimensions of learning mathematics with technology-enhanced learning?***

To answer this question, I conducted an extensive literature review of the three main learning theories, technologies used in mathematics learning, and the relationship between educational technology, pedagogy, and mathematics. The extensive literature review is presented in Chapter 2.

Q2 ***How can artifacts for mathematics learning be developed?***

Two artifacts were created and evaluated during this research;

(i) a *pedagogical INBECOM model* for mathematics learning advocating both behavioristic and constructivist perspectives

(ii) a story-based *UFractions mobile game for learning* of fractions

In Chapter 6 I elaborate on the development of the INBECOM (Integrating Behaviourism and Constructivism) pedagogical model, the underlying idea of this study.

From the perspective of software development, the Exploratory Software Development (ESD) (Trenouth, 1991) method with an iterative structure was used in the development of UFractions game. In this case, the purpose of ESD method is to examine a domain of application that is poorly understood. The UFractions used the previously developed Myst platform and the development process was iterative in the sense that the research team had previous experience of using the platform for story-based games (Laine, 2011). In subsection 4.2.1. I will describe in detail the development of the UFractions game.

The UFractions game was tested in five South African schools. The evaluation of the game employed a multi-method approach comprising qualitative and quantitative research strategies. The dominant method was a qualitative case study. Quantitative methods were used to extend the qualitative approach; A total of 105 Grade 8 mathematics learners and five teachers completed questionnaires during the evaluation sessions. After each test session, the teachers and three to five students per evaluation site were interviewed by a semi-structured interview method. The demographical characteristics of the sample, the testing procedure and the research instruments are explained more carefully in subsection 4.2.1 and papers I and III.

Q3 ***What are the considerations to be taken into account during the design of a mobile game for learning fractions in diverse contexts?***

The third research question calls for a contextualized approach, meaning a design principle taking seriously the expectations, needs and especially the strengths of the context where technology is used (Vesisenaho, 2007). This study used a re-contextualization process adapting a contextually designed technology into a new context (Bada, Duveskog, Suhonen, & Sutinen, 2009) and the UFractions game was adapted to Finland. The evaluation of the differences between South African and Finnish Grade 8 player experiences was made using a multi-method approach, comprising qualitative and quantitative strategies. The results are primarily derived from quantitative questionnaire data, and qualitative comments from the participants support the findings. The demographical characteristics of the sample and the testing procedure is explained more carefully in subsections 4.2.1 and 4.2.2, as well as in papers I, II and III.

In order to investigate how factors that interfere gaming (disturbance factors) and motivational factors could be taken into account in the design of games, the sample expanded to 305 participants and and one more country was added to the sample, namely Mozambique. A modified version of the questionnaire and interviews was used to measure also the level of technology integration. called technology integration evaluation tool (Laine, Sutinen, Nygren, & Joy, 2011). The part of the research conducted in Mozambique and the survey instrument used there are described in more detail in subsection 4.2.3 and paper V. A multi-method approach was used, as we analyzed both quantitative and qualitative data to find distracting and motivating factors.

Q4 ***What strategies do we need in order to compile a mobile game for learning for diverse contexts?***

The empirical tests of UFractions are evaluated to see what factors need to be considered in different contexts. The evaluation employed a multi-method approach comprising qualitative and quantitative research strategies. The dominant method was a qualitative case study. Quantitative methods were used to extend the qualitative approach.

The practical guidelines compiling a mobile game for game users and educators from the interplay of disturbance factors and motivations described in the previous research question methodology subsection.

Q5 ***How can artifacts, such as games for learning, address the different dimensions of learning mathematics?***

A multi-method approach, comprising of both qualitative and quantitative data collection strategies was employed when studying the functioning of games in the areas of different dimensions of mathematics. Different rationales to the story and different arguments about the mathematics in the game was examined using South African sample (n= 105) described in 4.2.1 and paper I.

In the development of the motivation taxonomy, on the other hand, a larger sample (n= 279) from three different countries, Finland, South Africa and Mozambique was used. This sample is described in detail in chapters 4.2.1, 4.2.2 and 4.2.3, as well as in paper IV. The quantitative data analysis gave an initiation to the taxonomy; six mobile game play motivations were identified and clustered from the dataset. Next, the qualitative data received from the interviews and the open-ended questions was analyzed with Atlas.ti software. The qualitative data-analysis supported the six previously found motivators, and revealed one more.

Based on the analysis of specific example artifacts the researcher investigated how they map onto different dimensions of learning mathematics.

Q6 *What is the relationship between motivation and disturbances during learning mathematics with games for learning?*

To identify factors that interfere gaming (disturbance factors, DFs) in UFractions, we first analyzed qualitative data sets from South Africa and Finland and found 16 DFs (Laine et al., 2011). Qualitative data from Mozambique was collected using an extended questionnaire and interviews, and analyzed using a specific technology integration evaluation tool (Laine et al., 2011). In this evaluation 22 DFs were identified including all but one of the previously discovered 16 factors. The part of the research conducted in Mozambique and the survey instrument used there are described in more detail in subsection 4.2.3 and paper V. Further research performed during the Mozambique stage is reflected in the thesis of Teemu H. Laine (Laine, 2011).

Q7 How does an approach of creating interventions and artifacts based on the taxonomy for dimensions of learning mathematics trigger affective learning?

To answer this question, I followed a sequential, equal status, multi-mode research design and methodology where the qualitative data were derived from the interviews with researchers, teachers and students, as well as from learning diaries, feelings blogs, and observations (311 documents) across three contexts (South Africa, Finland, and Mozambique). The qualitative data was quantitized (Saldaña, 2009), i.e. analysed deductively in an objective and quantifiable way as instances on an Excel™ spreadsheet for statistical analyses. All the data was explored from the affective perspective, by labelling the feelings participants experienced according to the affective levels of the Krathwohl *et al.* (1964) framework.

4 Research Narrative

As explained in the previous chapter, the research progressed in six stages (Figure 14) and the development of the artifacts can be divided into four iterations (Figure 15). In this chapter, I describe the different research contexts and then illustrate the research process.

4.1 Research contexts

The next three paragraphs will address three different research contexts. The cross-cultural and cross-countries approach was chosen to ensure wider generalisability of the research.

4.1.1 The South African context

South Africa is at the most southern part of Africa, and it consists of nine provinces with significant differences in size, socio-economic viability as well as population density. Each province, depending on which political party won most votes in the provincial election, will govern with its legislature, premier, and executive council. The nine provinces are known as the Eastern Cape, the Free State, Gauteng, KwaZulu-Natal, Limpopo, Mpumalanga, the Northern Cape, North West, and the Western Cape. South Africa is a medium-sized country consisting of a total land area of slightly more than 1.2 million square kilometers. The North West Province is one of the smallest and least affluent provinces. Over 58 million inhabitants have a wide variety of cultures, languages, and beliefs. Gauteng and KwaZulu-Natal have the largest share of the South African population. Of the eleven official languages, English is the most spoken in economic and public life. IsiZulu is the mother tongue of 22.7% of South Africa's population, followed by isiXhosa at 16.0%, and Afrikaans at 13.5%. South Africa's first democratic election took place in April 1994, with victory to the African National Council (ANC) with Nelson Mandela as president (Mid-year population estimates 2019, 2019; South Africa Gateway, 2020).

South Africa has 12.7 million learners, about 386 600 teachers, and 30 000 schools. The General and Further Education and Training Phases are provided by two types of schools: public schools and independent schools (private schools).

Education is governed by two national departments, namely, the Department of Basic Education (DBE), which is responsible for primary and secondary schools, and the Department of Higher Education and Training (DHET), which is responsible for tertiary education and vocational training. The central government provides a national framework for school policy, but administrative responsibility lies with the nine provinces. School life spans thirteen years from grade R through to grade 12. Under the South African Schools Act of 1996, education is compulsory, but not free in all cases for all South Africans from age 7 (grade 1) to age 15, or the completion of grade 9. Impoverished schools are excluded from school fees and are subsidized with feeding schemes. In 2014, the national average learner-educator ratio in schools was 29.8:1 (Department of Basic Education, 2016; South Africa - Education at a Glance, 2019; South Africa Gateway, 2020).

South Africa is rich in diversity in terms of income, culture, languages, and technology. On the one hand, South Africa is renowned for its technological innovation and development. On the other hand, the majority of South Africa's learners do not share in the expectations of the information age. South Africa performed less than adequate in the most recent International Mathematics and Science Study (TIMMS), where its learners were the second-lowest performers of all countries (Mullis, Martin, Foy, & Hooper, 2016). The Progress in Reading Literacy Study (PIRLS) indicates that grade 4 learners' reading skills by far do not meet the international standards (Mullis, Martin, Foy, & Hooper, 2017).

The results of the Second International Technology in Education Study (SITES 2006)—a longitudinal large-scale international comparative survey on the use of Information and Communication Technologies (ICTs) in schools (Law, Pelgrum, & Plomp, 2008) indicated that South Africa was the only country in the SITES 2006 study that could not provide students with full access to computers. South African schools' overall computer access (38%) remains dismally low when compared to other education systems in developing countries, such as Estonia (100%), Chile (96%), and Israel (96%) (Blignaut, Els, & Howie, 2010). Only about 15% of South Africa's mathematics and science teachers used ICTs in their teaching and learning (Blignaut et al., 2010). Recent statistics from the National Education Infrastructure Management System (NEIMS) shows that only about twenty percent of the ordinary operational schools had internet connectivity for learning and teaching purposes (National Department of Basic Education, 2019).

South Africa is a really important country for this research, because the UFractions game was developed there, and the first game evaluations were done in South African schools as explained in subsection 4.2.1.

4.1.2 The Finnish context

Finland is in northern Europe between Russia and Sweden, and the majority of the 5,500,000 population reside in southern Finland (about 1,400,000 in the metropolitan area of Helsinki). The geographical area of Finland is 338,145 km^2 and is the most sparsely populated country in the European Union, with 15.7 inhabitants per km^2. Official languages are Finnish and Swedish, although Finnish is the most common language, spoken by over 90% of inhabitants. Following 700 years of rule by Sweden, and then 100 years by Russia, Finland became an independent parliamentary democracy in 1917 and is now a member of the European Union (This is Finland, 2020).

Finnish students are compared high in international comparisons of science skills, such as in the OECD PISA (Programme for International Student Assessment) surveys testing mathematics, science, literacy, and problem-solving skills of 15-year-old students in over 40 countries every three years. PISA tests assess essential knowledge, not specific to any curricula. However, Finnish students' reading, mathematics, and science performance trend have declined after 2006 (OECD, 2019). The PISA 2015 survey also tested, in an interactive digital environment, the collaborative problem-solving skills of students, and the average score points for Finnish students were the seventh-highest among all the countries and economies taking part in the assessment (Ministry of Education and Culture, 2016). In Finland, the gender gap in reading was one of the widest across all PISA 2018 participating countries. However, the socio-economic background and region have a lower impact on Finnish students' performance than in the other PISA countries (OECD, 2019). The factors for success include professionalism of teachers and teacher education, emphasis on educational equity, long-term educational policy, the culture of trust, and a high level of cooperation, helping the educational system to work smoothly (Üstün & Eryılmaz, 2018).

The Finnish education system consists of one-year pre-primary education and nine-year primary education (comprehensive school), followed by upper secondary education and higher education. Students start their compulsory schooling at the age of seven and continue until they have accomplished all nine grades or are aged 17. Almost all Finnish students complete the basic education syllabus. Primary education in Finland is free of charge, including books, school meals, and health care. Most comprehensive schools are public schools, and local authorities monitor compulsory education. Primary education providers construct their curricula according to the instructions in the national core curriculum given by the Finnish National Agency for Education. Generally, teachers are highly qualified, and a Master's degree is required at all school levels (Finnish National Agency for Education, 2020).

According to the Second Information Technology in Education Study (SITES), the level of access to computers, and the internet is 100% in Finnish lower secondary schools (Kankaanranta & Puhakka, 2008). Nevertheless, SITES research also showed that 61% of science teachers and less than 48% of mathematics teachers used ICTs during a specific period during the school year. However, many teachers have never used ICTs in their lectures, indicating that pedagogical opportunities for ICT are still untapped (Kankaanranta & Puhakka, 2008). The Digital Age Project (2017–2018) evaluated the digitalization process in Finnish Comprehensive Schools. The project discovered that traditional resources, such as books, notebooks, and handouts, and methods still prevail in instruction, leaving little space for learners to boost their digital skills (Tanhua-Piiroinen et al., 2019). The active role of learners using technology is not yet realized in practice on the contrary to the competence goals of the national core curriculum. However, the digital competence of teachers markedly improved during the two-year evaluation, and opportunities for digitalization are now included in the common goals of schools and the overall planning of teacher's work.

The researcher herself is from Finland, so the need for research as well as the INBECOM model stems from the Finnish school world. The UFractions game was tested in Finland after it was evaluated in South Africa. Furthermore, the ActiveMath tutoring system needed in the INBECOM model were tested in Finland before testing the actual model. The game evaluations in Finland is described in subsection 4.2.2., the ActiveMath evaluations in subsecton 4.3, and the research setting for the fractions course using the INBECOM model is represented in subsection 4.4. Chapter 6 is dedicated to the INBECOM model and its development.

4.1.3 The Mozambican context

Mozambique is located on the east coast of southern Africa and divided into ten provinces. The Mozambican population is about 30 million people. Mozambique has 14 spoken languages, but Portuguese is the official language. It is sparsely populated, with 45 % of inhabitants younger than 15. After gaining independence in 1964, the civil war lasted for 16 years. Reconstruction got underway in 1992 after the peace treaty was signed. However, the country has been suffering from floods, drought, food shortages, and HIV/AIDS, as well as national debt and low life expectancy (Central Intelligence Agency, 2020). Although Mozambique remains a developing country, much has improved in the last 20 years, and the GDP per capita has been growing approximately 5% annually since 2006 (Fox, Santibañez, Nguyen, & André, 2012).

The Mozambican government has focused on developing education, and the public expenditure on education is 6.5% percent of GDP (Central Intelligence Agency, 2020; Fox et al., 2012). Primary school enrollment has increased, but school construction and teacher training enrollments still have room for improvement. Nowadays, the Mozambican structure of education is divided into three levels: Primary level with seven grades (1st to 7th); Secondary level with five grades (8th to 12th) and Higher/University level with 3, 4 or 5 years (Farrell & Isaacs, 2007). About 60.7% of the total population (age 15 and over) can read and write (Central Intelligence Agency, 2020). Alongside formal schooling grades, Mozambique has an adult literacy program. The educational system contains both public and private schools. Most primary students go to public schools. The private sector, however, is essential in upper secondary, about a third of students attend private schools (Fox et al., 2012).

The Ministry of Education and Culture policies emphasize the use of ICT, especially at the secondary level. Therefore, the government is supplying secondary schools with ICT resources. Currently, there is a general understanding that computers are essential resources to facilitate teaching activity, and teachers are trained to use these technologies (Matavele & Camundimo, 2009). Projects, such as SchoolNet Mozambique, NEPAD eSchools, and EPCI, aim at improving the use of ICT at schools in Mozambique (Farrell, Isaacs, & Trucano, 2007).

Mozambique was the third country where the UFractions game was tested and evaluated. The game tests in Mozambique are explained in subsection 4.2.3.

4.2 Research setting for the *UFractions* game

This subsection addresses the research setting for the UFractions game (Stage 2 in Figure 14).

4.2.1 Developing and testing the game in South Africa

The UFractions mobile game is a story-based game with a story and mathematical problems. The UFfactions game was created in collaboration with Dr Carolina Islas Sedano, Mikko Vinni and Professor Teemu H. Laine during January and February 2009. At that time, I was a visiting researcher at North-West University, South Africa, and I got the idea to make the fraction game suitable for the local context as well as to use wooden math rods in the game. Carolina Islas Sedano, Mikko Vinni and Teemu H. Laine had developed story-based games and a special Myst platform, which I thought would also be well suited for a math game (Laine, Vinni, Islas Sedano, & Joy, 2010). Teemu H. Laine was mainly responsible for the technical development of the game. I was responsible for the idea and designs for the UFractions game, as well as for story and math problems, and graphics like the pictures of leopards. Carolina Islas Sedano helped me especially with the designs. Local cultural experts and school teachers assisted me with contextualized for the South African context.

As a starting point we had a Myst platform that works on a smartphone and an idea for a math game. UFractions operates on most phones that support Java and WLAN through the Myst learning platform developed at the University of Joensuu, Finland (Laine et al., 2010). Myst-based mobile games had earlier been successfully used and tested at the SciFest science festivals in Joensuu, Finland (SciMyst) (Sedano, Laine, Vinni, & Sutinen, 2007). A main feature of the Myst platform is its usability in various locations with minimal customization. The Myst platform offers the following game-like features:

1. Context-sensitive problems, or *enigmas,* as we call them, which can involve queries with multiple choice or open answers, or 'take-a-picture' tasks.
2. *Battle-mode* in which the player solves enigmas against a count-down timer. The battle is to be played at the end of the game as a drill.
3. Interactive *help-feature* which allows a player to request help from another player through the mobile device. Context-sensitive hints are also available for enigmas.
4. Recording of data, *impressions*, through the mobile device's camera and text input mechanism. Sound and video recording features are also available.
5. *Story-based structure* that has one or more virtual characters – each having its own characteristics and ways to respond to the player.
6. *Guest book* which allows players to leave their comments and ideas of the learning experience. Guest book entries can be published on the game's website.
7. Synchronous integration between the players' activities and the *game's website.*

My task was to create a good story to the game that is suitable for South African context. The contextualization process of the game started with school visits and discussions with cultural experts. After the interviews, I transcribed interviews and wrote observations in my learning diary. The participants choose the main character to be a leopard and her cub, Senatla. I also visited national parks and observed wild animal behavior. Part of discussions especially with the teachers was to determine suitable level for the mathematics in the game. I wrote the story of the game and developed the math tasks to fit the context based on my observations and interviews (Figure 16 and Figure 17). Papers I and III describe the development of the game and the game structure and features in detail.

Figure 16: Story and problems connect mobile phones and math rods.

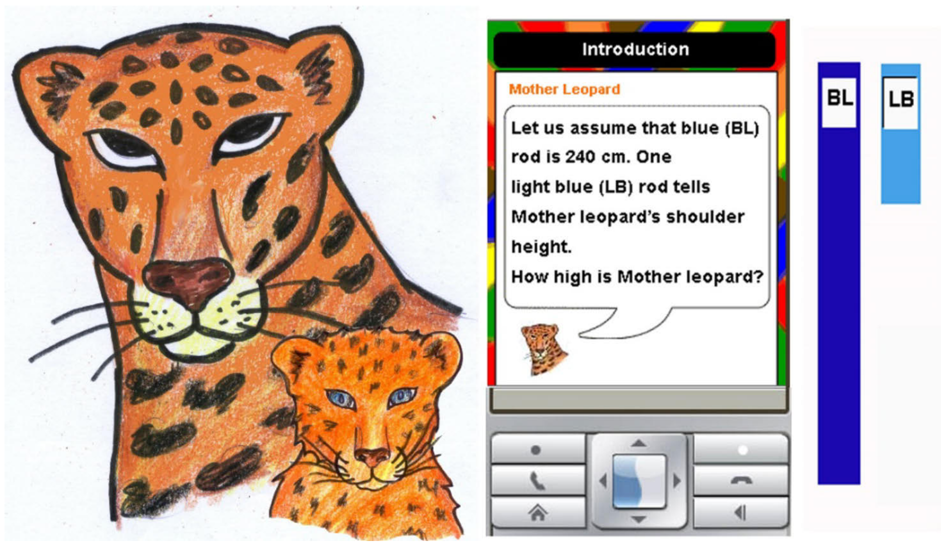

Figure 17: UFractions user interface and an example of UFractions problem.

Evaluating the *UFractions* game targeted at Grade 8 students was done in five purposefully selected secondary schools in the North-West Province, South Africa, in March 2009. Table 3 indicates the demographic characteristics of the five schools. When choosing schools, attention was paid to the fact that schools from socioeconomically different areas were included in the sample, the schools varied in size, and were located all over the province in both urban and rural areas. Altogether 105 students participated in the tests, and the group sizes ranged from 16 to 27 students. The respondents comprised 61 females and 44 males. The ages of the participants varied from 12 to 16.

Table 3: Demographic characteristics of the schools.

	Number of participants (males/females)	*Median age*
South African Schools		
Alabama Secondary	21 (8/13)	14
Lebone II	22 (11/11)	13.5
Seiphemelo Secondary	16 (6/10)	14
High School Zeerust	27 (11/16)	14
Zinniaville Secondary	19 (8/11)	13
Finnish Schools		
Arppen Koulu	32 (14/18)	14
Lieksan Keskuskoulu	31 (16/15)	14
Joensuun Normaalikoulu	18 (9/9)	14
Tietäväisen koulu	23 (14/9)	14
Mozambican Schools		
Kids Club at Polana Secondary	16 (11/5)	15.5
Maputo International School	54 (32/22)	11
Matola Secondary School	26 (12/14)	13

I visited the schools together with Teemu H Laine. Professor Seugnet Blignaut guided design of the experiments and Christo J. Els helped a lot with practical arrangements. Every evaluation session started by dividing the students into seven groups. Their first task was to create a game name for the group. The students then asked their parents to complete the informed consent forms to take part in the study. The next step was the pre-study questionnaires. The questionnaires included questions related to demographic data, mobile phone use, students' attitudes toward mathematics, regular study routines, and fraction skills. The research permit issued by the South African Ethics Committee is in Appendix 2 and the questionnaire is in Appendix 3.

Before playing the *UFractions* game, the researchers introduced the game idea with presentation slides, explaining the different game functions and the use of the mobile phones the researchers provided for the students to play on during the experiment. Examples of the problems demonstrated to the students how to use the color codes of the rods.

The students played the game for about forty minutes. Throughout the gameplay, two researchers observed the players' reactions to the game, paying particular attention to (1) problem solving using rods; (2) phone use; (3) discussions among students; (4) the players' general reactions to the game. They involved the class teachers in observing the gameplay, as well as observing their students' reactions to the game.

After playing the game, students and teachers reflected on their game playing experience by completing the questionnaires. By a semi-structured interview method, researchers interviewed the teachers and three to five students per evaluation site to collect data on their unique experiences and attitudes. They also probed questions on technical aspects and the usability of the game. The teacher's questionnaire is in Appendix 4, and the interview questions are in Appendix 5.

4.2.2 Testing the game in Finland

We completed evaluations together with Teemu H. Laine in four Finnish schools in March 2010. Table 3 exhibits the demographic characteristics of the four schools. A total of 104 students took part in the tests, and the group sizes ranged from 16 to 54 students. The respondents comprised 51 females and 53 males. The ages of the participants were homogenous, either 13 or 14.

Before the tests, the game was translated into the Finnish language. The testing procedure was the same as in South Africa.

Figure 18: Pupils engaged in game play in South Africa (March 2009) and Finland (March 2010) (Permission to use the image is included in the informed consent form)

4.2.3 Testing the game in Mozambique

Teemu H. Laine completed evaluations in two Mozambican schools in May 2011. **Table 3** indicates the demographic characteristics of the three schools. A total of 96 students took part in the tests, and the group sizes varied from 18 to 23 students. The respondents comprised 41 females and 55 males. The players' ages ranged from 10 to 32. At the moment of writing Paper IV related to Motivations for Play in the UFractions Mobile Game in Three Countries, the collection of data was still underway in Mozambique, and we used the sample size n = 70. We supplemented the sample size to make it homogenous with the other two samples in Papers V and VI. The age distribution in Mozambique was intentionally selected broader than 8th graders to measure perceptions by different age groups. Before the tests, the game was translated into Portuguese.

The testing procedure was the same as in South Africa. However, in Mozambique, we employed a modified version of the instrument, called a technology integration evaluation tool, targeting at measuring technology integration (Laine et al., 2011). The tool follows a mixed-method approach, including questionnaires and interviews for both students and teachers. These instruments measure aspects ranging from feelings and improvement suggestions to the applicability of the system to other contexts. Furthermore, the qualitative data aims at identifying disturbance factors affecting learners' experiences. The technology integration evaluation tool incorporates similar components from the instruments used for evaluating UFractions in South Africa and Finland. Specifically, demographics, background, usability, and motivation measures are similar. Laine investigated the technology evaluation tool in his PhD thesis extensively (Laine, 2011).

4.2.4 Generating theory related to the UFractions game

The researchers analyzed the gameplay experiences of all three testing contexts. The theory related to these findings is discussed in Subsections 2.2.1, 2.4.3 and 2.4.7, as well as in Papers I-III. Next chapter presents the research results, which also include the taxonomy of play motivations for the UFractions mobile game, to which Paper IV is dedicated. Additionally, I and Teemu H. Laine analyzed the connections between motivators and disturbance factors, presented as parts of the results in next chapter and Paper V. All these findings used the data presented in Sections 4.2.1-4.2.2.

4.3 Research setting for the ActiveMath tutoring system

This subsection addresses the research setting for the ActiveMath tutoring system (Stage 3 in **Figure 14**).

In this research, I used the ActiveMath intelligent tutoring system for teaching fractions. ActiveMath is a self-regulating, multi-lingual, web-based learning environment that integrates multiple mathematical functions. For instance, computer algebraic systems, functions plotter, concept map tool, semantic search, and notes function (Melis & Siekmann, 2004a). The ActiveMath system is intended for both students and teachers. The tools of the web-based system enable students to develop their mathematical knowledge, increase their level of cognition, enable students to realize their learning potential, assist students in compensating for absenteeism, and facilitate students to think about their thinking (metacognition). Figure 19 shows the user interface of ActiveMath. Teachers using the system can help students of different cognitive abilities, design their instruction, provide immediate feedback, inspire gifted students, as well as incorporate elements from the existing content to achieve a specific learning goal. Erroneous fraction examples encompassed in the system are drawn from the real world, and they are individually selected and prepared to coach German students for PISA (DFKI & Saarland University, 2007).

In 2008, I visited the University of Saarbrücken in Germany, where the system is developed. The ActiveMath system includes the fraction content using erroneous examples with an adaptive error-detection and error-correction help. I explored the fraction content and translated it into Finnish, incorporating the Finnish translation in the ActiveMath system (Tsovaltzi et al., 2010).

In March 2010, I applied the ActiveMath tutoring system in Finland with 36 students from grades 9 and 10. The experiment included a pre-questionnaire, a pre-test, a familiarisation, an intervention, a post-test, and a post-questionnaire. I was interested in seeing whether the use of erroneous examples in ActiveMaths contributed towards the development of the cognition of fractions in mathematics teaching and learning. The ActiveMath research group had conducted research related to the learning of fractions using erroneous examples in German schools and obtained positive results (Tsovaltzi et al., 2010). In Tzovaltzi's research, the effect of erronous examples on learning mathematics is studied in particular, and how the tutoring system would be best used in the case of erroneous examples. However, the thesis does not focus on this specific topic because the research would have become too broad. Therefore, these 36 students are not included in the sample of this dissertation. Instead, I used the ActiveMath system when teaching the fraction course organized using the INBECOM model. Chapter 6 contains a detailed description of the INBECOM model, and the next subsection reports the research setting for the fraction course. Also, I observed the ActiveMath experiments in Finland and wrote my observations in a learning diary. I analyzed this learning diary when I studied the affective experiences of the research participants. Research setting for this analysis is presented in Subsection 4.5 and all primary documents are in Table 5.

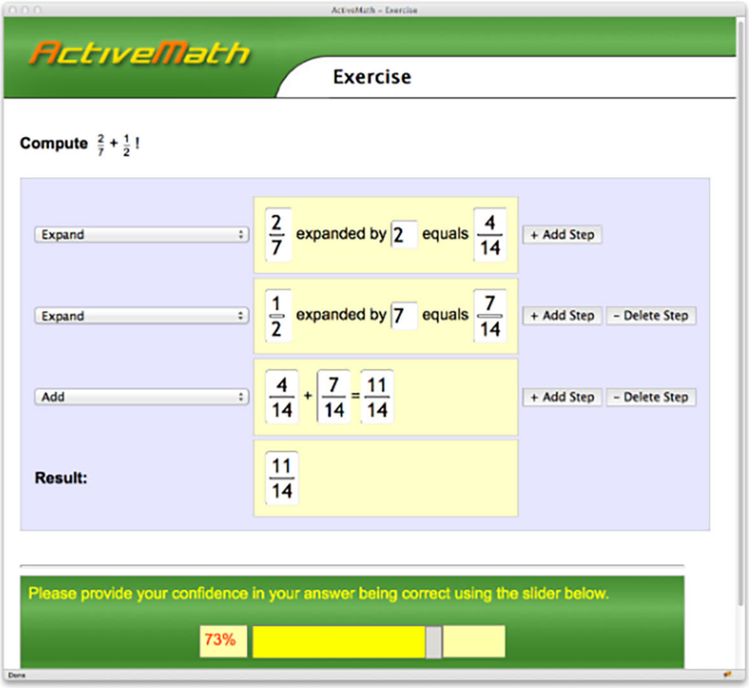

Figure 19: User interface of ActiveMath (Andres, Heeren, & Jeuring, 2013).

4.4 Research setting for the Fraction course using the INBECOM model

This subsection addresses the research setting for the Fraction course using the INBECOM model (Stage 4 in Figure 14).

I taught a course on fractions organized following the INBECOM model at the Folk High School in Kitee, Finland, during Autumn 2011. Twenty-one students participated in the course. The course lasted six weeks and consisted of eleven contact sessions. Table 4 presents more detailed course contents and methods. The course sessions were divided into three elements:

1) Game element: The students play a problem-based mobile game *UFractions* based on fractions as a mathematical concept. The students design their own mobile game stories and fraction problems.

2) Teacher involvement: The teacher interacts with students by explaining, visualizing, and discussing.

3) Intelligent tutoring system: The students learn concepts of fractions through the tutoring system (ActiveMath). Activities comprise theory and practical exercises.

Table 4: Fraction course schedule.

Learning session	Duration (min)	Content	Learning methods
1.	90	▪ course description ▪ familiarization with ActiveMath and web blog	Teacher explaining. Writing about general feelings toward mathematics and course expectations.
2.	90	▪ fraction as a part of a whole: $\frac{a}{b}$ (out of 1) ▪ fraction as a relative part: $\frac{a}{b}$ (out of c) ▪ numerator, denominator ▪ interpret numeric, symbolic and graphical presentations	30 minutes of teacher teaching, 30 minutes ActiveMath, 30 minutes writing about feelings to blog.
3.	135	▪ familiarization with UFractions game (basics of fractions)	Playing UFractions.
4.	90	▪ fraction as a relative part: $\frac{a}{b}$ (out of c)	40 min debate, 30 min teacher instructing, 20 min blog.
5.	90	▪ mixed numbers ▪ proper fractions ▪ improper fractions	30 min teacher instructing, 50 min writing game story and doing own exercises with rods.
6.	90	▪ simplification, extending ▪ equivalent fractions, unlike fractions	30 min Active Math, 30 min groups, show their stories and rod exercises to teacher, 30 min blog+ActiveMath.
7.	90	▪ comparing fractions	30 min teacher instructing,30 min writing game story, 30 min ActiveMath+blog.
8.	135	▪ adding and subtracting like fractions	30 min teacher instructing,30 min writing game story, 30 min ActiveMath+blog.
9.	90	▪ adding and subtracting unlike fractions	30 min teacher teaching, 30 min writing game story, 30 min ActiveMath+blog.
10.	90	▪ test	Traditional test with pen and paper.
11.	135	▪ presenting the game stories ▪ feedback ▪ self-assessment	Presentations to the whole group. Feedback and self-assessment using a blog.

The students wrote self-reflections on their experiences and feelings in the *"How-am-I-feeling?*-Blog" to achieve learning objectives and compelling learning experiences. In a learning diary, I captured my reflections on teaching experiences.

The INBECOM model and its development (Stage 5 in Figure 14) are documented in Chapter 6.

4.5 Research setting for affective learning experiences

The affective learning experiences of the INBECOM project participants are reported in the next chapter as part of the answer to RQ7. Paper VI focuses on Quantizing affective data as project evaluation on the use of UFractions and ActiveMath system. The research participants comprised the researchers, teachers, and students from three countries: South Africa, Finland, and Mozambique. The study sample was obtained by data collected during the project 2007-2014, including plans, reports, questionnaires, interviews, and observations. Table 5 lists the primary documents used and the related participants. The total number of analyzed documents was 311. For the analysis of the affective learning experiences, I also used the data from 11 students not involved in the data sets used in the analysis related to research questions Q1-Q6 (Papers I-V), because those students were not Grade 8 students, but interested in mobile game development.

I completed the data analysis in two stages. To explore the affective learning process during the project; the qualitative data analysis to identify and evaluate the process; and the quantitative data analysis, to find significant differences between the groups and between the affective learning stages, as well as to validate the findings of the qualitative analysis (Table 12). The qualitative data were analyzed using an overview analysis, codes having been predetermined according to the five different levels of Bloom's *affective domain of learning, receive, respond, value, organize and internalize* (Krathwohl et al., 1964).

Table 5: Primary documents and participants.

Participants	Data Sources as Primary Documents	Number of participants
Researcher	Project plan	1 researcher
Researcher	Description of the INBECOM model	1 researcher
Researcher	Interview of a researcher in Indigenous Knowledge Systems	1 researcher
Researcher	Interview of a professor in Setswana	1 researcher
SA students	Interview of students after game tests in SA	20 students
SA teachers	Interview of teachers during *UFractions* game tests	5 teachers
South African teachers	Questionnaire to teachers, SA *UFractions* tests	5 teachers
Finnish students	Questionnaire, *UFractions* tests	78 students
SA students	Questionnaire, *UFractions* tests	116 students
SA researchers	Questionnaire	7 researchers
Finnish teachers	Interview of teachers during *UFractions* game tests	5 teachers
Finnish students	Interview of students during *UFractions* game tests	26 students
Mozambican students	Questionnaire to players, Mozambique *UFractions* tests	69 students
Mozambican teachers	Questionnaire	2 teachers
Mozambican teacher	Interview of teachers during *UFractions* game tests	2 teachers
Finnish students	*How-I-am-feeling?-blog* during mathematics course	21 students
Researcher	Experiment notes of ActiveMath tests	1 researcher
Researcher	Learning experiences as learning diary during the project	1 researcher

5 Findings

I will now discuss each research question and responses in turn. This chapter will also summarize the results reported in publications included in this thesis (**Table 2**).

Q1 What are the dimensions of learning mathematics with technology-enhanced learning?

Chapter 2 presents a literature review used to answer this question. Firstly, the primary learning paradigms, behaviorism, cognitivism, and constructivism, are defined and discussed. Considering the use of learning theories for designing and using educational technology, I introduce two views that inspired me to do this research. Ertmer and Newby (1993) suggest that instructional designers must intelligently choose the appropriate methods based on information gathered from the learners' present competence level and the type of learning task. According to Cronje (2006), learning events can contain both behavioristic and constructivistic elements. He suggests that these two approaches are situated at right angles to one another and presented as rectangle lines (Figure 3). This integrative viewpoint of learning theories would lead to more realistic and meaningful learning and teaching strategies.

I address the dimensions of learning by describing motivation theories and Bloom's taxonomy classifying learning according to three domains: cognitive, psychomotor, and effective. The various technological tools used in the teaching of mathematics are presented as well as their typical use and relation to the learning theories. More generally, issues related to the use of technology in teaching and technology integration into classrooms are discussed. Furthermore, the level of technology integration, the level of students' cognitive process, and the level of teachers' knowledge are intertwined (Figure 7).

Throughout the study, the goal was to mirror the results of the empirical research to these three dimensions (Figure 12). Hence, in Chapter 7, I define more precisely a dimension taxonomy of learning mathematics with technology-enhanced learning based on the answers to other research questions.

Q2 How can artifacts for mathematics learning be developed?

In Chapter 2, various technological tools for learning mathematics were presented, as well as their connections to learning theories. The theory of games used in learning mathematics was also summarized. Using this information as a starting point, a mathematical mobile, story-based game UFractions was developed.

In Subsection 4.2.1 and papers I and III, the development and contextualization of the UFractions mobile game in South Africa is described. The features of the *Myst* pervasive mobile learning platform the UFractions utilizes are presented, as well as the process of creating a storyline with appropriate mathematical problems to UFractions for the South African context. The *UFractions* game is expressed in detail.

According to the observations made during the South African interventions, all the students enjoyed playing the UFractions. Quantitative data shows participants' contentment with the game; almost all the participants thought that it was fun to play with mobile phones, and, compared to ordinary mathematics class activities, the gameplay was exciting. Students actively argued about math problems. After gaming, students were eager to view the web pages to determine if Senatla had survived and how their group's totals compared to those of others.

The quantitative analysis shows that most liked game activities were problem-solving, using the mobile phone, playing with the rods, and reading the story (Figure 20).

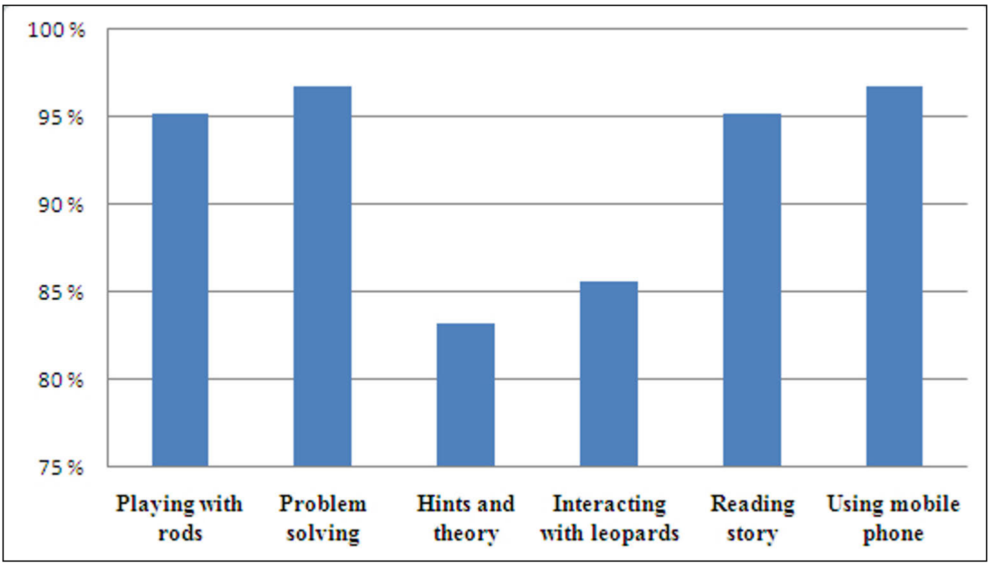

Figure 20: Most liked game activities according to quantitative analysis.

Part of the research focused on the novel way of combining manipulates and mobile game work in a real classroom situation. Quantitative data shows that students liked playing with the math rods (96.2%), and almost all the participants (92.4%) felt that they wanted to play more with the math rods. The majority of players (96.2%) also indicated that math rods helped them to understand fractions (Table 6). Qualitative data obtained from interviews support the view that rods are helpful to the students. The students explained various ways in which math rods helped them to solve problems. The teachers indicated that the use of maths rods is effective and motivational, and additionally, combining manipulates, and the game enhances the learning process in mathematics.

Table 6: South African students' feedback related to the use of mathematical rods.

	Strongly agree	Agree	Disagree	Strongly disagree	I do not have an opinion
I wanted to play more with math rods	68	29	4	2	2
I liked playing with the math rods	69	32	2	2	0
Math rods helped me understanding the fractions	78	23	1	1	2

Playing experiences showed that the usability of UFractions is good, and students can use the phone easily as a playing tool. Players' constructive feedback helps the future development of UFractions.

As part of the answer to this research question, Chapter 6 illustrates the development and features of the pedagogical INBECOM model supporting meaningful and useful mathematics learning.

Q3 What are the considerations to be taken into account during the design of a mobile game for learning fractions in diverse contexts?

The research contexts are described in Subsection 4.1. Papers I, II and III focused particularly on South-African and Finnish contexts and the role of culture in learning mathematics.

In Subsections 4.2.1 and 4.2.2 and Paper II, a case study of the UFractions game is presented. The game was developed to support learners in South Africa, and the same game was subsequently delivered to a class of Finnish learners at the same grade level only by translating the game into Finnish. This process is called reverse transfer, in contrast to a typical technology transfer that takes a technology created in a technology-familiar environment (such as Finland) to a technology-alien environment (such as many places in South Africa).

The reverse transfer process, taking technology from a context where the use of technology is, on average, relatively low (South Africa) to a context with advanced use of technology (Finland), facilitated us to analyze the contextual factors of technology for learning. In the traditional setting of transferring technology from the North to the South, it is difficult to assess how much of possible obstacles in using technology are the result of being unfamiliar to it rather than the cultural or pedagogical preferences inbuilt in the use of technology from a different context. Contrary to that, the reverse transfer process resulted in a scenario where pupils with profound expertise in technology were given technology with only a cultural or pedagogical bias, not technical.

Two aspects of the context were anayzed: the cultural and pedagogical ones, and the effects of technology integration were identified. To evaluate the adaptation of UFractions to the *cultural context,* we considered the elements of the language used as well, as the story and the player's immersion (concentration on the story) in it. Immersion can lead to a flow state, which in turn can be interpreted as a deep, intrinsically motivated, and therefore effective learning experience (Csikszentmihalyi & Hermanson, 1995).

As expected, there were several differences between the Finnish and the South African data sets. The South African pupils were very positive about all evaluated aspects as only a few pupils gave negative answers. Finnish pupils, on the other hand, were slightly negative about most aspects, particularly those that concerned the story and the players' immersion in it. One of the key findings regarding cultural context adaptation was that there was a significant difference in the level of immersion between the two groups. We can conclude that many Finnish students, due to lack of contextualization, may not have reached the flow state at all.

The role of pedagogical context adaptation was measured by three statements measuring the comprehension of problems, the use of concrete manipulatives in learning fractions, and the general effectiveness of UFractions as a learning tool. The two sets of students considered the presented problems as easy to understand. South African pupils highly valued the use of concrete manipulatives to connect the fractions theory to physical objects, as 96% of them considered the rods useful in learning fractions. Respectively, 74% of the Finnish pupils reported the rods helping them in understanding fractions, and 13% did not give an opinion. The educational value of the game was investigated by measuring how much the players felt that they learned fractions during the game session. This metric is far from comprehensive, but it may indicate educational effectiveness as well as the pedagogical suitability of UFractions for the target groups. The South African data set revealed that 95% of the pupils reported that they learned while playing, and 80% thought that they learned a lot (strongly agreed).

On the other hand, only 33% of the Finns reported that they learned fractions while playing, 40% did not report any learning at all, and 28% had no opinion. These results indicate that the game and its challenges were useful in the South African context and that they were also useful for some of the Finnish pupils. As the cultural context adaptation results above suggest, the Finnish pupils were not immersed in the story as much as the South African pupils, hence rendering their learning processes ineffective. Additionally, 51% of the Finnish teams played through at least two levels of the game. In contrast, in the case of South African teams, this figure was 19%, suggesting that the challenges were easy for at least some of the Finnish pupils, resulting in challenges being solved routinely.

The aim of technology integration analysis was to see how natural the use of integrated technology was to the pupils. The phone was considered to be an easy tool for playing in both data sets (Finland 83%, South Africa 93%). The motivation / fun factor[21] for using the mobile technology was higher in South Africa (95%) than in Finland (78%). For some pupils there might have been a degree of novelty value in using the phone for playing in South Africa, because the mobile phone ownership in South African data set was 63% and in Finnish data set 100%. Fraction rods as a learning technology were enjoyed by most of the pupils; 64% of Finns and 96% of South African.

These results can be considered as an indication that a learning tool cannot only be (reverse-) transferred without proper context adaptation or re-contextualization. Instead, a full re-contextualization process is required covering at least the cultural and pedagogical aspects of the context of the users. This might also, strongly, indicate the difficulties for importing educational technology from the North to the South, without (re-)contextualization.

Empirical studies of the UFractions game were conducted in a third country, namely Mozambique. The procedure of the research in Mozambique is described in the Subsection 4.2.3 and paper V. We deepened the evaluation of the UFractions game in different contexts by also looking at disturbing factors perceived by students, and the *disturbance factor (DF)* was defined as an element of a learning system, having a negative effect on the learner. Some of the factors disturbing the game were related to the students' skill level, and therefore we used Vygotsky’s Zone of Proximal Development (ZPD) to define some of the disturbance factors (Csikszentmihalyi, 1991) . It means the area or level of cognitive activity at which the learner is able to act with the support of a qualified instructor, but not independently. Guidelines for game developers from the noticed disturbance factors were developed (Table 7). Because development is a resource-intensive operation, we can

[21] Question “It was fun to play with the phone”.

maximize the effort by concentrating on the good sides of the game and make them stronger while eliminating critical disturbances that affect many motivators. It is important to keep in mind that diminishing one disturbance factor might strengthen other disturbance factors. For example, making the story shorter because a few players noted it to be too long may cause more players to consider the game too short. Therefore, necessary precautions must be taken and constant evaluation applied to ensure that the changes do not produce undesired effects.

Table 7: Guidelines for game developers based on the UFractions disturbance factors.

Target group	Guideline	Disturbance Factors
Developers	Allow end-users in different contexts to create stories to promote ownership and improve contextualization	Disturbing content, monotony, wrong age group
	Include multiple modalities of media and presentation templates that can be chosen by the user from game preferences	Monotony, Inappropriate graphics, Inappropriate sounds
	Create multiple levels of challenge to accommodate players with different abilities	Beyond ZPD, Below ZPD
	Enable gameplay with players' own devices to reduce usability issues related to new technology	Inconvenient interaction with the phone, small screen
	Level test before playing	Beyond ZPD, Below ZPD

Q4 What strategies do we need to compile a mobile game for learning for diverse contexts?

After developing the UFractions game, it was in South African schools in 2009. At that time, the low number of students majoring in mathematics at South African universities and inadequate mathematical background of university students was serious concerns for the socio-economical development of the country (Engelbrecht & Harding, 2003). South African youth already used a lot of mobile phones, but they were not used much for learning purposes in schools (Kreutzer, 2009). Therefore, we wanted to test how the mobile game would be received in South African schools and what kind of factors should be taken into account when compiling it.

According to this empirical study, teacher's attitudes towards mobile gaming are positive and they would like to use more mobile games in their teaching and learning. Especially the active participation of players was mentioned by many teachers, as for example *"...the most of the children were involved from the beginning and they stayed involved right through the whole game. There was no one that was disappearing in the group, they were all arguing…"*.

However, according to the qualitative analysis of the data, mobile gaming is not yet practical in South African schools from the teachers' point of view because of the following five significant challenges: *time, big classroom sizes, lack of resources, lack of compatible games,* and *language* (Paper III). Figure 21 shows examples of teachers' comments related to these five challenges.

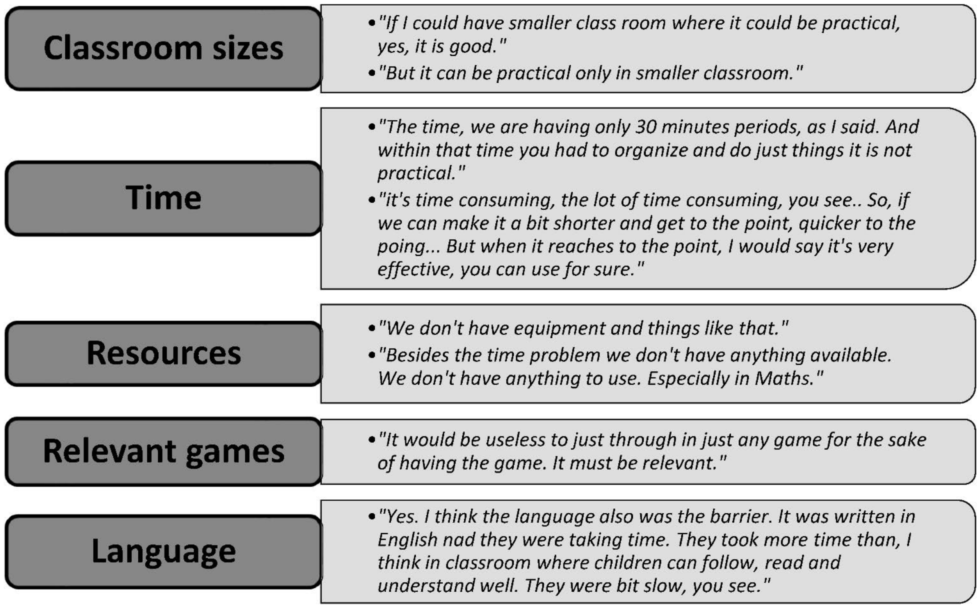

Figure 21: South African teachers' comments about the challenges for mobile gaming.

The evaluation of students' game playing was presented in the answer to research question Q2. As a conclusion from the South African experiences, mobile gaming could become a suitable learning strategy for South African schools. Students engaged in mobile gaming and teachers would like to use mobile games in their teaching and learning. As a whole, these findings from the first evaluations of UFractions are similar to other empirical studies conducted at the time on mobile game playing in South African schools (Ford & Batchelor, 2007; Roberts & Vänskä, 2011). However, challenges will most probably change over time – for example technology becomes cheaper.

Creating well designed, practical educational games that can be used in affordable mobile devices, is an important step in implementing mobile games in South African schools, as well as in other contexts. Therefore, to answer the question what strategies do we need to compile a mobile game for learning for diverse contexts we analysed also the data from empirical tests in Finland and Mozambique from the viewpoint of disturbance factors, and the guidelines for game users and educators were developed (Table 8). Recommendations for teaching pedagogical strategies given in Subsection 7.4 are also related to playing educational mobile games in different contexts.

Table 8: Guidelines for game developers, users, and educators based on the UFractions.

Target group	Guideline	Disturbance Factors
Users	Work as a team: one member reads the story, others handle rods. Roles can be changed after some time	Lack of peer support, Harassment
	Read the instructions carefully to avoid losing points because of misunderstanding	Unclear instructions, Lack of scaffolding
	Choose the game level based on your skills. Choose level 1 if you are unsure	Beyond ZPD, Below ZPD
Educators	Ensure that the students possess the necessary prerequisite knowledge for learning the topics.	Beyond ZPD, wrong age group.
	Reconsider using another game or level if the students already master the topics covered in the game or level.	Below ZPD, the wrong age group.
	Explain the use of auxiliary tools (e.g. fraction rods) to students before playing the game for the first time.	Inconvenient interaction with rods, Unclear instructions
	Divide students into teams and assign roles to each member (see above)	Lack of peer support, Harassment
	Participate in technology training to learn how to solve technical and usability problems	Technical faults, Inconvenient Interaction with rods and phone

Q5 How can artifacts, such as games for learning, address the different dimensions of learning mathematics?

Based on the evaluation of the Ufractions game, story-based mobile gaming brings many dimensions to learning. Using cluster analysis, I investigated what the students had commented about the story of the game in particular, and what they had told about mathematics. Students identified themselves with the story of Mother leopard and her cub Senatla, and the story induced ethical, physical, and cognitive rationales (Paper I). Participants solved actively real-life fraction problems using mathematical rods and gave compelling, functional, and action-oriented arguments for liking the mathematics in the game (Table 9).

Table 9: Rationales related to the story and different arguments about mathematics in the game.

Ethical rationale	Cognitive rationale	Physical rationale
“I enjoy playing with leopard because I was helping them."	*“I enjoyed knowing and learning about them.”*	*“I enjoyed feeding Senatla and playing with him."*
"I enjoyed playing the game because I want to help the wildlife."	*"I enjoyed learning how they survive in the wild and what they eat."*	*“When they run fast, I want to compete with them."*
Affective argument	**Action-oriented argument**	**Functional argument**
"I liked the fractions a lot."	*"Hands-on"*	*“It enabled me with my maths."*
"Adding the things together."	*"I enjoyed solving the problems."*	*“It exercises your brain."*
"I do like more think like fractions cm, kg, km, and so on."	*“The problem solving was great!”*	*“To make use of my mathematical brain.”*

In Paper IV, we initiate the development of a taxonomy for the variety of play motivations in the UFractions game through quantitative data analysis of the data set collected through the questionnaires: altruism, challenge, curiosity, fantasy, relations, and technology. The basis for finding these motivations was the theory of the subject as described in section 2.4.3 and Table 1, and of course the setting of the questions in the questionnaire. Next, we analyzed the qualitative data. Interviews were manually transcribed and then both interviews and the open-ended questions were thematically coded using Atlas.ti software (Boyatzis, 1998). The qualitative data-analysis identified the same six-play motivators that we identified from the quantitative data analysis. However, there were several cases in the coded material that did not relate to any of these six motivators. Example sentences for such cases are: “I enjoyed the way I was learning” and “... The thinking process”. Hence, the qualitative data-analysis revealed an additional seventh play motivator for mobile educational games, namely *cognitive restlessness*.

Altruism

According to Malone (1981), fantasies in computer games almost certainly awaken some emotional needs players want to satisfy. The story of the leopards and especially helping the cub appeal to altruism, which is close to empathy, but a more generic approach. In biology, *altruism* means the active donation of resources to one or more individuals at the cost to the donor (Wilson & Keil, 1999). More generally, altruism is a deliberate pursuit of interest or welfare in others or fictive characters, and altruistic people help others unselfishly. Altruism is the opposite of egoism.

Challenge

Challenge is one of the major causes of the flow experience. The game should be designed so that its structure allows increasing or decreasing the level of challenges the player is facing so that the player’s skills match the level of the game missions (Csikszentmihalyi, 1991). The game is challenging when it provides goals with uncertain attainment (Malone, 1981). According to Alessi and Trollip (2001, p. 279), "challenge differs from a goal in that challenge is what one has to overcome or succeed to reach a goal." In an intrinsically motivating environment, goals should be personally meaningful, and the attainment of a goal should be demanding. Players also need some feedback to know how well they are achieving their goals (Malone, 1981).

Cognitive Restless

Cognition refers to the process of thought. Cognitive restlessness is about the desire to gain knowledge, to process information, and acquire knowledge using different methods. Games embody the process of cognitive disequilibrium, a concept defined by psychologist Jean Piaget in the late 1960s (Piaget, 1969). People experience cognitive disequilibrium when there is a discrepancy between something new and previously acquired knowledge. This discrepancy produces a state of disequilibrium, driving them to eliminate it, in other words, to learn something new to achieve equilibrium. According to Van Eck (2006), the extent to which games accomplish in creating cognitive disequilibrium without exceeding the capacity of the player to succeed largely determines the engagement of the game. Games should create a continuous cycle of cognitive disequilibrium and resolution. This is obtained via assimilation (players attempt to fit new information into existing slots or categories) or accommodation (players must modify their existing model of the world to accommodate new information that does not fit into an existing slot or category).

<u>Curiosity</u>

The degree to which games can arouse and then satisfy players' curiosity is one of the essential features of intrinsically motivating games (Malone, 1981). According to Alessi and Trollip (2001), curiosity compels students to seek new knowledge and motivates them to learn beyond what they currently know or to explore further the game. Curiosity and challenges are closely related; they both often depend on the environment's adjustment to the learner's level of understanding and ability (Alessi & Trollip, 2001; Malone, 1981). Malone (1981) identifies two types of curiosity. In essence, *sensory curiosity* is curiosity related to images, sounds, or other sensory input in the game.

In contrast, *cognitive curiosity* is curiosity about information, and it is evoked by presenting just enough information to make players' knowledge seem incomplete or inconsistent to motivate them to learn more to complexify their cognitive structures better-formed. Malone and Lepper (1987) mention Cuisenaire Rods as an example of instructional material designed to stimulate sensory curiosity. Educational games should both arouse a person's curiosity and satisfy it through learning the information embedded in the game (Alessi & Trollip, 2001). Informative feedback is also a factor of curiosity (Alessi & Trollip, 2001; Malone, 1981), and in the UFractions game, the feedback was both surprising and constructive to support sensory and cognitive curiosity.

<u>Fantasy</u>

Many games involve fantasies to make them more interesting and more educational (Malone, 1981). The degree of fantasy can vary substantially from a precise representation of reality to a more unrealistic imaginary story. Malone defines two types of fantasies in games: *Extrinsic fantasies* depend on the use of the player's skills, not vice versa, so that the same fantasy could be used with completely different kinds of problems. *Intrinsic fantasy* skills also depend on the fantasy so that "problems are presented in terms of the elements of the fantasy world, and players receive a natural kind of constructive feedback" (Malone, 1981, p. 361). Malone also claims that intrinsic fantasies are both more exciting and more instructional than extrinsic fantasies (Malone, 1981, p. 361). The UFractions' story of leopards can be considered as an intrinsic fantasy because the game's math problems are related to leopards' life and leopards themselves give feedback to players.

Relations

Relations are related to the social dimension, in other words, to a preference for group work versus individual work. From an early age, students are socialized into various beliefs, values, customs, and orientations of their family, friends, and community. People differ in the extent to which they promote individual values (for example, power, achievement, and stimulation) versus collectivistic values (for example, benevolence, tradition, and conformity) (DeVito, 2001).

Technology

Technology refers to the educational technology with a variety of tools supporting students´ learning process. The term technology deals with material objects, and they can be modern technical devices such as cellular phones but also tangible manipulatives like math rods.

My taxonomy has similarities with existing taxonomies presented in Table 1, such as those of Malone and Lepper: *challenge, fantasy*, *curiosity*, and *control* (Malone & Lepper, 1987). The differences between the taxonomies could be due to the differing contexts. Malone and Lepper studied educational games generally. Contrary to them, I studied a story-based mobile game with math rods. The motivation *altruism* derives from the mission of the game— to help the leopards. Moreover, the use of mobile phones and rods generates motivation *technology*. Cognitive restlessness and challenges are similar to some extent, but I decided to keep them separate because many respondents emphasized the importance of the learning process. Thus, the motivation challenge is more related to reaching and accomplishing goals, and cognitive restlessness means enjoying the way of learning.

Q6 What is the relationship between motivation and disturbances during learning mathematics with games for learning?

Understanding engagement in games provides significant opportunities for developing motivating educational games. However, even good games may induce disturbances in the learner. In Paper V, we go further than only presenting the results and discussion related to the motivation aspects and disturbance factors of the playing experience in the UFractions game. Namely, we define the dynamics between these two important game features.

Table 10: Disturbance factors identified by the technology integration evaluation tool (adapted from (Laine et al., 2011).

Disturbance factor	***I***	***Evidence***
Too long game	A	"The game is very big. It must have been a bit shorter" (Male, 13, Indian)
Too short game	A	"I thought they could have a bit...maybe a bit longer the game." (Male, 12, Mozambican)
Beyond ZPD	A	"There were some fractions that were difficult to solve." (Male, 13, Mozambican)
Below ZPD	A	"For learning purpose maybe you should make it a little harder but as a game it is ok.", (Male, 12, Indian)
Wrong age group	A	"Maybe it would be better for younger kids because it's this story of two leopards, so it would be from 8 to 11." (Female, 13, Indian)
Lack of scaffolding	A	"Sometimes when you were doing a question and you keep on not understanding I think there should be like where you can go to the next question if you can." (Male, 11, Mozambican)
Conflicting content	A	"I was surprised because I had some answers that I was sure were correct but somehow they were wrong" (Male, 11, English)
Too much story	A	"Too much reading and after a while it gets boring" (Female, 13, Mozambican)
Monotony	A	"A part that I didn't like was that it was always about leopards. If we had lots of settings with maybe gorilla and rhino we could all learn the lives of lots of animals which shows you lots of different fact. (Male, 11, Irish)
Too educational	A	"It was nice but the thing is like it's not something I wanna do on a weekend or something. Maybe if you're bored..." (Male, 12, Indian)
Harrassment	A	"The thing was that two people would play it so one person would just take the phone and the other person will take it. The other person would have taken it and I couldn't have read so that was sort of a disadvantage. (Female, 11, Korean)
Lack of peer support	A	"Disadvantage is that maybe no one would be there to explain to you" (Female, 13, Indian)
Disturbing content	A	"The story of Senatla is not very good because the father of Senatla did not care for Senatla. Senatla was living with her mother..." (Female, 17, Mozambican)
Punishment	A	"[I disliked] When we got questions incorrect" (Male, 11, Mozambican)
Lack of animation	P	"I'd just say more animations into the story, kind of hide the fact that it's about fractions. […] (Male, 12, Indian)
Inappropriate graphics	P	"The screen was a bit too...all the colors around it and...it kind of...not too many colors but all the colors around it were kind of distracting. It could be one plain color maybe." (Male, 11, Irish")
Inappropriate sounds	P	"Make it more lively with sound" (Male, 13, Mozambican) "If you're gonna improve it, maybe you should like...let's say if someone has troubles reading it you should have voice over" (Male, 12, Indian)
Inconvenient Interaction with rods	P	"I wouldn't advise to use them because sometimes they make it complicated." (Female, 15, Mozambican)
Inconvenient interaction with phone	P	"One thing that I really didn't find that much interesting was using the phone. That wasn't that much fun but I think that's all really.[…] There were buttons and everything. I think it would be easier if you use something like maybe a calculator or something." (Male, 11, Mozambican)
Technical faults	P	"Once it turned down...it quit by itself but then we were on track again." (Male, 11, Irish)
Small screen	P	"The phone's screen was a bit too small so I couldn't see." (Female, 11, Korean)
Unclear instructions	p	"I didn't like some parts because I didn't quite understand some questions. Like about four questions but the rest was ok." (Male, 12, Mozambican)

The connections between motivations and disturbance factors (DFs) are essential to acknowledge because this knowledge may help us to diminish the DFs and thereby increase motivation while preventing undesired side effects. To identify DFs in UFractions, the qualitative data sets from South Africa and Finland were analyzed which led to the discovery of the 16 DFs (Laine et al., 2011). Laine (2011) developed the technology integration evaluation tool that was tested in the Mozambican context (more information in Subsection 4.2.3). In this evaluation 22 DFs were identified including all but one of the previously discovered 16 factors. Table 10 presents these 22 DFs together with integration type (I: Active or Passive), and evidence samples from Mozambican data supporting the DFs.

To understand the interplay of DFs and motivations in the UFractions game play, the data from empirical tests in South Africa, Finland, and Mozambique was analysed. The total sample in was 305 students. The relationships between identified motivations and DFs in the UFractions game are described in Table 11. Each motivation relates to a set of DFs, typically having adverse effects on the player's motivation. Exceptions are DFs marked with asterisks: they may affect the respective motivations positively.

Table 11: Connections between motivations and disturbance factors in the UFractions.

Motivation	Disturbance factors
Altruism	Too long game, wrong age group, too much story, monotony, too educational, *disturbing content
Challenge	Too long game, below ZPD, beyond ZPD, wrong age group, lack of scaffolding, *conflicting content, monotony, punishment, unclear instructions
Cognitive restlessness	Too long game, too short game, below ZPD, beyond ZPD, wrong age group, lack of scaffolding, *conflicting content, too much story, monotony, too educational, disturbing content, unclear instructions
Curiosity – sensory	Lack of animation, inappropriate graphics, inappropriate sounds, inconvenient interaction with rods
Curiosity – cognitive	Too long game, below ZPD, beyond ZPD, *conflicting content, too much story, monotony, unclear instructions
Fantasy	Too long game, wrong age group, too much story, monotony, too educational, *disturbing content
Relations	Harassment, lack of peer support
Technology – mental	Inappropriate graphics, inappropriate sounds, lack of animation
Technology – physical	Small screen, inconvenient interaction with phone, inconvenient interaction with rods, technical faults

We also categorized the identified motivations on the axes of active-passive technology integration and cognition-affection (Figure 22). For example, altruism and fantasy are related to affective domain of learning, and the mobile game integrates the contextual resources to support these motivations via suitable story and characters, in other words game is actively integrated. Instead, the technology motivation is related to passive integration, where technology is the object of integrationr. Supporting motivations for both cognition and affection is essential. Should we only emphasize the motivators connected to knowledge, the game would turn into a bad example of a serious game which would lack the emotional attachment that makes many games addictive. In another scenario, if we would reduce the value of knowledge and emphasize motivators connected to feelings, the game would be less educating and more drama-like. Finding a balance between cognition and affection depends on such factors as the game type, context, topic, and available technology.

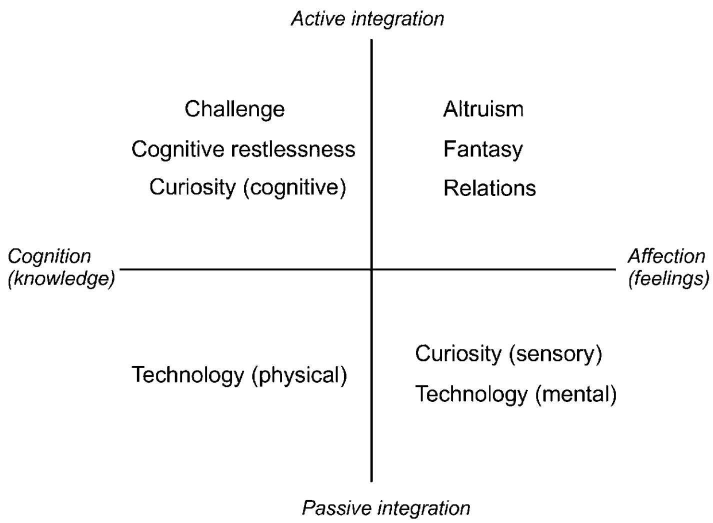

Figure 22: Motivations in the dimensions of active-passive integration and cognition-affection. (Paper V).

We applied (Edmondson, 2008) idea related to psychological safety hindering performance from the field of leadership and management, to the motivations and disturbance factors. Six different *learning zones* of playing were defined (Figure 23) by estimating the intensity of DFs and motivational intensity and placing them onto a 2x3 matrix. The intensity of a DF indicates the learners' irritation level and the importance of its effect on the gameplay. In gaming, motivational intensity means the degree of player motivation on the scale from low to high. When the DFs are too intensive, they become destruction factors or even crucial to make the game unplayable.

We identified three zones of learning with high motivational intensity:

1) FLOW ZONE: With the low intensity of DFs, the game is played in the flow state, enabling effective content learning.

2) CREATIVE ZONE: With the medium intensity of DFs, students not only learn well the game content but involve themselves in co-designing the game and enhance their learning.

3) IRRITATION ZONE: With high intensity of DFs, despite irritation and weak content learning, students may learn some other necessary skills. For example, to handle information and solve problems, and to apply self-made or ready-made computer programs as part of studying mathematics. These are considered as the learning objectives in the Finnish National Core Curriculum for Basic Education (Finnish National Agency for Education, 2014).

Correspondingly, there are three zones of learning when motivational intensity is low:

4) ROUTINE ZONE: With the low intensity of DFs, students get on playing. The gameplay takes place routinely without disturbances, resulting in moderate learning of game content.

5) APATHY ZONE: With the medium intensity of DFs, students lack sufficient motivation to overcome the DFs on their own. They expect the teacher to assist, quitting quickly, and learning poorly.

6) FRUSTRATION ZONE: With high intensity of DFs, students become frustrated, which is a severe impediment to learning the content.

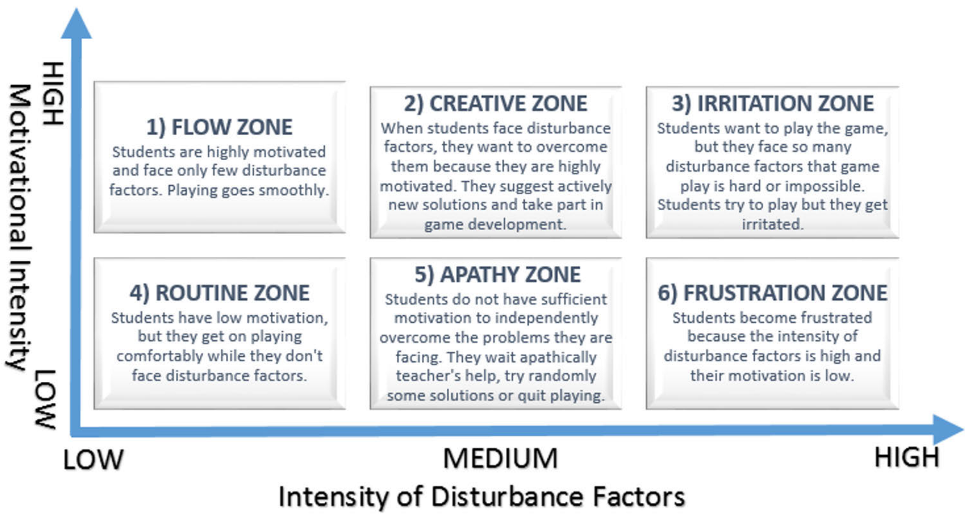

Figure 23: Six different zones of learning according to motivational intensity and intensity of disturbance factors (Paper V).

From the learning zones, we notice that DFs with medium intensity may nourish the learning process in several ways:

- Affective learning: learning content is perhaps better remembered when the learning environment triggers strong emotions
- Critical thinking: DFs create disequilibrium, which can help the learner examine the subject of the lesson critically.
- Creative problem solving: if the problem cannot be resolved quickly due to DFs, then new, creative ways to solve the problem must be figured out. This, however, concerns more extensive problems than the UFractions exercises.

Q7 How does an approach of creating interventions and artifacts based on the taxonomy for dimensions of learning mathematics trigger affective learning?

Paper VI presents the multi-method evaluation of the INBECOM project conducted according to the affective learning experiences domain of Krathwohl. The purpose of the study was twofold:

(i) to explore the affective learning experiences of the three groups of participants (researchers, teachers, and students) and

(ii) to determine the significance of the relationships among the affective learning experiences of the three groups of participants across three contexts (South Africa, Finland, and Mozambique).

Table 12: The themes and code density of the qualitative analysis.

Krathwohl's Levels of Affective Learning									
Level 1 Receive (15 codes)		**Level 2 Respond (14 codes)**		**Level 3 Value (8 Codes)**		**Level 4 Organize (11 codes)**		**Level 5 Internalize (5 codes)**	
Be open to experience	187	Assist	66	Justify	227	Arrange	2	Act	4
Acknowledge	214	Become exited	58	Argue	145	Build	49	Display	14
Ask	2	Cite	0	Challenge	186	Compare	32	Influence	31
Attend	8	Clarify	69	Confront	6	Contrast	15	Practice	29
Identify	9	Contribute	11	Criticize	123	Defend	21	Solve	74
Discuss	9	Interpret	8	Debate	19	Develop	202		
Do	180	Perform	128	Persuade	2	Formulate	3		
Feel	509	Present	5	Refute	7	Modify	15		
Focus	1	Provide references	191			Prioritize	12		
Follow	4	Question	7			Reconcile	17		
Hear	4	React	21			Relate	11		
Listen	9	Respond	88						
Read	9	Seek clarification	16						
Retain	7	Write	17						
Participate	8								
TOTAL:	**470**		**685**		**715**		**379**		**152**

N=3405

The qualitative data (Table 5) was deductively analyzed following the adequate levels of Krathwohl and subsequently quantitized as instances on an Excel™ spreadsheet for statistical analyses. We encountered effective learning codes across all five levels (receive, respond, value, organize, and internalize) of all the participating research groups involved in the study. During the deductive coding and analysis process of the data, we employed the illustrative verbs of the Krathwohl framework. Table 12 lists the structure of the qualitative analysis according to the themes and code density.

The highest density of codes was related to the lowest level of Bloom's taxonomy of affective learning, namely the *receive* stage. The effective learning occurred among students, teachers, and researchers are summarized in Figure 24.

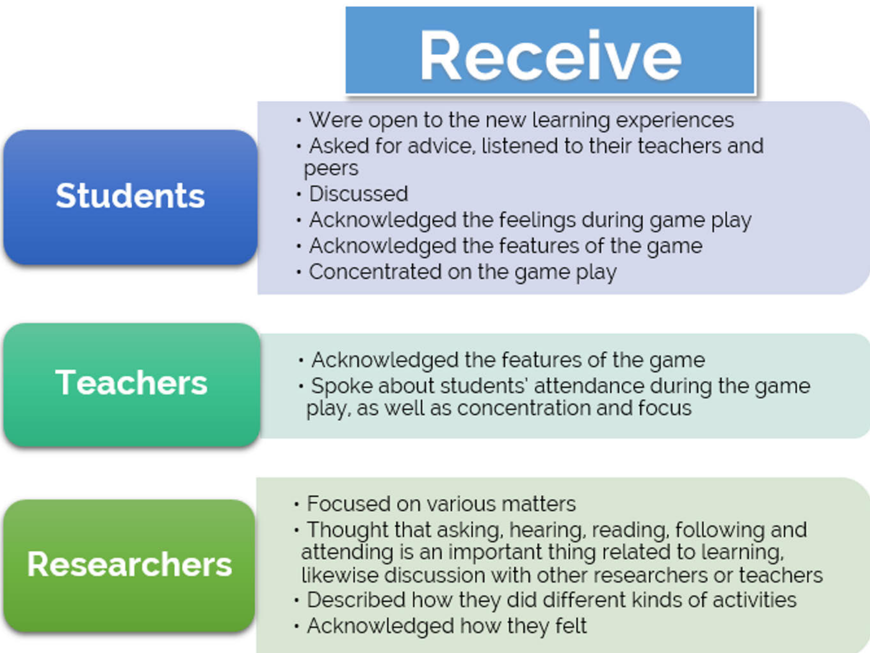

Figure 24: Affective learning at receive stage.

Regarding the second response stage, responding, the students became animated, interpreted, and gave references. Students contributed to the UFractions gameplay for development. Compared to the comments at the receive stage, the students reacted more and showed more active participation. The teachers' comments related to the responding stage display how students got animated of gameplay. Teachers clarify and interpret issues, as well as provide examples. Researchers appreciated assisting each other. Considering research, they sought clarification, contributed, interpreted, and gave references and showed interest in research outcomes. Researchers also showed active participation by making questions, reacting, and responding. Figure 25 shows a wrap-up of affective learning on the response stage.

Regarding the value stage, students saw the benefit of using ICT and playing games. They accepted or committed stances of action by making justifications and arguing. Students also expressed their personal opinions by noticing challenges and criticizing. Teachers attached values by advocating their own opinions. They argued the effects of the game.

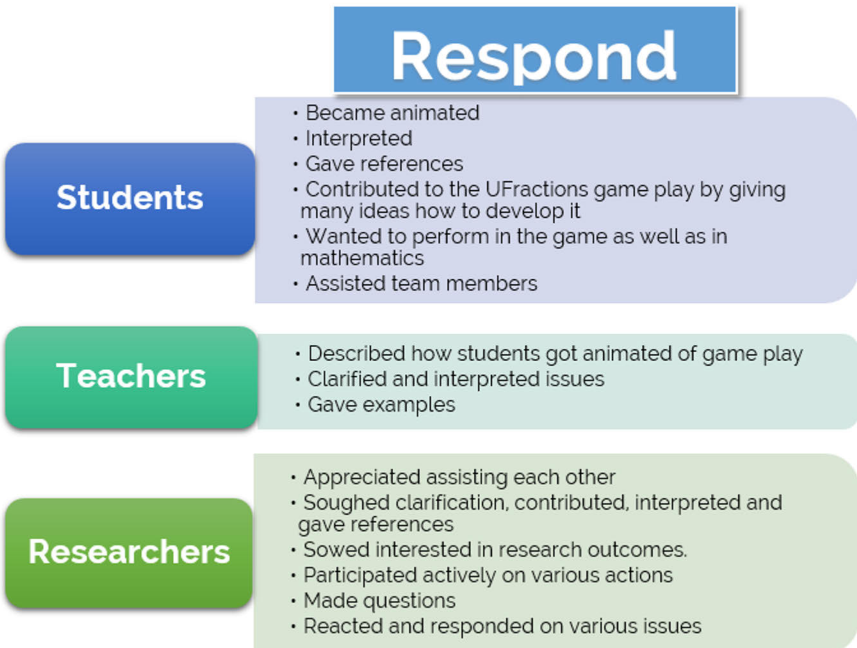

Figure 25: Affective learning at respond stage.

Moreover, they found challenges, debated, and criticized. Researchers shared their perspectives by justifying, arguing, and debating. Furthermore, they found challenges in the use of ICT as well as conducting research. Different dimensions of affective learning at the value stage are summarized in Figure 26.

Related to the organizing stage, students created knowledge and established relationships by making comparisons. Students participated in the game development and also had a broader perspective of technology use in schools. Teachers organized or conceptualized values by making comparisons and developing ideas related to gaming and the use of ICT in the schools. Researchers compared and contrasted things. Their comments demonstrate clearly how the organization stage is about adapting new information to an existing schema. They determined how the acquired information makes sense by theorizing issues. Figure 27 gives an overview of effective learning at the organizing stage.

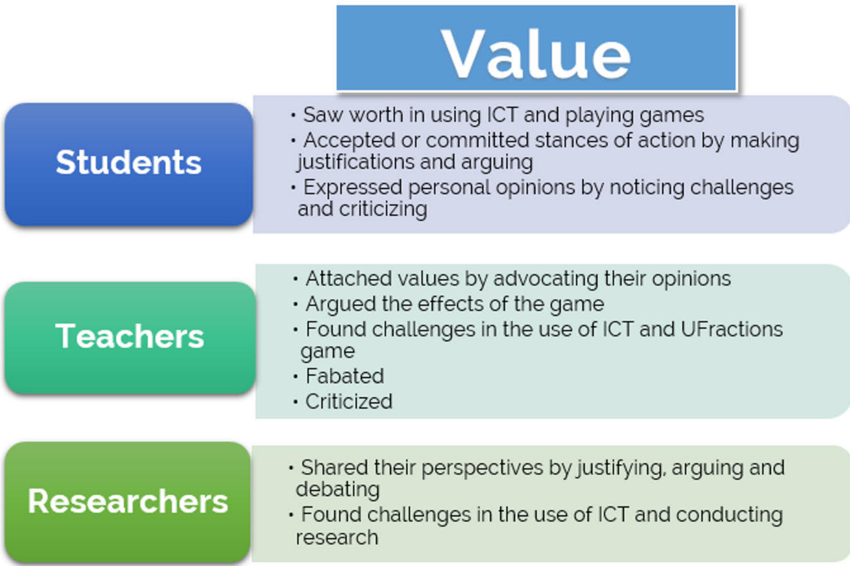

Figure 26: Affective learning at value stage.

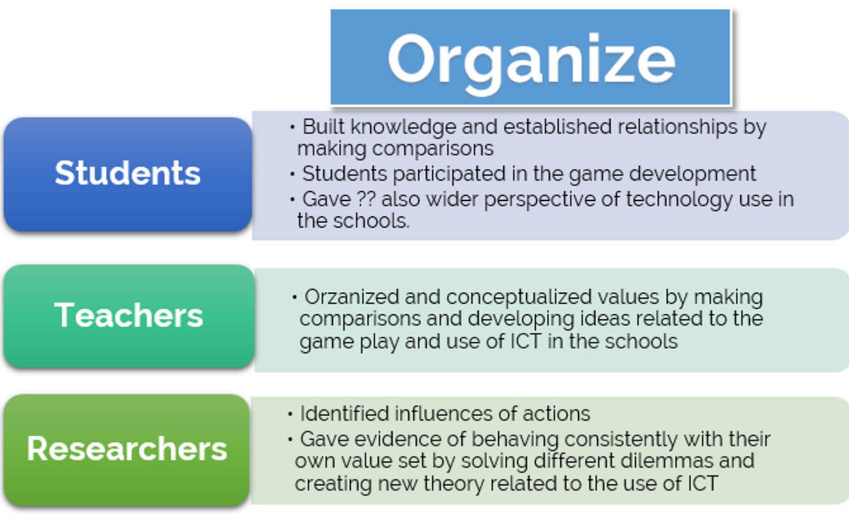

Figure 27: Affective learning at organize stage.

Considering the most complicated stage, internalization, students’ new knowledge integrated into their schema by recognizing the influences of learning mathematics and playing the UFractions game. Students exhibited new behavior, attitude, or belief. Additionally, they practiced and solved issues. Teachers perceived the influence of technology and gave self-reliantly comments on the possible use of the game and further development.

Regarding the internalizing stage, researchers' comments showed that the adoption of a belief system and philosophy are related to identifying influences of actions. Researchers also give evidence of behaving consistently with their own value sets by solving different dilemmas and creating new theories related to ICT use. Affirmation related to affective learning at the internalizing stage is shown in Figure 28.

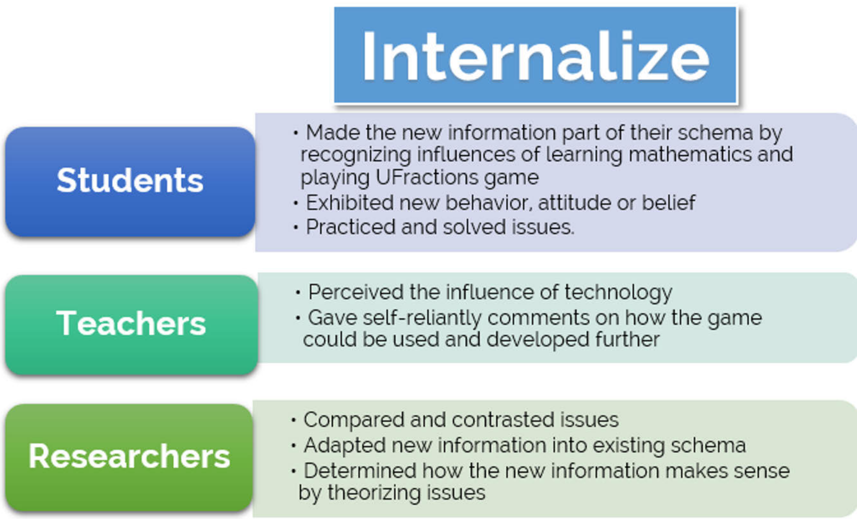

Figure 28: Affective learning at internalize stage.

To determine the significance of the relationships among the affective learning experiences of the three groups of participants in the INBECOM project, we performed various statistical procedures. The distribution of the score variables was established and presented as a Box-and-Whisker plot (Figure 29). The horizontal scales indicated the value of the scores, and the vertical lines marked the highest and lowest values in the data (Cramer & Howitt, 2004). It also graphically illustrated the distribution of the qualitatively observed effective levels, confirming the hierarchical nature of the levels (Krathwohl et al., 1964), and substantiating that engagement originated from the less complicated practical level. This Box-and-Whisker plot substantiates the values of the quantitized data indicated in **Table 12**.

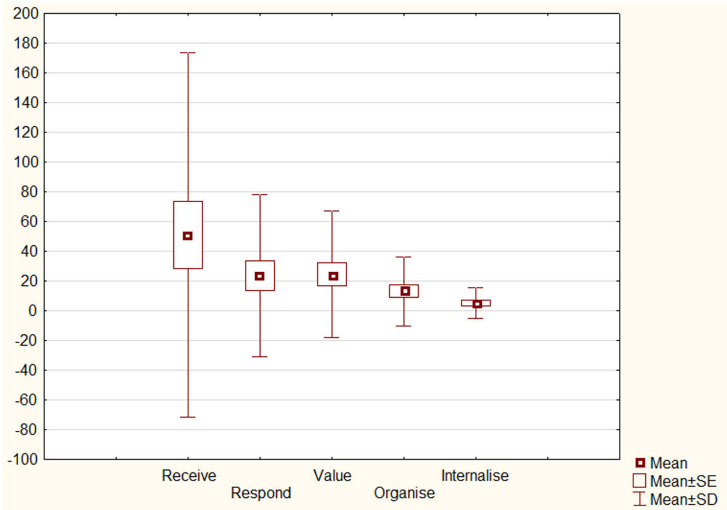

Figure 29: Box-and-Whisker plot of the quantitized data.

The affective levels organize and internalize statistically less significant in the whole group than receive, as can be seen from the means and standard deviation in **Table 13**.

Table 13: Descriptive statistics of the five effective levels as represented in the Box-and-Whisker plot.

Variable	Average rank	Sum of ranks	Mean	Standard deviation
Receive	4.017	116.500	51.068	122.494
Respond	3.069	89.000	23.620	54.347
Value	3.810	110.500	24.655	42.401
Organize	2.637	76.500	13.069	23.162
Internalize	1.466	42.500	5.241	10.453

In order to determine the statistical relationships among the five effective levels, the researchers performed an ANOVA, Mauchly's test of sphericity with a p=0.000 indicated significant differences between the different effective levels. The Greenhouse-Geisser values (f=105.434, p=0.000) indicated that there were significant differences between at least two of the affective levels (Cramer & Howitt, 2004) (Table 14). Receive showed practically significant relationships with respond (d=0.54), value (d=0.53), organise (d=0.76), and internalise (d=0.92).

Table 14: Effect sizes of the relationship among the five effective levels.

	Respond	Value	Organize	Internalize
Receive	0.54 [a]	0.53 [a]	0.76 [a]	0.92 [a]
Respond	—	—	—	—
Value	0.01	—	0.22	0.37
Organize	—	0.23	—	0.61
Internalize	—	0.39 [b]	—	—

[a] medium to a large effect (practical significance)
[b] Medium effect (tends toward practical significance)

Participants from the three groups were all immersed in learning aspects in the project, but not in the same manner and at different levels. *There was a significant relationship between receive and all the other four effective levels.* Before participants can respond, value, organize, or internalize, they must be open to various learning experiences, focus on the learning at hand, discuss the learning with their fellow research participants. Then to retain the learning content, they have to identify with the concepts of the learning and participate in the learning activities. Figure 29 illustrates the hierarchical nature of the taxonomy of affective learning experiences (Krathwohl et al., 1964) of the project participants.

To determine the statistically significant relationships among the three groups of participants (students, teachers, and researchers) during the evaluation of the effective levels, the researchers performed comparisons among the means of the five effectiveness levels. All the relationships were statistically significant (p=0.000) (Table 15).

Table 16 lists the effect sizes of the relationships among the three groups for the five affective levels which indicate a practically significant relationship for receive between teachers and students (d=0.43); for a value between teachers and students (d=0.50); and for a value between researchers and teachers (d=0.52). Paper VI gives a more detailed description of the statistical analysis.

Table 15: Descriptive statistics of the relationships among the three groups of participants.

	Receive	Respond	Value	Organize	Internalize
Students	0.482	0.199	0.195	0.057	0.035
Teachers	0.270	0.152	0.396	0.170	0.019
Researchers	0.342	0.231	0.184	0.164	0.081
f value	930.196	390.540	683.526	337.001	80.160
p-value	0.000	0.000	0.000	0.000	0.000

Table 16: Effect sizes of relationships among the three groups for the five effective levels.

	Teachers	Researchers
Receive		
Students	0.43 [a]	0.29
Teachers	–	0.15
Researchers	–	–
Respond		
Students	0.12	0.08
Teachers	–	0.20
Researchers	–	–
Value		
Students	0.50 [a]	0.03
Teachers	–	0.52 [a]
Researchers	–	–
Organize		
Students	0.26	0.24
Teachers	–	0.02
Researchers	–	–
Internalize		
Students	0.08	0.23
Teachers	–	0.31
Researchers	–	–

[a] medium to large effect (practical significance)

The analyses indicated that the three groups of project participants were all subjected to effective learning experiences. The discussion of the results, therefore, related to the three groups of project participants.

Derived from the results in Table 15, the students participating in the project presented the highest mean (0.482) for receive which implies that they were open to playing the *UFractions* game and ActiveMath, communicated with the other participants, engaged with the learning content, and explored while playing the UFractions game and ActiveMath. Respond (0.199), and value (0.195) offered similar results, which suggest that the students were involved with the activity, responded through their reflections in the How-Am-I-Feeling-blog and learning diary, and committed to finishing the UFractions game and ActiveMath tests. Organize (0.057) and internalize (0.035) had the lowest results. These results indicate that they did not make comparisons, execute modifications, solve complex problems, or adopt a specific view of mathematics. However, the overall results indicate that the students experienced learning-centered emotions while playing the UFractions game. This is an indication that the researchers intelligently integrated the technology with the design of the UFractions game and the ActiveMath tests.

Table 15 indicates that teachers, in comparison with students, achieved a higher mean (0.396) for value and less for receive (0.270). Even though the teachers participating in the project were open to experiences, attentive to the environment, and prepared to listen to others, they primarily adopted the role of an educator who reflects, challenges, criticizes, persuades, commits to specific goals, and expresses opinions. The teachers responded (0.152), and they organized (0.170) by adjusting the subject content to the appropriate level. However, the teachers who participated in the project seldom internalized (0.019) while they solved complex problems, nor adopted a specific worldview, nor influenced others to adopt their philosophy.

The researchers achieved the highest mean for receive (0.342) (Table 15). The learning diary, blog, experimental notes, and interviews with the students and teachers during the project created an opportunity for the researchers to be aware of and attentive to the environment. The researchers listened to the opinions of the participants in order to adjust the subject content of the *UFractions* game and the ActiveMath tests for students to engage in effective learning emotions. The researchers responded (0.231) by interpreting the suggestions of the teachers and students as well as including the written reflections. Even though the researchers organized (0.164) to a lesser extent, the first researcher modified the subject content of the UFractions game based on her interaction with the other participants in the project. Value (0.081) obtained the lowest results, which indicates that the first researcher did not intend to adopt a worldview or philosophy.

The teachers and researchers observed the students' interactions. The students listened, asked for advice, posed questions to the teachers and researchers, focused on their learning, interacted with the mobile game, discussed their learning with their peers, and shared their feelings with the researchers and on the blog. The students were younger than the teachers and researchers, and therefore their value systems were less mature than those of the teachers and researchers. However, through learning behavior, commitment to play and complete the game, and responses to the researchers in the blogs, they contributed towards the value of effective learning. The students' role was crucial during the introduction and the adaption of the new mathematics concepts. Fractions are difficult concepts frequently taught with stereotypical representations. Through the compilation of the UFractions mobile game, the designers triggered students' mathematical thinking during play of the fantasy stories game (Kafai, 1995). The INBECOM project intervention enabled students to learn about fractions playfully. The teachers, as well as researchers, were more familiar with the manipulation of fractions due to extensive experience, as well as their diverse roles as teachers and researchers in the project. The teachers participated mostly in mobile game development and experiments, and to a lesser extent, in evaluation and theory generation.

The participants from the three groups who contributed to the experiments and the teaching and learning sessions were open to new experiences, willing to listen and to experience emotions. They became aware of what the project offered, but rarely contributed ideas and posed questions that related to the new model for technology-enhanced learning intervention in schools. The participants' affective learning inclined more towards the level of receive than the level of value (d=0.53, medium effect), and more towards the level of receive than the level of organizing (d=0.76, medium effect) (Table 14). There were also practically significant differences between the levels of receive and internalize (d=0.92). This confirms that the participants' affective learning occurred mostly on the level of receiving. From Table 12, it becomes clear that the participants received more than they responded, valued, organized, or internalized. For the probability level of p=0.00, we conclude that there were significant differences between the variances of the differences (Table 12). This result is in line with what we expected since learning in all domains firstly happens at the least complicated process level. Without interaction on this level, no learning can occur. Interestingly, there were no similar significant effects between other levels of affective learning.

6 Generating the Pedagogical Model INBECOM

This section introduces the different knowledge types produced by Design Science Research (DSR) during the generation of the pedagogical model INBECOM (Integrating Behaviorism and Constructivism in Mathematics). To provide more explicit information on this project's development activities and to explicate research contribution to the knowledge base, the three cycles of different actions (the relevance cycle, design cycle, and rigor cycle) are described, followed by an overview of a few specific modes of design theorizing[22].

6.1 Different knowledge types of DSR projects

According to Drechsler and Hevner (2018), DSR projects produce both project design knowledge and contributions to the propositional and prescriptive human knowledge bases (Figure 30). Project design knowledge is project-specific, probably untested, conjectural, and temporary. Human knowledge bases are divided into Ω—descriptive knowledge and λ—prescriptive knowledge. Furthermore, prescriptive knowledge base includes solution design knowledge as distinct from solution design entities.

Chapter 5 included results published in scientific publications as part of both descriptive knowledge and prescriptive knowledge. This chapter elaborates on the development of the INBECOM pedagogical model, the underlying idea of this study. Drechsler and Hevner (Drechsler & Hevner, 2018) include i-processes, meaning implementation/intervention/instantiation processes, as a separate solution design entity apart from meta-artifacts[23]. Hence, it is valuable to describe more than just the finished result, the refined model.

[22] The term *design theorizing* refers to all the activities when utilizing or contributing knowledge in the context of design.

[23] *Meta-artifacts* are artifacts that lead to the development of other art

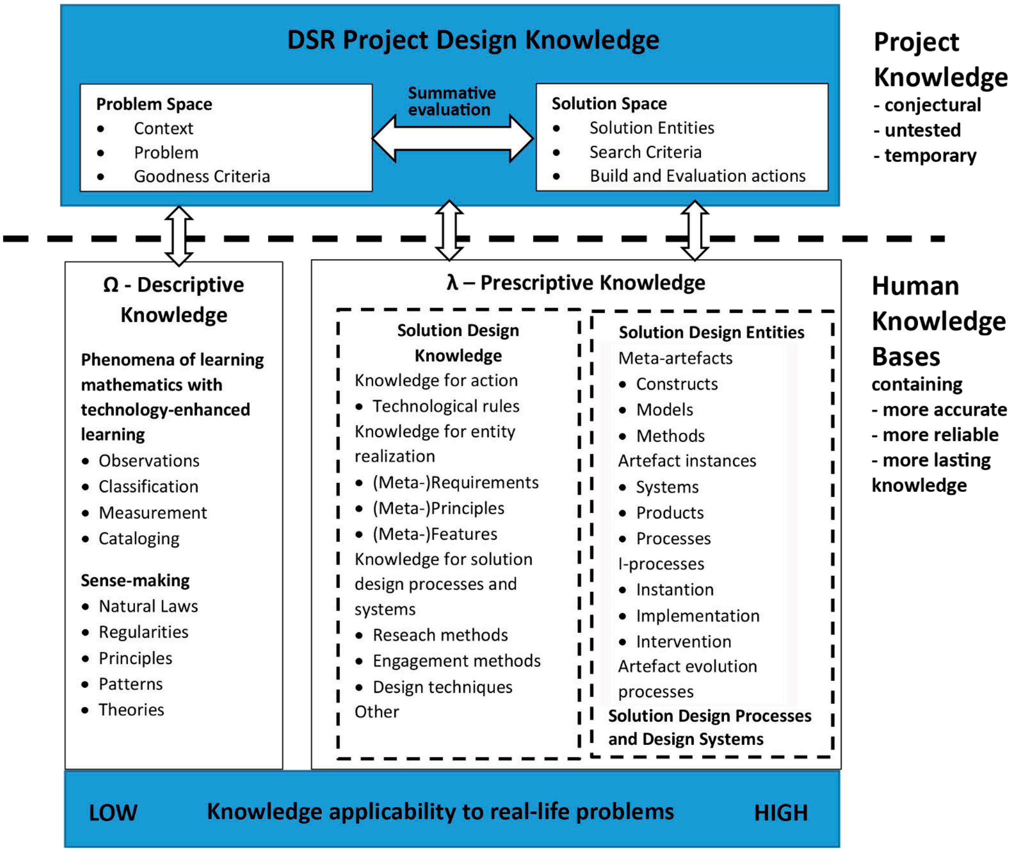

Figure 30: Different knowledge types of this project (Drechsler & Hevner, 2018).

6.2 Evolution of the INBECOM pedagogical model

The evolution of the INBECOM model (Integrating Behaviorism and Constructivism in Mathematics) and the factors influencing its development will be explained in the next Subsection.

6.2.1 INBECOM 1.0

I presented the first version of INBECOM model to the ActiveMath research group of Saarland University, Germany, DFKI GmbH (German Research Centre for Artificial Intelligence) in 2008. The INBECOM 1.0 model is described next.

The students participating in the math lesson would be divided into three groups. If a lesson according to the INBECOM model would last 45 minutes, then each group would work for 15 minutes on the following activities (Figure 31):

1) The ActiveMath intelligent tutoring system teaching students concepts of fractions.
2) A drill-and-practice game offers entirely mechanical practicing activities concerning the concept to be learned. This game is behavioristic.
3) A problem-based mobile game using concrete mathematical tools gives students more complex tasks to be solved. A constructivist game.

After 15 minutes of work, the groups changed activities enabling each student to attend every activity. The assumptions were that these short periods of work would guarantee concentration of the students, and using two games would increase their motivation.

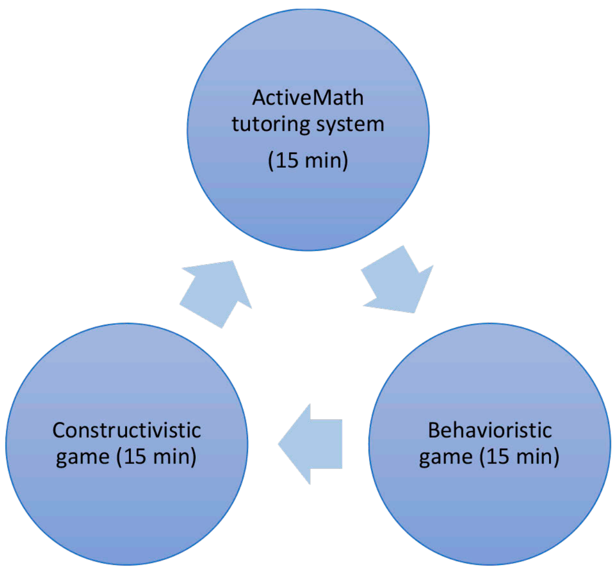

Figure 31: INBECOM Model 1.0,

The German researchers were willing to start collaboration and translation of the fraction content in the ActiveMath tutoring system began. ActiveMath research setting is described in detail in Subsection 4.3. The next step in the research was to determine a behaviorist and constructivist game to be used testing the INBECOM model.

6.2.2 INBECOM 2.0

In 2009, there existed many behavioristic games, for example Cheese Factory-drill-and-practice-game, where a learner had to find a suitable piece of cheese to make a whole cheese, fractions indicating cheese pieces. However, these were simple games, and the maximum playing time of one game would be about 10 minutes. Furthermore, a single game typically covered only a part of the concept of the fraction.

Also, it became apparent that the fraction content of the ActiveMath tutoring system included many drill-and-practice exercises, an example shown in Figure 32. Furthermore, the erroneous examples approach of the ActiveMath group was impressive. Erroneous examples are solutions comprising errors that students have to detect and or correct. Traditionally, student errors are seen as a sign of inefficiency of a sequence of instruction, a tool to diagnose learning difficulties, and for remedial exercises. However, Borasi (1994) suggested that errors can be used to (i) enhance students' understanding of mathematical content; (ii) encourage critical thinking about mathematical concepts, (iii) facilitate problem-solving activities; (iv) motivate reflection and inquiry about the nature of mathematics and (v) engage students in higher-order thinking, which enable them to understand, analyze and control their cognitive processes.

Moreover, the best way to address common errors in mathematics is to introduce them during teaching and learning so that students can explore the associated definitions, theorems, and concepts. This idea corresponds to the experimental problem-solving approach proposed by Polya (2004). Research in pedagogical investigations indicates that error culture in schools improves conditions for learning and stimulates the performance of students (Melis, 2004). With error detection and correction, a deeper conceptual understanding and procedural precision can develop as students tend to self-explain, which in turn enables them to learn (Adams et al., 2014; Melis, 2004). In addition, using conflictive animations in teaching programming has indicated that students improve their metacognitive skills and knowledge when compared to using standard animations for debugging (Moreno, Sutinen, & Joy, 2014).

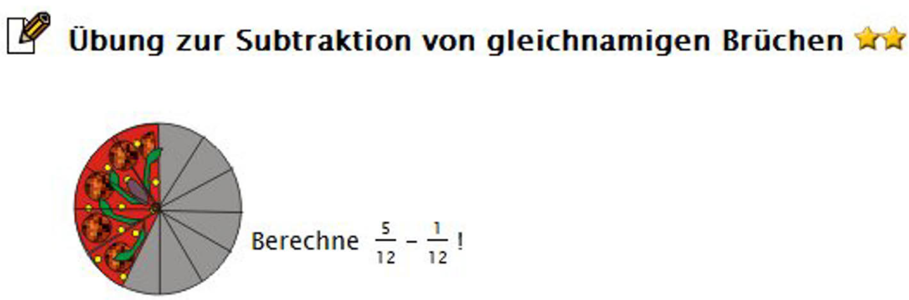

Figure 32: Drill-and-practice exercise example from the ActiveMath tutoring system.

Therefore, the INBECOM 1.0 model was changed in two respects: (i) instead of a behavioristic game there would be drill-and-practice exercises in ActiveMath, enhanced by a particular game-like setting offering mechanical practicing activities for the concept of the fraction, and (ii) instead of learning a concept only by theory and visualizations, there would also be correct and erroneous examples.

A new description of the INBECOM model was developed (Figure 33). The students in an INBECOM 2.0 lesson would be divided into three groups. Every group would work for 15 minutes on the following activities:

1) Students use the tutoring system ActiveMath. Students interact with both correct and erroneous examples.
2) Students do drill-and-practice exercises in ActiveMath enhanced by a particular game-like setting that offers mechanical practicing activities for the concept to be learned.
3) Students play a problem-based mobile game that uses concrete mathematical tools. The game is constructivist and called UFractions.

According to the plan, after fifteen minutes of work, the groups would change their activities with every student attending each work point within 45 minutes, the typical duration of one class.

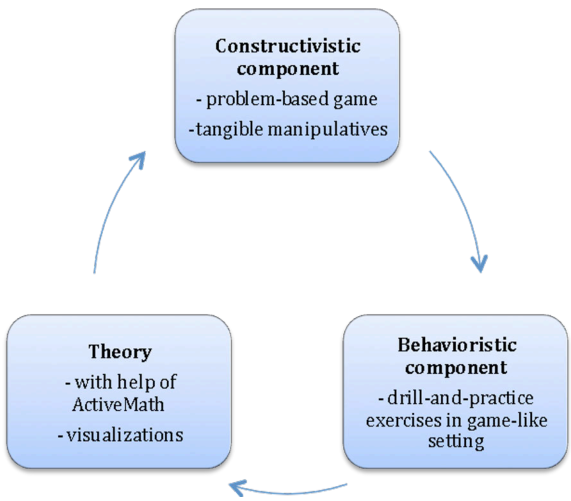

Figure 33: INBECOM Model 2.0.

6.2.3 INBECOM 3.0

The INBECOM 2.0 model would have been suitable for evaluation for one or two lessons. However, the aim was to teach the whole fractions course using the

INBECOM model, and the ActiveMath system contained enough content for a whole course. Together with the ActiveMath research group, it was decided to design and instruct a 20-hour course for 10th-grade students based on the INBECOM model in the Folk High School of Kitee, Finland. Therefore, the INBECOM 2.0 model had to be developed further. There were also other reasons for the improvement, which will be discussed next.

The Ufractions game was meant to be constructivist. However, during the experiments with the students, it became evident that although its tasks had been designed according to constructivist principles, they began to look similar, and were almost drill-and-practice when repeated. It included many closed questions and tasks that did not require creativity. A decision was made to make the game part of the course more constructivistic: students would be allowed to plan their games, in other words, to make their own stories and exercises to the game. To be able to do that, they would be playing the UFractions game for about 1.5 hours at the beginning of the course.

Also, it became apparent that because the work points would be in different classrooms and signing into the different systems would take time, the time per work point had to be extended from 15 minutes to 30 minutes. The course structure was hence changed to 90 minutes per class.

The new description of the INBECOM model is shown in Figure 34. The students in an INBECOM 3.0 lesson would be divided into three groups. Every group would work for 30 minutes with the following activities:

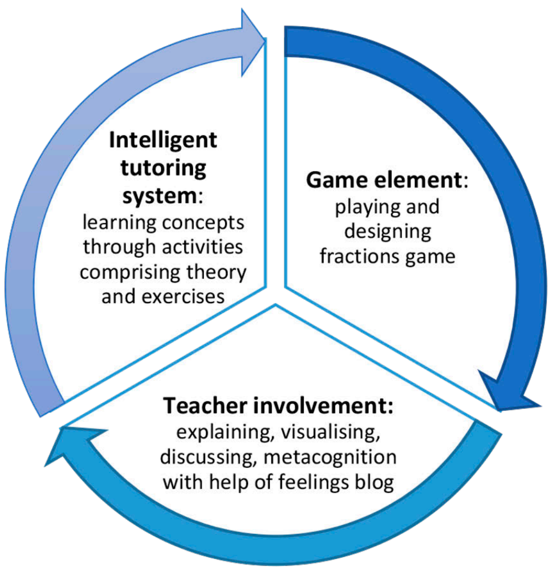

Figure 34: INBECOM Model 3.0.

The course sessions are divided into three elements:

1) Game element: Students play a problem-based mobile game *UFractions* based on fractions as a mathematical concept. Students design their own mobile game stories and fraction problems.
2) Teacher involvement: The teacher interacts with students by explaining, visualizing, and discussing. Metacognition[24] With the help of Feelings Blog.
3) Intelligent tutoring system: Students learn concepts of fractions through the tutoring system (ActiveMath). Activities comprise theory and practical exercises.

In 2010, I organized and taught the fractions course using the INBECOM 3.0 model. The research setting of the course is discussed in Section 4.4. in more detail. The sample demographics differed substantially from the plan made beforehand. The group of 10th graders consisted of mainly immigrants, for example, from Africa, and the few Finnish students had considerable difficulties with maths learning. The course plan was made for remarkably different students, namely native Finns aged 16-18. Because of this, some of the course content had to be facilitated and some of the more difficult elements had to be removed..

Also, there were problems with using ActiveMath. Translating the fraction content and putting the content into the system succeeded, but the automatic correction of the exercises did not work correctly. Moreover, the structure and focus of the ActiveMath research group changed, and hence, they could not offer extensive technical support. Thus, parts of the ActiveMath content using a unique advanced technique of showing sequences of erroneous examples had to be removed. Therefore, some lessons included correcting of erroneous examples during the ActiveMath work point.

Observations were written in the learning diary during the whole course. The students embraced the *game element* of the INBECOM model. However, there was a need for improvement. 21 students were divided into seven groups. Even though students liked to play the UFractions, all students liked to create their own stories to the game. Word processing software and ordinary drawing tools were used to write the story. These tools were quite slow and there was a need to use something to make storyline writing easier. Three groups out of seven were excited and able to create the story by themselves. Others needed much support. One timid boy came to explain his story broadly after one lesson. He said that he would like to make an outstanding story for a mobile phone game. It was great to see him so excited. Designing the exercises possible to be solved with math rods was even more demanding than designing the story. However, the students realizing how to do exercises also learned

[24] Facilitate students to think about their thinking and learning.

very well the concept of fractions. Game subjects were, for example, traveling, cats, criminals, Hitler, as well as their own experiences.

Teacher involvement was significant in supporting the whole learning process. The teacher could concentrate on teaching sessions to some extent because students needed help with computers or how to write a storyline to the game. Hence, using quite ordinary paper copies of theory and assignments was necessary. When one group of students was filling in the written material, the others were assisted in different classrooms.

The students wrote about their experiences in the "How am I feeling today? Blog" (Figure 35). To make them reflect on their learning, they were given a few open questions concerning the tasks completed during that day. The blog included a category of seven main feelings (anger, fear, joy, love, neutral, sadness, surprise) to choose from while they were posting their comments. The most common feelings the students chose were "joy" and "neutral." Signing in the web blog was difficult; remembering the passwords was amazingly difficult for immigrants. The immigrants liked to write the blog, but understanding their text was not easy for the teacher, and the text was concise.

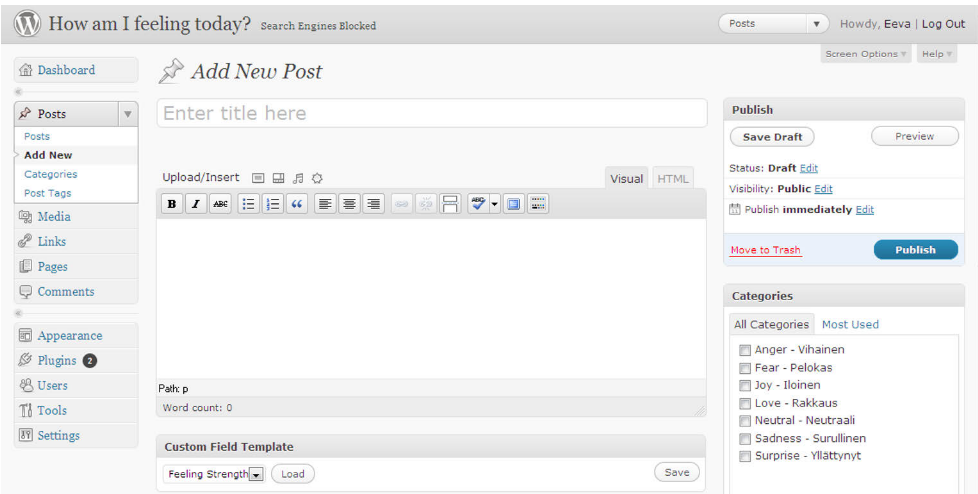

Figure 35: Interface of How am I feeling today? blog.

The ActiveMath tutoring system was otherwise easy to use, but some of the students did not read and concentrate properly on the tasks in the system. They were only guessing the answers and clicking the mouse to proceed further, the reason being the difficulty of the tasks or lack of interest. On the other hand, some students concentrated intensely on the tasks, and their progress was slow. Therefore, progress levels varied significantly. Some of the older students wanted to use pen and paper instead of the computer interface because of inadequate computer skills. The tutoring system also had a different password than the blog, and this caused problems in login.

As a whole, the course went well, and the feedback of the course was positive. Especially the game element was mentioned in the feedback of several students as the best part of the course. The typical comments from students' blog posts related to the different parts of the INBECOM model are shown in Table 17.

It can be concluded that the INBECOM model motivated the students considerably. However, the use of the model was demanding for learners with special needs and immigrants with low language skills. The ActiveMath software proved to be too complicated for them, and creating their own story and fraction problems to the game needed teacher support more than anticipated.

Table 17: Students' comments related to the fraction course.

Game Element	Teacher Involvement	Intelligent Tutoring System
“The most effective learning method was the mobile phone game.” *"There was no worst part of the course, but I found it difficult to make my own story to my own game!"* *“I learned well by playing the game.”* *"It was nice to do the game, and of course, to play it!"*	*"Well, it was easy when the teacher was teaching with the blackboard and used the fraction sticks. Fraction multiplications were easy. Nothing was especially tricky, although I did not get ten for the exam, ☺ “* *“I learned by good teaching of the teacher and by listening.”* *"There could have been fewer copies."* *"The best part was the copies, and the teacher was excellent.*	*“I would like to have more time with computers.”* *"In the beginning, there were easy questions, then more difficult, and then again, easy ones. I prefer that from easier to more difficult."* *“I think that I learn more and I would like to do more exercises.”*

This chapter describes one pedagogical model combining behaviorist and constructivist approaches to learning to teach mathematics using educational technology. In addition to Cronje (2006), several researchers suggest blending of these two learning theories, such as Weegar and Pacis (2012), for online learning in an effort to best meet the learning styles for all students, and Sidney (2015), for the 21st century skills. However, only a few researchers have suggested a concrete model on how these approaches could be combined in teaching.

One example of concrete models of these kind is a multimedia-based cognitive tool for solving word problems involving fractions by Ahmad et al. (2005). Their behaviorist-cognitivist- constructivist approach includes the following constructs:

- *Multimedia-based Information Visualization*; Using animation, graphics, audio and text.
- *Exploratory* – Solving problems independently.
- *Scaffolding* – Usage of certain support tools.

- *Reflection* – Reflecting on the concepts and procedures of fraction calculation.
- *Self –paced* – Controlling one's own learning, and repetition of steps.

The above research and model is based on the fact that problem solving in mathematics needs both basic skills and higher level skills.

Another concrete example is a mathematical learning strategy for Indonesia by Weegar and Pacis (2012), using blended learning. Their model combines face-to-face learning and e-learning, having the following elements: 1) face-to-face learning, 2) independent learning, 3) application, 4) tutorials, 5) collaborations, and 6) evaluations. This model has proven successful, as students can evaluate their own skills and progress at their own pace.

The INBECOM model presented in this study complements well the existing studies. There should be more similar concrete experiments and their evaluations to enable teachers to choose the pedagogy suitable for their students.

7 Conclusions and Recommendations

The final chapter first summarizes the results, followed by a discussion of what I learned from this research. Lastly, the recommendations for both further research, policymaking, and practice are presented.

7.1 Summary

The focus of this research was to determine and examine the dimensions of technology-enhanced mathematics learning. Using a design science research approach, I created two artifacts; (i) *a novel pedagogical model INBECOM* advocating innovative behaviorist-constructivist perspective towards mathematics teaching and learning, and (ii) *a story-based context-aware mobile game UFractions* using mathematical manipulatives. Like other DSR studies, this study was done iteratively and cumulatively by developing different types of new knowledge through the design of artifacts. I conducted all research activities in interaction with an established knowledge base and a practice environment involving many different contexts in different countries—Finland, South Africa, Germany, and Mozambique. This cross-cultural and cross-countries approach was chosen to ensure wider generalisability of the research. The central part of the research was carried out during 2009-2014, and the research was conducted in five stages consisting of

1. idea and planning of an educational model,
2. development, experiments and evaluation of the UFractions mobile game,
3. content and translations, experiments and evaluation of the ActiveMath intelligent tutoring system
4. organizing fraction course using the INBECOM model and
5. theory generating of the INBECOM model.

The seven chapters of the thesis describe the progress of the research. In the Introduction, I explain the background and need for this research and the personal story of my research journey. Chapter 2 explains the problem space through the literature review. The third chapter defines the research design and the methodologies used. Chapter 4 illustrates the research progress in detail. Chapter 5 presents a summary of the results published in Papers I – VI, including partly **Ω**-detailed knowledge and partly λ–prescriptive knowledge. Chapter 6 adds the human knowledge base by describing the development of the INBECOM pedagogical model, in other words, i-process, and meta-artifacts as a separate solution design entity of this study (Drechsler & Hevner, 2018).

In terms of the research questions of the study, I summarize the essential elements of the results as follows:

Q1 What are the dimensions of learning mathematics with technology-enhanced learning?

The answer to this question is illustrated in Figure 36. The literature review in Chapter 2 indicated three main dimensions of learning mathematics with technology-enhanced learning: (i) Domains of Learning, (ii) Orientation of Learning, and (iii) Motivation of Learning (Figure 12). Next, I will describe how my research supported this view, and what kind of evidence I found to confirm and complement these dimensions.

<u>Domains of Learning</u>
Bloom's taxonomy classifies learning according to three domains: cognitive, psychomotor, and affective. Technology-enhanced learning based on different learning theories affects different levels of these domains, and this study confirmed all these three domains being part of learning. However, the evaluations of the UFractions game indicated that it would be important to take into account also the dimension regarding cooperation with others, as well as the dimension in which a person's internal learning processes are examined. I define these domains of learning *interpersonal* meaning the learning related interaction between you and other people, and *intrapersonal* denoting learning related activities occurring within the individual mind or self. These are both inspired and supported by Vygotsky's socio-constructive theory (Hausfather, 1996). Dillenbourg's (2016) viewpoint, supporting my research findings, underscores a central challenge in the future of artificial intelligence in education, involving the development of computational models that account for individual and social dimensions of human learning. Despite individual cognitive processes occurring within our brains, they operate in tandem with social interactions.

Evidence related to these two new dimensions during the study were, for example, the following:

- Playing the game in the groups was effective in sense of argumentation considering mathematical problems and learning by doing together (interpersonal) (Paper I)

- Every student was able to participate and proceed at their own level (intrapersonal), aiming at common goal (interpersonal). (Paper I)
- Considering social dimension, students who enjoyed group work, also indicated that they learned better than usually (interpersonal). (Paper III)
- One of the indicated Ufractions motivation for playing the game is "cognitive restlessness" that is about the desire to gain knowledge, to process information and acquire of knowledge using different methods (intrapersonal). Furthermore, the motivation “relations” refers to cooperation with other students (interpersonal). (Paper IV)

Therefore, the five domains of learning are:

1) cognitive, 2) psychomotor, 3) affective, 4) interpersonal, and 5) intrapersonal

<u>*Orientation of Learning*</u>

To understand the dimensions of mathematics learning, I discussed in Chapter 2 the three most essential learning theories of the 20th century; behaviorism, cognitivism, and constructivism. I shared the view that combining behaviorism and constructivism would lead to more motivating and meaningful teaching and learning strategies. Furthermore, the level of technology integration, the level of students’ cognitive process, and the level of teachers’ knowledge are intertwined.

In Chapter 6, the development of the INBECOM model was presented, as well as the empirical tests of fraction course organized according to the model. The results of the qualitative evaluation confirm the view that successful instructional practices have features that are supported by both constructivism and behaviorism. In next chapter, this is discussed further.

<u>*Motivation of Learning*</u>

In Chapter 2, motivation theories and their connection to mobile games and technology-enhanced learning was described. The first Ufractions game evaluations showed that the game play induced *ethical, physical, and cognitive rationales* (Paper I). A taxonomy of play motivations was originated from the deeper evaluations of UFractions mobile game; *altruism, challenge, cognitive restlessness, curiosity, fantasy, relations*, and *technology* (Paper IV).

The test results of the UFractions game were also analyzed from the point of view of looking at the interplay of motivating factors and disturbance factors. Six different zones of learning according to motivational intensity and intensity of disturbance factors were characterized (Figure 23) (Paper V). Therefore, I suggest that it is crucial to take into account motivations and DFs of the technology being used in the mathematics classes. In other words, this is one complementary perspective to the motivation dimension of learning.

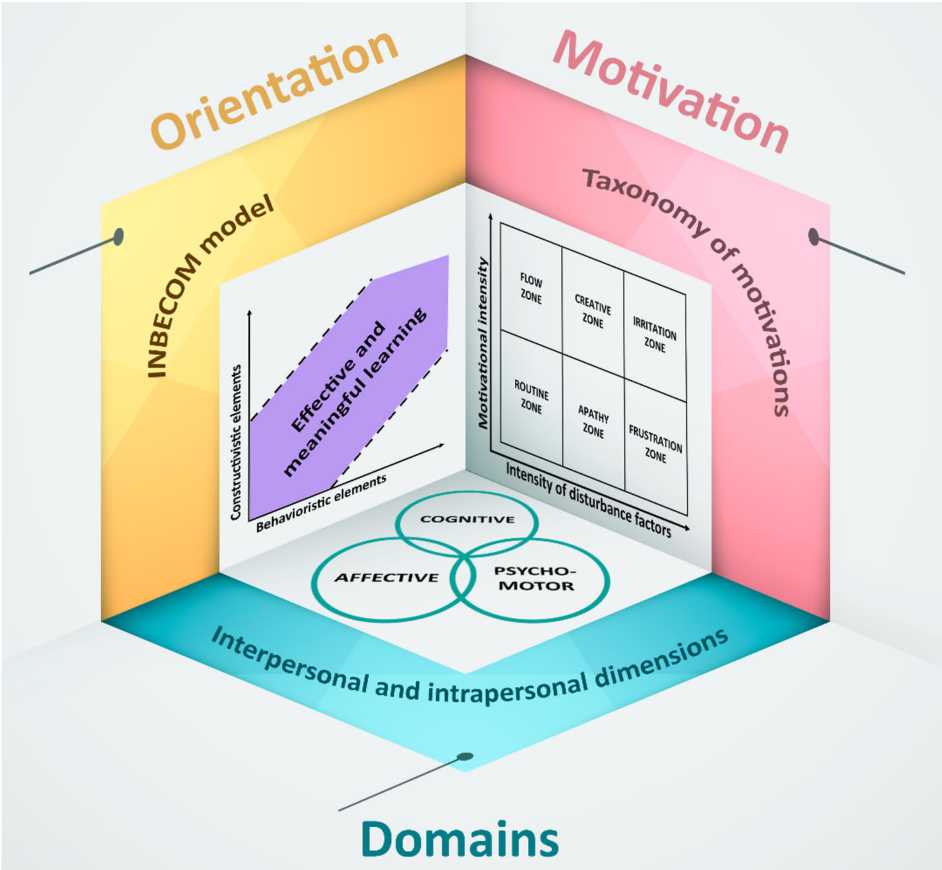

Figure 36: Dimensions of Learning Mathematics via Technology.

Q2 How can artifacts for mathematics learning be developed?

To enhance mathematics learning, I developed a mathematical mobile, story-based game UFractions in South Africa with the help of cultural and technical experts, as well as local school teachers using the co-design principle. Equally, In Chapter 2 I presented game fundamentals and the motivation for using tangible manipulatives. In Chapter 5, the results of testing the game prototype was illustrated. Empirical tests indicate that combining concrete manipulatives and mobile phones are an effective and meaningful way to learn the abstract concept of fractions in developing contexts, increasing active student participation.

The Ufractions game was used as a part of the pedagogical INBECOM model combining behaviorism and constructivism and, at the same time, supporting different dimensions of mathematics learning. In addition to the game element, the model includes teacher involvement and the use of an intelligent tutoring system. The model was developed according to design science principles by reflecting the model to existing theories and testing its functionality with students. The development process is presented in Chapter 6.

Q3 What are the considerations to be taken into account during the design of a mobile game for learning fractions in diverse contexts?

Mathematics was born out of the practical needs and problems, but over the centuries it has become abstract. Mathematics taught in schools can be very alienated from practical life. In this study, we wanted to develop mathematics teaching by comparing and analyzing empirical observations in different cultures. Empirical tests of UFractions mobile game were conducted in three countries.

One point of view of analysing the results is called reverse transfer; the game was developed in South Africa and subsequently delivered to a class of Finnish learners at the same grade level only by translating the game into Finnish. The reverse transfer process made it possible for us to investigate the contextual factors of technology for learning. We were interested in two aspects of the context: the cultural and pedagogical ones, and identified the effects of technology integration. The results indicated that the pupils in the Finnish context had significantly more problems or complaints in using the UFractions game than their South African counterparts. We interpreted the results so that reverse transfer is insufficient for a functional learning environment. Instead, a full re-contextualization process is required, covering at least the cultural and pedagogical aspects of the context of the users. Furthermore, the technology integration results suggest that the technology worked well in both contexts. However, this is only one example of the reverse transfer process, and the validity of this claim would need further research.

Q4 What strategies do we need in order to compile a mobile game for learning for diverse contexts?

South Africa has adopted mobile telephones. The question was why mobile learning is not used more in South African classrooms? According to this empirical study, teacher's attitudes towards mobile gaming are positive and they would like to use more mobile games in their teaching and learning. However, mobile gaming is not yet practical in South African schools because of the following five major challenges: *time, big classroom sizes, lack of resources, lack of compatible games,* and *language*. These challenges will most probably diminish over time—for example, technology becomes cheaper. These challenges also apply only to South Africa, and may be at least partly different in other countries.

Therefore, to answer the question what strategies do we need to compile a mobile game for learning for diverse contexts we analysed also the data from empirical tests in Finland and Mozambique from the viewpoint of disturbance factors. We derived practical guidelines compiling a mobile game for game users and educators from the interplay of disturbance factors and motivations (Table 8).

Q5 How can artifacts, such as games for learning, address the different dimensions of learning mathematics?

Based on the evaluation of the Ufractions game, story-based mobile gaming brings many dimensions to learning. Students identified themselves with the story of Mother leopard and her cub Senatla. The story also induced ethical, physical, and cognitive rationales. Participants solved actively real-life fraction problems using mathematical rods and gave compelling, functional, and action-oriented arguments for liking the mathematics in the game (Paper I).

A taxonomy for a variety of play motivations in the UFractions game was developed: altruism, challenge, curiosity, fantasy, relations, cognitive restlessness, and technology (Paper IV). Added to that, we illustrated how the various features of the game affect or create these motivators.

Games for learning are also part of the INBECOM model presented in Chapter 6 and developed during this research. Positive results of the evaluation of the model suggest combining behaviourism and constructivism leading to effective and meaningful learning (Figure 36).

Q6 What is the relationship between motivation and disturbances during learning mathematics with games for learning?

In Chapter 5, I analyzed the dynamics between game motivations and disturbance factors (DFs) in the UFractions mobile game (Paper V). Each motivation relates to a set of DFs (Table 11), which typically have adverse effects on the player's motivation. By becoming aware of these relations, we can design more motivating educational games and give guidelines for game developers, users, and educators. Different motivations and DFs can be taken into account in developing, setting up, and using a mobile educational game.

We defined six learning zones by estimating the intensity of DFs and motivational intensity and placing them onto a 2x3 matrix (Figure 23). From these zones, we discovered that the medium intensity of DFs nourishes the learning process. Thus, DF is not purely an undesirable element of a learning system, having a negative effect on the learner. However, it can also be an element that challenges the learner to be more creative and broaden their perspective.

Q7 How does an approach of creating interventions and artifacts based on the taxonomy for dimensions of learning mathematics trigger affective learning?

In Paper V, we conducted a multi-method evaluation of the whole INBECOM project from affective learning levels of Krathwohl et al. (1964). We concluded that: (i) the research participants not only received information but actively participated in the learning process; responded to what they learned; associated value to their acquired knowledge; organized their values; elaborated on their learning; built abstract knowledge; and adopted a belief system and a personal worldview; and (ii) affirmation of affective learning at all five levels was recognized among the three groups of participants.

The results show that affective learning mostly took place at the receiving level, indicating that the participants received more than they responded, valued, organized, or internalized. There was also a significant effect of research participants about receive; students' affective learning occurred more at the receiving level than that of the teachers, and teachers' affective learning emerged more at the value level.

7.2 Discussion

This dissertation describes dimensions of learning mathematics by technology. The chosen action design research method benefited the progress of the study. ADR research makes research contributions to both the application environment and the knowledge base, providing solutions to practical problems that emerged from the problem space. Hence, the research problems became more specific after we had become familiar with the environments in which empirical research was conducted. However, I specifically wanted to do research that would provide solutions to both practical problems and broaden the abstract theory. What was essential and essential to me as a researcher was that my own learning process progressed during these years. If I had done more straightforward research and focused more deeply on a narrower problem, I would not have acquired such a holistic understanding of the use of technology as a tool for teaching mathematics in different settings. ADR offered the opportunity for flexible and open-minded research.

I examined the affective learning of the three groups, in other words, students, teachers, and researchers attending the project. The use of the Krathwohl's framework provided a novel and valuable insight into project evaluation methodology, and also contributed towards the planning and conducting of technology-enhanced learning projects. I have indicated that the affective domain of learning provided an in-depth perception of the learning and research outcomes of the INBECOM project. I did not focus on finding out the pedagogical effectiveness of the UFractions game or the INBECOM model. However, during the fraction course organized according to the INBECOM model, the teacher's and students' experiences indicate meaningful and useful learning of fractions. Hence, successful instructional practices have features that are supported by both constructivism and behaviorism (Figure 37). Discussions with Johannes Cronje, in May 2018, confirm these findings.

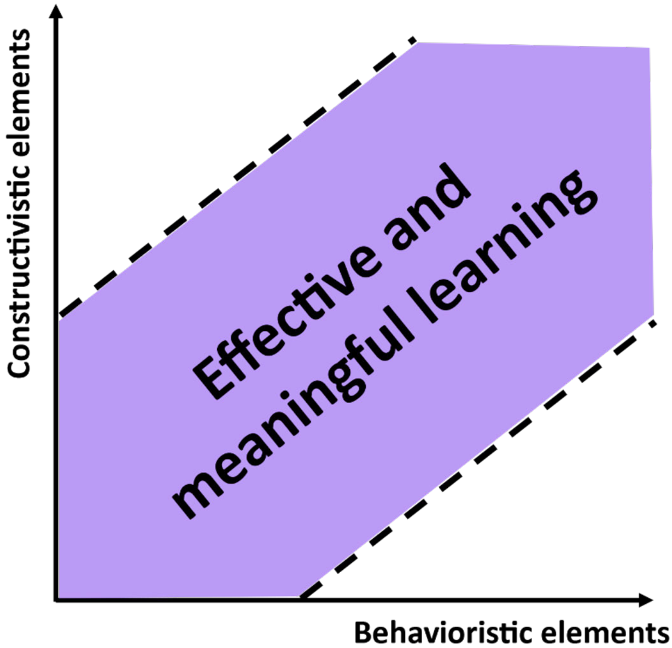

Figure 37: Choosing activities for effective and meaningful learning.

A taxonomy of play motivations encountered in the UFractions mobile game, *altruism, challenge, cognitive restlessness, curiosity, fantasy, relations*, and *technology,* has similarities with existing taxonomies, such as those of Malone and Lepper's: *challenge, fantasy*, *curiosity* and *control* (Malone & Lepper, 1987). I did comparison of motivation taxonomies in subsection 2.4.3 and Table 1. The differences between the taxonomies are probably based on differing contexts; the studied games are different, and not necessarily even learning games. In addition, the target groups of the studies and their sizes vary. Malone and Lepper studied educational games generally. Contrary to them, we studied a story-based mobile game with math rods. The motivation *altruism* derives from the mission of the game – to help the leopards.

Moreover, the use of mobile phones and rods generates motivation *technology*. Cognitive restlessness and challenges are similar to some extent, but we decided to keep them separate because many respondents emphasized the importance of the learning process. Thus, the motivation challenge is more related to reaching and accomplishing goals, and cognitive restlessness is about enjoying the way of learning.

To date, there is no clear consensus on the factors for playing digital games. Indeed, the answer depends on whom we are asking and the game in question; there are different types of games and players. Understanding player motivation facilitates computer game design. Especially in the field of serious games, the understanding of players' demands helps game designers to direct players to learn effectively. There is a demand for future studies to establish a holistic view of player motivation in educational games.

Considering the disturbance factors, we could separate negative and positive disturbance factors (DFs) for a more extensive analysis. Negative DFs are frustrating elements crucially affecting the functioning of the game, such as problems with the network or the bugs that excessively interfere with gaming. Positive DFs are factors that do not have destructive effects on gameplay. Negative DFs might cause more frustration than creativity. By contrast, positive DFs are constructive in the sense that they lead students to the creative zone.

A similar idea has been applied in the field of leadership and management by Edmondson (2008) . The competitive imperative of learning suggests shifting from "execution-as-efficiency" to "execution-as-learning," which means that instead of relentlessly and efficiently executing the consistent production and delivery of goods or services, the companies should create a psychologically safe environment where mistakes are approved. Workers are encouraged to solve problems by using the best available knowledge collaboratively. The workers are learning while they create effective processes. In game-based learning, from learning, this means that ready-made polished learning games are not just "executed," but instead, the game is allowed to be unpolished, i.e., include DFs. In addition to learning game content, the students are solving problems related to the game design, thus learning problem-solving skills, design principles, and collaborative learning.

The importance of the affective domain in education is widely acknowledged, and there are many theoretical and empirical variables to define and measure in order to scale effective learning (McCoach, Gable, & Madura, 2013). Instructional-design theories have addressed the affective domain (Martin & Reigeluth, 2013). Despite this, comparing the results of this study with current research was demanding since few studies refer to affective learning in technology-enhanced learning. Rovai, Wighting, Baker, and Grooms (2008) developed and validated a self-report instrument, named *CAP Perceived Learning Scale* for measuring learning in the cognitive, affective, and psychomotor domains in the use of technology at the higher education level. However, the CAP Perceived Learning Scale does not measure learning at all five levels of Krathwohl's effective criteria but considers nine questions relating to students' attitudes. Rowell (2015) measured the affective learning dimension of Open Educational Resources (OER) using three items included in the OER perceptions survey. According to this study, students have a greater sense of self-reliance as a result of their enrolment in a course that employed OER compared to a course that did not. Relating to the effective learning of mathematics, Lim and Chapman (2015) identify priority affective variables within and across the three affective variables of (i) motivation, (ii) attitudes, and (ii) anxiety—the key sub-constructs that educators should focus on while engaging with technology-enhanced learning strategies. Kiili and Ketamo (2017) investigated the use of a digital game-based mathematics test concerning effective learning. They noted that game-based assessment could lessen test anxiety as well as increase school satisfaction.

7.3 Limitations

Limitations of the study relate to the following aspects. We involved only two technological tools in the research, the *UFractions* mobile game, and the *ActiveMath* intelligent tutoring system. Other games and tutoring systems were not considered. For example, there may still be other disturbance factors and motivations to be identified and guidelines for designing and compiling mobile games aroused from the qualitative research findings on the UFractions game. Moreover, an analysis of affective learning could also have produced different results, at least in some respects, if other technological solutions had been used.

Although the study took place over a multitude of years from 2007-2014, it made use of a snapshot evaluation; a longitudinal analysis could have provided more in-depth insight. Development and evaluation of the UFractions game and ActiveMath content were resource-intensive so far that the INBECOM model was only tested once in one mathematics course. The functionality of the pedagogical model was significantly influenced by the participating students' background and proficiency in mathematics including the low level of their ICT skills.

Regarding the reverse transfer process where the game was created for a South African context and then reverse-transferred to Finnish context, the student feedback collected during the game testing was quite different between countries and both the cultural and pedagogical context adaptation of the game was higher in South Africa. This result could be partly influenced by the novelty effect; at that time in South African schools, mobile games were not used much, which were commonplace in Finland. Furthermore, the students' different language skills and the "No opinion" option often chosen by Finnish students contributed to the result.

7.4 Recommendations

Based on this research, I would suggest the following recommendations for teaching practice and pedagogical strategies.

1. Use cognitive, psychomotor and affective domains of learning taking into account interpersonal and intrapersonal aspects of learning

Mathematics can be learned cognitively by thinking and understanding, effectively by liking and even loving, and psychometrically by acting. These three can be used as a measurement tool to address a learning outcome, but they should also be used as a basis for designing educational goals and practical activities. Lower levels of learning are necessary to be able to learn more complex and profound skills. Therefore, *use every stage of the learning domains***.** While planning technology-enhanced learning, take into consideration that distinct technologies support learning differently, considering the learning domain and the stage of learning. Take into account *interpersonal and intrapersonal aspects of learning;* individual and social perspectives on learning activities are both important in mathematics education.

2. Choose activities based on both constructivism and behaviorism

Successful instructional practices have features that are supported by both constructivism and behaviorism (**Figure 37**). Students' skill levels and learning objectives contribute to the choice of activities. The diversity of learners must be taken into account when choosing the methods and technologies to use. Choosing which method to use can make a big difference in whether students also learn generic skills and problem-solving.

3. Acknowledge the connections between motivations and disturbances when using technology

The connections between motivations and disturbance factors (DF) are essential to recognize because this knowledge may help to diminish the DFs and thereby increase motivation while preventing undesired side effects. Do not be afraid of disturbance factors because they might even make the learning experience deeper when the motivation is high. Considering mobile gaming, we identified three zones of learning with high motivational intensity:

1) FLOW ZONE: With the low intensity of DFs, the game is played in the flow state, enabling effective content learning.

2) CREATIVE ZONE: With the medium intensity of DFs, students not only learn well the game content but involve themselves in co-designing the game and enhance their learning.

3) IRRITATION ZONE: With high intensity of DFs, despite irritation and weak content learning, students may learn some other necessary skills. For example, to handle information and solve problems, and to use self-made or ready-made computer programs as part of studying mathematics.

7.5 Future research

The reverse transfer could be a future trend of learning technology development meaning development of the core concept and technology in a technology-alien context and then re-contextualizating them for the needs of a technology-familiar context. The advantage of this model is that designers and developers must take into account serious challenges (e.g., pedagogical, technical) that they face in the technology-alien context. By meeting these challenges, the end product is likely to be more robust than in the case of an ordinary technology transfer. To investigate more the effectiveness and the benefits of reverse transfer, we will continue developing technologies in countries such as South Africa and Tanzania, and then reverse-transferring or re-contextualizing them to technology-familiar contexts. This process, where an artifact is designed as a joint exercise within a set of selected contexts to enrich the result is also called inter-contextual design (Havukainen, Gupta, Wolhuter, Sutinen, & Laine, 2016).

The UFractions mobile game was improved after this research was conducted, but this development process was not part of the dissertation research. Further development of the game took into account experience of testing the original UFractions game in three countries. The things that worked well in the game remained, and the 8th graders critical opinions and wishes were considered, as for example more story paths and animal characters to the story, and mathematics tasks other than fractions. Technology-related challenges that came up in the evaluations were solved with newer technology. The following two main development-evaluate iterations have been conducted:

In 2016, we introduced the ScienceSpots AR concept for creating context-aware storytelling games with augmented reality (AR) (Laine, Nygren, Dirin, & Suk, 2016). ScienceSpots AR was designed to be extensible and portable to support learning across various science disciplines and contexts. To investigate its feasibility, we prototyped the ScienceSpots AR concept with a geometry learning game in Leometry and evaluated it with Korean elementary school students. The development of Leometry started with the analyzes of the UFractions game, producing the following design guidelines for Leometry: 1) storytelling, 2) pedagogical grounding, 3) appealing user interface with cartoonish and lighthearted drawing style, 4) challenges with scaffolding hints, 5) immediate feedback, 6) characters based on real-world examples, and 7) real-world connections. We reused the leopard characters from the UFractions and wrote a story with geometry challenges. In Leometry, we did not use math rods, and the AR features did not contain any pedagogical objectives as they aimed at demonstrating the platform's AR capabilities, for instance, disarming a virtual bomb and a virtual trap as a part of the game adventure.

In 2019, we conducted a multidisciplinary project involving computer scientists, artists, designers, and pedagogical experts to create a story-based AR mobile game Tales & Fractions (Kim et al., 2019). Technological advances made it possible that the new game works on tablets, wooden sticks became virtual, the story of the leopards was lengthened, other characters from the savannah became part of the story, and mathematical problems were reconsidered. The completed Tales & Fractions was evaluated at two elementary schools in Sweden with 56 students. The evaluation results showed good reception among the participants, but they also revealed several improvement points, such as technical bugs, difficulty imbalance in some tasks, and issues with user interaction. We are going to develop the game further based on the first empirical tests.

The findings of this thesis and the further development of the UFractions game provide clues for future research on the pedagogical and motivational effects of the combination of the AR manipulatives, and the robust story focus forms an exciting research avenue for us to pursue in the future. In the future of ubiquitous learning, powerful mobile technologies with sensing capabilities will grant us new pedagogical possibilities to take learning from classrooms to the pockets of the learners while maintaining close links to the surrounding contexts.

In conclusion, the results of this research related to pedagogy and the improvement of mathematics teaching practices will enable future development and evaluation of the INBECOM model, for example:

- to evaluate the INBECOM model concerning the learning process in mathematics and different levels of learning
- to evaluate how different learners with different skills and backgrounds experience the model with these different types of activities
- to evaluate how different types of activities affect the learners' motivation

One possibility is to create a comprehensive, integrated system adapting its behavior to individual students based on the information acquired by the system during the learning process. The system could also include, for example, a pretest as well as a personality test assessing the learning style, for instance, the Myers-Briggs Type Indicator (MBTI) based on Carl Jung's theory of psychological type (Adewale, Agbonifo, & Lauretta, 2019). The integrated system could then offer students the most suitable components of the INBECOM model in an effective and meaningful way. Personalization could contain the creation of a virtual avatar with a specific personality and learning style for each student. Another more straightforward solution would be a set of compatible games that can be integrated on-demand because big systems, as ActiveMath, are hard to design and maintain.

Also Aylward and Cronjé (2022) suggest extension of the four-quadrant model presented in subsection 2.1.4 (Figure 3) by adding learner mastery to the domains of knowledge, and teaching and learning methods. They propose the metrics for determining a learning curve. When the curve is mapped to the four quadrants of learning, teachers and learners know what type of knowledge is in question, and what method of teaching should be used. It is interesting that their analysis also examines internal and external factors of learning. I also suggested interpersonal and intrapersonal dimensions to be important domains of learning. However, my view of external factors‘ effects referred more to the importance of cooperation and learning together.

However, I consider it really important that the level of substance competence of the learners is taken into account - and that is a shortcoming in my own model.

Our examination of different learning zones raises further questions like: Are the best mathematics learning games those that create high motivation and include only some DFs? Or those that also include many DFs to inspire the players beyond the explicit content? To build theoretically and empirically grounded conceptual framework for different learning zones related to educational digital games, a systematic motivation and DF examination of the variety of games is needed, as well as a study of dimensions of learning in different zones.

Regarding effective learning, the study raised several issues. Examples are: (i) how adequate levels of learning are intertwined with cognitive levels of learning while learning mathematics in a technology-enhanced learning environment; (ii) how pedagogical models which take into account both cognitive and affective aspects of learning support deep learning; (iii) how motivation for learning relates to the different levels of affective learning; (iv) how affective learning concepts could be integrated into mathematics technology-enhanced learning classrooms; and (v) how to examine teachers' affective learning while integrating new pedagogical skills into their teaching.

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I seized the opportunity and became enthusiastic about developing something new and getting new challenges to my life. However, my PhD journey was not as straightforward as doctoral studies usually are. These about ten years of my life were far more interesting and multidimensional than I would ever have imagined. However, life also brought me many other experiences, some demanding and others sorrowful, which took some time off studying.

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Appendices

APPENDIX 1: Author's contribution.

The author was the main contributor to all the manuscripts except for article II. The followings are detailed description of author's contributions in each of the article:

Paper I.

Practical work

I created the UFfactions game in collaboration with Dr Carolina Islas Sedano, Mikko Vinni and Professor Teemu H. Laine. Laine was responsible for the technical development of the game. The candidate was responsible for the idea and designs for the UFractions game. Christo J. Els, Professor Blignaut and Professor Laine helped the candidate with the practical arrangements of the game evaluations in South Africa.

Data processing

The candidate was responsible for data processing.

Paper II.

Practical work

UFfactions game was created in collaboration with Dr Carolina Islas Sedano, Mikko Vinni and Professor Teemu H. Laine. Laine was responsible for the technical development of the game. The candidate was responsible for the idea and designs for the UFractions game. Christo J. Els, Professor Blignaut and Professor Laine helped the candidate with the practical arrangements of the game evaluations in South Africa. Professor Laine and the candidate made together the practical evaluations in North Karelia.

Data processing

The candidate and Professor Laine made the data processing together.

Writing of the manuscript

The candidate was responsible for writing the section ”2.4 Technologies for mathematics education” and took part in the writing of other parts of the paper.

Paper III.

Practical work

UFfactions game was created in collaboration with Dr Carolina Islas Sedano, Mikko Vinni and Professor Teemu H. Laine. Laine was responsible for the technical development of the game. The candidate was responsible for the idea and designs for the UFractions game. Christo J. Els, Professor Blignaut and Professor Laine helped the candidate with the practical arrangements of the game evaluations in South Africa. Professor Laine and the candidate made together the practical evaluations in North Karelia.

Data processing

The candidate was responsible for the data processing. Dr. Suria Ellis from Statistical Consultation Service, North-West University helped with statistical analysis.

Paper IV.

Practical work

UFfactions game was created in collaboration with Dr Carolina Islas Sedano, Mikko Vinni and Professor Teemu H. Laine. Laine was responsible for the technical development of the game. The candidate was responsible for the idea and designs for the UFractions game. Christo J. Els, Professor Blignaut and Professor Laine helped the candidate with the practical arrangements of the game evaluations in South Africa. Professor Laine and the candidate made together the practical evaluations in North Karelia. Professor Laine made the practical evaluations in Mozambique.

Data processing

The candidate and Professor Laine made together the data processing.

Paper VI.

Practical work

UFfactions game was created in collaboration with Dr Carolina Islas Sedano, Mikko Vinni and Professor Teemu H. Laine. Laine was responsible for the technical development of the game. The candidate was responsible for the idea and designs for the UFractions game. Christo J. Els, Professor Blignaut and Professor Laine helped the candidate with the practical arrangements of the game evaluations in South Africa. Professor Laine and the candidate made together the practical evaluations in North Karelia. Professor Laine made the practical evaluations in Mozambique. The candidate made practical arrangements for testing of the ActiveMath system and the fraction course in Finland.

Data processing

The candidate was responsible for the data processing. Professor Blignaut and professor Leenderz gave guidance on the qualitative analysis. Professor Leenderz and Statistical Consultation Service, North-West University, did majority of the statistical analysis.

APPENDIX 2: The research permit issued by the South African Ethics Committee.

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2 Maart 2009

Beste Marietjie

Etiekaansoek

Etieknommer: NWU-00007-09-S2

Projekhoof: Prof Seugnet Blignaut (Student: Eeva Turtiainen)

Projektitel: Increasing motivation towards mathematics through educational games

Die bogenoemde etiekaansoek is deur die etiekkomitee beoordeel en nadat seker sake onder die aandag van die projekleier gebring is en dit reggestel is en onduidelikhede uitgeklaar is kan volle magtiging aan die projek verleen word.

Die etieknommer kan dus gewysig word na en 'n etieksertifikaat kan dus uitgereik word:

NWU-00007-09-**A**2

Vriendelike dank

JLdeK Monteith
Voorsitter Etiekkomitee Fakulteit opvoedingwetenskappe

APPENDIX 3: The questionnaire for students.

Prototype Test Day - March 2009 (Students)
Questionnaire for Development of UFractions Mobile Math Game

Please help us with your feedback

We thank you that today you voluntarily accepted to test our application prototype. This questionnaire will help us analyse how to improve the game we are developing. For that, it is important that you answer all the questions as honestly as possible. All the answers you give to these eleven topic sections are confidential and will be used for research purposes anonymously.

General information

i) Date:
ii) School:
iii) Home language:
iv) Cultural group:
v) Gender: □ Male □ Female
vi) Age:
vii) Game Nick- Name:

Part I – Before Playing

Question topic 1
Skills considering mobile phones

a) Do you have a mobile phone? □ No □ Yes (if **No**, go to **Question topic 2**)
b) What is your phone model: ___________________________

Estimate how much you use the following functions of your phone:

	Several times a day	Once a day	Once a week	Never	My phone doesn't have it
i) SMS					
ii) Talking					
ii) Taking photos					
iv) Other (what):					

Question topic 2

a) How do you feel about Maths?

b) What do you usually do in Maths class?

c) What do you think is expected of you in the Maths class?

d) What do you expect from yourself in the Maths class?

e) What emotions do you feel in the Maths class?

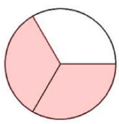
f) Do you find fractions difficult?

Question topic 3
Some questions considering fractions:

1. a) Look at the rectangle. What fraction is shaded grey?

Answer: __________

b) Look at the circle. What fraction is shaded grey?



Answer: __________

2. What is 8 in this fraction $\frac{5}{8}$. Thick the
   a. the numerator
   b. the denominator
   c. the quotient
   d. the sum

3.

a. What is $\frac{1}{4}$ of 16? Answer:______

b. What is $\frac{3}{5}$ of 20? Answer:______

4.

a. Turn this into a mixed or whole number: $\frac{10}{6}$ Answer:______

b. Turn this into an improper fraction: 3 ½ Answer:______

5. Calculate

a. $\frac{1}{4}+\frac{2}{4}=$ _________

b. $\frac{4}{5}-\frac{1}{5}$ = _________

c. $\frac{1}{2}+\frac{1}{3}=$ _________

Part II – After Playing

Question topic 4

While playing with leopards today, which one was your motivator to keep on playing:

	Strongly agree	Agree	Disagree	Strongly disagree	I don't have an opinion
a)I wanted to know what will happen next	1	2	3	4	5
b) I found the story of leopards interesting	1	2	3	4	5
c) I wanted to feel I was helping mother leopard	1	2	3	4	5
d) I was curious to see what I can learn about mathematics	1	2	3	4	5
e) I wanted to solve all the questions correctly	1	2	3	4	5
f) I wanted to play more with Cuisenaire Rods	1	2	3	4	5
g) I wanted to learn more about leopards	1	2	3	4	5
h) I wanted to learn more about fractions	1	2	3	4	5

Question topic 5

a) What did you like/enjoy about playing with Leopards?

b) What did you dislike or find difficult about playing with Leopards?

c) What would you suggest to improve in the game ?

Question topic 6
Which of these game activities did you like?

	Strongly agree	Agree	Disagree	Strongly disagree	I don't have an opinion
a)I liked playing with the Cuisenaire Rods	1	2	3	4	5
b) I liked answering the questions	1	2	3	4	5
c) I liked reading the hints and mathematical theory	1	2	3	4	5
d) I liked taking the pictures	1	2	3	4	5
e) I liked interacting with leopards	1	2	3	4	5
f) I liked reading the story	1	2	3	4	5
g) I liked using the Mobile Phone	1	2	3	4	5

Question topic 7
What are your opinions about the following statements?

	Strongly agree	Agree	Disagree	Strongly disagree	I don't have an opinion
a) It was fun to play with the phone	1	2	3	4	5
b) The screen was too busy	1	2	3	4	5
c) The size of the text was big enough	1	2	3	4	5
d) It was easy to use the phone as a tool for playing	1	2	3	4	5
e) Typing the colour codes of the rods was easy	1	2	3	4	5
f) Game helped me when I got stuck	1	2	3	4	5

Question topic 8
What are your opinions about the following statements?

	Strongly agree	Agree	Disagree	Strongly disagree	I don't have an opinion
a) Compared to ordinary class this was exciting	1	2	3	4	5
b) The problems were easy to understand	1	2	3	4	5
c) Cuisenaire Rods helped me understanding the fractions	1	2	3	4	5
d) Language was easy to understand	1	2	3	4	5
e) Leopards' comments after solving problems encouraged me	1	2	3	4	5
f) I will think of leopards from now on whenever I do fractions	1	2	3	4	5

g) It was important to be able to make my own choices	1	2	3	4	5
h) I felt important as I was saving the leopard	1	2	3	4	5
i) I would also like to meet other wild animals in the game and help them	1	2	3	4	5
j) I enjoyed playing together with my classmates	1	2	3	4	5
k) I learned a lot of fractions while playing	1	2	3	4	5
l) The hints and theory helped me to understand the problems	1	2	3	4	5

Question topic 10
What are your opinions about the following statements?

a)I think today's mathematics lesson (playing the game) was: □ Boring □ Interesting □ Other:
b) I want to try this kind of games again. □ Strongly agree □ Agree □Disagree □Strongly disagree
c) Today's experience affected my attitude towards mathematics positively: □ Strongly agree □ Agree □Disagree □Strongly disagree □I don't have an opinion
d) I learned better by playing than in usual mathematics lessons □ Strongly agree □ Agree □Disagree □Strongly disagree □I don't have an opinion

Question topic 11

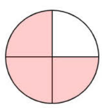
After such a long questionnaire, do you have any further comments for us in order to improve this prototype?

Question topic 12
Some questions considering fractions:

1. a) Look at the rectangle. What fraction is shaded grey?

Answer: __________

b) Look at the circle. What fraction is shaded grey?



Answer: __________

2. What is 5 in this fraction $\frac{5}{8}$
   a. the numerator
   b. the denominator
   c. the quotient
   d. the sum

3.
   a. What is $\frac{1}{4}$ of 20? Answer:______
   b. What is $\frac{3}{4}$ of 40? Answer:______

4.
   a. Turn this into a mixed or whole number: $\frac{7}{4}$ Answer:______
   b. Turn this into an improper fraction: $2\frac{1}{3}$ Answer:______

5. Calculate
   a. $\frac{3}{5}+\frac{1}{5}=$ _________
   b. $\frac{3}{4}-\frac{1}{4}$ = _________
   c. $\frac{1}{5}+\frac{1}{2}$ = _________

Thank you very much for all your insights and time!

APPENDIX 4: The questionnaire for teachers.

Prototype Test Day - March 2009
Questionnaire for Development of UFractions Mobile Math Game

Please help us with your feedback

We thank you that today you voluntarily accepted to test our application prototype. This questionnaire will help us analyse how to improve the game we are developing. For that, it is important that you answer all the questions as honestly as possible. All the answers you give to these eight topic sections are confidential and will be used for research purposes anonymously.

General information

i) Date:
ii) School:
iii) Home language:
iv) Cultural group:
v) Gender: ☐ Male ☐ Female
vi) Age:
vii) Game Nick- Name:

Part I – Before Playing

Question topic 1
Skills considering mobile phones

Do you have a mobile phone? ☐ No ☐ Yes (if **No**, go to **Question topic 2**)
What is your phone model: ___________________________

Estimate how much you use the following functions of your phone:

	Several times a day	Once a day	Once a week	Never	My phone doesn't have it
a) SMS					
b) Talking					
c) Taking photos					
d) Other (what):					

Question topic 2

a) What methods do you usually use in the Maths classes?

b) What do you think about using games as a teaching tool? Can they be effective?

c) Do you use ICT in math classes? How do you use it or how would you like to use it?

d) How do you think your students understand the concept of fractions and basic calculations (adding, subtracting and multiplication)?

Part II – After Playing

Question topic 3

While students played with leopards today, which one do you think that was students' motivator to keep on playing:

	Strongly agree	Agree	Disagree	Strongly disagree	I don't have an opinion
a) They wanted to know what will happen next	1	2	3	4	5
b) They found the story of leopards interesting	1	2	3	4	5
c) They wanted to feel they were helping mother leopard	1	2	3	4	5
d) They were curious to see what they can learn about mathematics	1	2	3	4	5
e) They wanted to solve all the questions correctly	1	2	3	4	5
f) They wanted to play more with Cuisenaire Rods	1	2	3	4	5
g) They wanted to learn more about leopards	1	2	3	4	5
h) They wanted to learn more about fractions	1	2	3	4	5

Question topic 4

a) What did you like about UFractions game?

b) What did you dislike or find difficult about playing UFractions game?

c) What would you suggest to improve in the game?

Question topic 5
What are your opinions about the following statements?

	Strongly agree	Agree	Disagree	Strongly disagree	I don't have an opinion
a) Students liked to play the game	1	2	3	4	5
b) The problems in the game were relevant	1	2	3	4	5
c) The problems in the game were interesting	1	2	3	4	5
d) It was easy for students to use the phone as a tool for playing	1	2	3	4	5
e) I would like to use this kind of game in my math class	1	2	3	4	5
f) The game seemed to be effective as learning tool	1	2	3	4	5
g) The problems in the in the game were easy	1	2	3	4	5
h) The idea of using Cuisenaire rods in the mobile phone game is good	1	2	3	4	5

Question topic 6

What are your opinions about the following statements?

	Strongly agree	Agree	Disagree	Strongly disagree	I don't have an opinion
a) Compared to ordinary class this was exciting	1	2	3	4	5
b) The problems were easy to understand	1	2	3	4	5
c) Cuisenaire Rods helped students understanding the fractions	1	2	3	4	5
d) Students learned a lot about fractions while playing	1	2	3	4	5
e) The hints and theory helped students to understand the problems	1	2	3	4	5
f) It would be difficult to assess the students learning outcomes while playing	1	2	3	4	5
g) The scoring of the game was reasonable	1	2	3	4	5
h) Students that have normally difficulties in mathematics, still had them as much as usually while they were playing	1	2	3	4	5
i) I would also like to test other similar games	1	2	3	4	5
j) It is interesting to learn new ways of teaching	1	2	3	4	5

Question topic 7

What are your opinions about the following statements?

a) I think today's mathematics lesson (playing the game) was: ☐ Boring ☐ Interesting ☐ Other:
b) I think that today's experience affected my students' attitudes towards mathematics positively: ☐ Strongly agree ☐ Agree ☐Disagree ☐Strongly disagree ☐I don't have an opinion
c) Students learned better by playing than in usual methods: ☐ Strongly agree ☐ Agree ☐Disagree ☐Strongly disagree ☐I don't have an opinion

Question topic 8

a) After such a long questionnaire, do you have any further comments for us in order to improve this prototype?

Thank you very much for all your insights and time!

APPENDIX 5: The interview questios for students and teachers.

Interviewing the students

The following questions are asked:

1. Do you like playing games?
2. What kind of games do you usually play?
3. Do you think that UFractions is a game?
4. Would you like to play games more often in math lessons?
5. How do you experience *UFractions* game in terms of motivation?
6. How do you experience *UFractions* game in terms of effective learning?
7. Did Cuisenaire Rods help you to find solutions to the problems?

Interviewing the teachers

The following questions are asked:

8. How do you experience *UFractions* game in terms of motivation?
9. How do you experience *UFractions* game in terms of effective learning?
10. What do you think about using concrete manipulatives like Cuisenaire Rods in math lessons? Are they effective learning tools?
11. What do you think about using games in general in math lessons?
12. Would you like to use digital educational games in math lessons?
13. What kind of aspects combining manipulates and game brings to the learning process in mathematics?
14. Were there any students with learning disabilities in this testing group? Were they able to play the *UFractions* game? How did they manage?
15. How do you describe the socio-economic background of your students?

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