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ON THE PROPERTIES AND COSMOLOGY OF Q-BALLS

by

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The universe is full of magical things patiently waiting for our wits to grow sharper.

-Eden Phillpotts

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Contents

Abstract

The properties and cosmological importance of a class of non-topological solitons, Q-balls, are studied. Aspects of Q-ball solutions and Q-ball cosmology discussed in the literature are reviewed. Q-balls are particularly considered in the Minimal Supersymmetric Standard Model with supersymmetry broken by a hidden sector mechanism mediated by either gravity or gauge interactions.

Q-ball profiles, charge-energy relations and evaporation rates for realistic Q-ball profiles are calculated for general polynomial potentials and for the gravity mediated scenario. In all of the cases, the evaporation rates are found to increase with decreasing charge.

Q-ball collisions are studied by numerical means in the two supersymmetry breaking scenarios. It is noted that the collision processes can be divided into three types: fusion, charge transfer and elastic scattering. Cross-sections are calculated for the different types of processes in the different scenarios.

The formation of Q-balls from the fragmentation of the Affleck-Dine -condensate is studied by numerical and analytical means. The charge distribution is found to depend strongly on the initial energy-charge ratio of the condensate. The final state is typically noted to consist of Q- and anti-Q-balls in a state of maximum entropy.

By studying the relaxation of excited Q-balls the rate at which excess energy can be emitted is calculated in the gravity mediated scenario. The Q-ball is also found to withstand excess energy well without significant charge loss. The possible cosmological consequences of these Q-ball properties are discussed.

List of papers

This thesis consists of the introductory review part and the following research papers:

I T. Multamäki and I. Vilja,
Analytical and numerical properties of Q-balls,
Nucl. Phys. **B574** (2000) 130.

II T. Multamäki and I. Vilja,
Q-ball collisions in the MSSM: gravity-mediated supersymmetry breaking,
Phys. Lett. **B482** (2000) 161.

III T. Multamäki and I. Vilja,
Q-ball collisions in the MSSM: gauge-mediated supersymmetry breaking,
Phys. Lett. **B484** (2000) 283.

IV K. Enqvist, A. Jokinen, T. Multamäki and I. Vilja,
Numerical simulations of fragmentation of the Affleck-Dine condensate,
Phys. Rev. D **63** (2001) 083501.

V T. Multamäki,
Excited Q-Balls in the MSSM with gravity mediated supersymmetry breaking,
Phys. Lett. **B511** (2001) 92.

Summary of the research papers

Paper I: Analytical and numerical properties of Q-balls
Q-ball solutions are studied in three different types of potentials: two polynomial potentials and a potential corresponding to a D-flat direction in the MSSM with SUSY broken by a gravity mediated mechanism. The profiles and charge-energy relations of Q-balls in these potentials are calculated. Analytical criteria are derived to check whether stable Q-balls exist in the thick-wall limit for a general polynomial potential. Evaporation rates are calculated numerically in the perfect thin-wall limit and for realistic Q-ball profiles. In each case the evaporation rate is found to increase with decreasing charge.

Paper II: Q-ball collisions in the MSSM: gravity-mediated supersymmetry breaking
Q-ball collisions are studied in the Minimal Supersymmetric Standard Model where supersymmetry has been broken by a gravitationally coupled hidden sector. It is found that the relative phase difference between the colliding Q-balls determines the type of the collision process at velocities $v \lesssim 10^{-2}$. The total cross-sections, fusion cross-sections and charge transfer cross-sections are calculated by numerical simulations on a two dimensional lattice. The charge transfer cross-section, as well as the total cross-section, is found to be larger than the geometric cross-section whereas the fusion cross-section is smaller than the geometric one.

Paper III: Q-ball collisions in the MSSM: gauge-mediated supersymmetry breaking
Collisions of Q-balls are simulated numerically on a two dimensional lattice in the MSSM where SUSY has been broken at a low energy scale via a gauge mediated mechanism. The total and fusion cross-sections are calculated over a range of charges. The total and geometric cross-sections appear to converge with increasing charge. The fusion cross-section for large, cosmologically interesting, Q-balls is noted to be larger than 60% of the geometric cross-section.

Paper IV: Numerical simulations of fragmentation of the Affleck-Dine condensate
The fragmentation of the Affleck-Dine condensate into Q-balls is studied by numerical and analytical means. On the basis of analytical arguments, it is expected that the number of Q-balls and anti-Q-balls in the final state is strongly dependent on the initial ratio of energy and charge density of the condensate, $x = \rho/mq$. If $x = 1$ only Q-balls are expected to appear whereas if $x \gg 1$, an almost equal number of Q-balls and anti-Q-balls is expected. Furthermore, if the initial energy density of the condensate is large compared to the charge density, the final state should be in a state of maximum entropy with most of the balls small and relativistic. The expected behaviour on the basis of analytical arguments is seen in numerical simulations on a two dimensional lattice. The total charge in the final state in Q-balls is found to be almost equal to the total charge in anti-Q-balls and much larger than the original charge in in the condensate for $x \gg 1$.

Paper V: Excited Q-balls in the MSSM with gravity mediated supersymmetry breaking

The dynamics of excited Q-balls are studied in the context of the MSSM with gravitationally broken SUSY. The rate at which Q-balls can emit excess energy is calculated for a range of charges and initial excess energies. The emission rate is found to be weakly dependent on the charge and initial excess energy in the Q-ball and to be suppressed by $\mathcal{O}(10^{-2})$ compared to the dynamical scale of the field, m. It is also noted that a Q-ball, at least in this scenario, is a very robust object that can withstand large amounts of excess energy without losing a significant amount of its charge. The significance of these Q-ball properties for thermal dissociation and dissolution processes is discussed.

Chapter 1

Introduction

Supersymmetry has over the last decades been one of the most studied theories that attempts to understand nature at high energies, regardless of the lack of direct experimental evidence of its existence. In addition to solving technical difficulties associated with quantum field theories, it also provides us with a rich phenomenology at varying scales of energy. In the present day collider experiments the obvious objective is to search for hints of supersymmetric particles but supersymmetry can also reveal its presence in other ways. Especially the extreme conditions present in the very early moments of the universe may have restored supersymmetry as an unbroken symmetry of nature, which can have far reaching cosmological consequences, up till present day.

The field theoretical content of supersymmetric theories is such that extended objects can typically exist. Localized bound states of supersymmetric particles that are non-dispersive in time are naturally present in many supersymmetric extensions of the Standard Model. These lumps of scalar particles, Q-balls, can have a number of phenomenological implications both at present and especially in the early universe. Understanding their properties is necessary to explore the possible role that Q-balls may play in cosmology.

In this work, the properties and cosmological significance of Q-balls in supersymmetric theories are discussed, in addition to studying Q-balls at a more general level. Analytical and numerical methods are utilized to understand the qualitative and quantitative features of Q-balls and their importance to cosmology.

The introductory review part is organized as follows: In Chapter 2 non-topological solitons are introduced as a general class of extended objects to which Q-balls belong. General features of Q-ball solutions are discussed in Chapter 3. Q-balls in supersymmetric theories are studied in Chapter 4 along with a brief look at different supersymmetry breaking mechanisms. The possibility of producing Q-balls by cosmological means is discussed in Chapter 5 and different properties of Q-balls are described in Chapter 6. In Chapter 7 aspects of Q-ball cosmology and detection possibilities are discussed. The work is summarized in Chapter 8.

Chapter 2

Non-topological solitons

Stable, localized bound states that have a finite shape and that can travel with a constant velocity are of interest in many different branches of physics. Possibly the most famous of such solutions is the Korteweg-de Vries soliton [1] that describes the motion of solitary waves in shallow water in one spatial dimension. Such localized solutions in three dimensions are obviously intriguing as models of extended objects. However, a stable, localized solution in more than two spatial dimensions is severely restricted by a theorem stating that no time-independent solution of this kind exists [2], *i.e.* the equation

$$\nabla^2\phi - \partial_t^2\phi = \frac{\partial f(\phi)}{\partial\phi} \tag{2.1}$$

has no time-independent, localized solutions for any $f(\phi)$. Put in another way, any such configuration must be time dependent.

The appropriate solutions in relativistic local field theories that have been considered in the literature can be classified into two types: topological solitons[1] and non-topological solitons [3]. The feature that distinguishes the two types from one another is the requirement of distinct vacuum states. In the topological case the boundary condition at infinity for a soliton state differs from the boundary condition of the physical vacuum state, implying degenerate vacuum states. In the case of non-topological solitons, a degenerate vacuum is not needed as the boundary conditions at infinity are equal for the soliton and vacuum states.

Non-topological solitons as solutions to classical field theories were first introduced and studied in [4, 5]. The general class of non-topological solitons has since then been split into a variety of types that correspond to different types of potentials and fields. Typically the models investigated involve one or two scalar fields, but also fermion and gauge fields have been studied. The types of different non-topological solitons discussed in the literature include Q-balls [6, 7, 8] (abelian and non-abelian), abnormal nuclei [9], quark nuggets [10], cosmic neutrino balls [11], soliton stars [12], Q-rings [13], and Q-lumps [14]. Fermionic non-topological solitons have been studied as models of hadrons [15] and also in connection to possible Q-balls in superfluid ^3He [16]. Some examples of the rich variety of Q-balls in different types of theories, other than Q-balls associated with supersymmetry that this work mostly discusses, are reviewed in Sect. 6.4.

[1]It should be noted that the word soliton is used very loosely in this context to describe stable, localized bound states that are non-dispersive. In a strict sense of the word, these objects are generally not solitons as they do not conserve their shapes and charges in collisions.

A typical general model that can have non-topological soliton solutions, consists of a (real) scalar field, σ, that couples to another field, ϕ, that is either a complex scalar field (see *e.g.* [3, 17]) or a fermion field (see *e.g.* [15]). The ϕ field carries a conserved quantum number and is "confined" by the non-zero vacuum expectation value of the σ field, σ_0. As a concrete example, consider a two scalar theory that is described by the Lagrangian density [17, 18]

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi^* + \frac{1}{2}(\partial_\mu\sigma)^2 - h\phi\phi^*(\sigma-\sigma_0)^2 - U(\sigma), \tag{2.2}$$

$$U(\sigma) = \frac{\lambda_1}{8}(\sigma^2-\sigma_0^2)^2 + \frac{\lambda_2}{3}\sigma_0(\sigma-\sigma_0)^3 + \Lambda, \tag{2.3}$$

where ϕ is a complex scalar field, σ is a real scalar field, h is the coupling constant and λ_i, Λ are constants. This theory can have non-topological soliton solutions. The constants are chosen such that in addition to the local minimum at $\sigma = \sigma_0$ there exists a global minimum at $\sigma = \sigma_-$ such that $U(\sigma_-) = 0$. The Lagrangian is invariant under $U(1)$ transformations of the ϕ field and hence there is a conserved Nöther charge, Q, in the system. As an example, consider the potential depicted in Fig. 2.1 ($\lambda_1 = 1$, $\lambda_2 = 1/3$, $\Lambda = 15/8$). Due to the coupling term in the Lagrangian, the ϕ field is

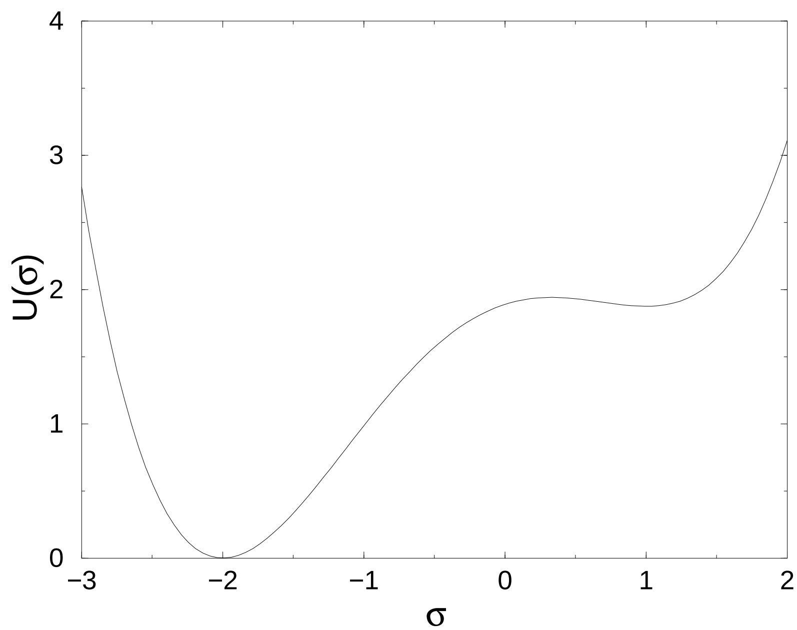

Figure 2.1: An example potential for theories with non-topological soliton solutions

massless at the local minimum, $\sigma = \sigma_0 = 1$, and massive, $m_\phi^2 = h(\sigma_- - \sigma_0)^2$ at the true minimum, $\sigma = \sigma_- = -2$. A configuration of massless ϕ particles near σ_0 can be trapped due to the mass gap for sufficiently large charge. The σ field hence acts as a confining field for the ϕ particles. Such a configuration can be energetically stable (to put more precisely, there exists a time-dependent spherical configuration of ϕ with charge Q whose energy is less than that of free ϕ scalars carrying an equal amount of charge) [17, 18].

Chapter 3

Q-balls

A class of non-topological solitons that has been studied more recently by several authors is the solution first discussed in [4, 7] and later named as Q-balls by Coleman [6]. The term Q-ball was associated to a class of non-topological solitons where solitons are described only by a single scalar field. In the context of the example in the previous chapter, a single field now plays the role of both the σ and ϕ fields so that the field is self-containing. This obviously sets limitations for choosing the potential $U(\phi)$ since now one clearly must require that $U(\phi) = U(|\phi|^2)$ and that localized solutions exist in the theory. In the following, the Q-ball solution is considered in more detail in order to gain some insight into the different aspects of Q-balls.

Consider a scalar field theory in D space dimensions with a global $U(1)$-symmetry and described by the Lagrangian density

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* - U(\phi^* \phi). \tag{3.1}$$

By Nöther's theorem there exists a conserved current

$$j^\mu = i(\phi \partial^\mu \phi^* - \phi^* \partial^\mu \phi) \tag{3.2}$$

and a corresponding conserved charge

$$Q = \int j^0 \; \mathrm{d}^D x. \tag{3.3}$$

The other conserved quantity, energy, is given by

$$E = \int [|\dot{\phi}|^2 + |\nabla \phi|^2 + U(|\phi|^2)] \; \mathrm{d}^D x. \tag{3.4}$$

We wish to find the minimum energy solution in the sector of fixed charge *i.e.* we want to minimize E while keeping Q fixed. This is conveniently done by using a Lagrange multiplier *i.e.* we wish to minimize

$$E_\omega = E - \omega[Q - i \int (\phi \partial^0 \phi^* - \phi^* \partial^0 \phi) \; \mathrm{d}^D x], \tag{3.5}$$

where ω is the Lagrange multiplier. Now we need to find the extremum of E_ω while varying ϕ and ω independently. Let us first vary ϕ, $\phi \to \phi + \delta\phi$, $\phi^* \to \phi^* + \delta\phi^*$, while keeping ω fixed and requiring that $\delta E_\omega = 0$. Doing the variation, integrating by parts, and using the equation of motion of the field ϕ, we find that the condition $\dot{\phi} - i\omega\phi = 0$ must hold. Hence we find that the field can be written as

$$\phi(t, \mathbf{x}) = e^{i\omega t}\varphi(\mathbf{x}), \tag{3.6}$$

where φ can be chosen to be real due to the gauge invariance. The charge and energy of the configuration (3.6) are

$$Q = 2\omega \int \varphi^2 \, \mathrm{d}^D x, \tag{3.7}$$

$$E = \int [\omega^2\varphi^2 + (\nabla\varphi)^2 + U(\varphi^2)] \, \mathrm{d}^D x, \tag{3.8}$$

and the equation of motion is

$$\nabla^2\varphi + \omega^2\varphi - \varphi\frac{dU(\varphi^2)}{d\varphi^2} = 0. \tag{3.9}$$

It now remains to extremize

$$E = \int [(\nabla\varphi)^2 + U_\omega(\varphi^2)] \, \mathrm{d}^D x + \omega Q, \tag{3.10}$$

where

$$U_\omega(\varphi^2) \equiv U(\varphi^2) - \omega^2\varphi^2, \tag{3.11}$$

with respect to variations of φ and ω (recalling that Q is fixed). We wish to find localized solutions that vanish at infinity and study their energies in order to determine the ground state of the theory.

The spherical rearrangement of function φ, where φ is positive and vanishing at infinity, is defined as the spherically symmetric monotone decreasing function φ_R, for which condition

$$\mu\{x|\varphi_R(x) \geq M\} = \mu\{x|\varphi(x) \geq M\}, \tag{3.12}$$

where M is a positive number and μ the Lebesgue measure, holds [6]. We can choose φ to be positive due to the gauge freedom and require that it vanishes at infinity. Clearly, such a rearrangement leaves the charge and the integral of $U_\omega(\varphi)$ unchanged. In addition, the spherical rearrangement theorem [20] states that

$$\int (\nabla\varphi)^2 \mathrm{d}^D x \geq \int (\nabla\varphi_R)^2 \mathrm{d}^D x. \tag{3.13}$$

We can hence minimize (3.10) by choosing φ to be spherically symmetric and monotone decreasing. Equation (3.9) now becomes

$$\frac{d^2\varphi}{dr^2} + \frac{D-1}{r}\frac{d\varphi}{dr} + \omega^2\varphi - \varphi\frac{dU(\varphi^2)}{d\varphi^2} = 0. \tag{3.14}$$

Localized, smooth solutions of (3.14) must have the properties $\varphi(r) \to \infty$ as $r \to 0$ and $\varphi'(0) = 0$.

So far we have not set any other requirements for the potential U than the $U(1)$-symmetry. By convention we choose $U(0) = 0$, require that the origin is a global minimum, and that the tree level mass of a free ϕ scalar is m, that is

$$\left.\frac{dU(|\phi|^2)}{d|\phi|^2}\right|_{\phi=0} = m^2. \tag{3.15}$$

For Q-balls to exist in the theory, the potential must be such that $U(|\phi|^2)/|\phi|^2$ has a minimum that is not located at the origin [6], *i.e.* that

$$\min\Big(\frac{U(|\phi|^2)}{|\phi|^2}\Big) \equiv \frac{U(|\phi_m|^2)}{|\phi_m|^2} < m^2,\ \phi_m \neq 0. \tag{3.16}$$

In other words, we require that the potential U grows more slowly than $m^2|\phi|^2$ over some range of ϕ (there is an additional technical restriction which assures that the minimum of $U/|\phi|^2$ is not at $\phi = 0$ or at $\phi \to \infty$ [6]).

We can get an intuitive understanding of the solutions of Eq. (3.14) and the conditions for the potential $U(|\phi|)$ by studying a mechanical analogy: Consider a point particle moving in the potential $-\frac{1}{2}(U(\varphi^2) - \omega^2\varphi^2) \equiv V_\omega(\varphi^2)$. Equation (3.14) then becomes

$$\frac{d^2\varphi}{dr^2} + \frac{D-1}{r}\frac{d\varphi}{dr} + 2\varphi\frac{dV_\omega(\varphi^2)}{d\varphi^2} = 0, \tag{3.17}$$

which can be thought to describe the motion of the particle in the potential V_ω, with φ describing the "position" of the particle and r the "time". Integrating (3.17) with respect to r we get

$$\frac{1}{2}(\frac{d\varphi}{dr})^2 + V_\omega(\varphi) - V_\omega(\varphi_0) = -(D-1)\int_0^r \frac{1}{\tilde{r}}(\frac{d\varphi(\tilde{r})}{d\tilde{r}})^2 d\tilde{r}, \tag{3.18}$$

where it has been assumed that the particle starts at rest at $\varphi(0) \equiv \varphi_0$. From (3.18) it is clear that the "energy" of the particle, $\varphi'^2/2 + V_\omega$, is conserved only for $D = 1$ and that for $D > 1$, the system is dissipative.

A generic effective potential that satisfies the conditions required for Q-balls to exist in the theory is depicted in Fig. 3.1. The solution that satisfies the conditions

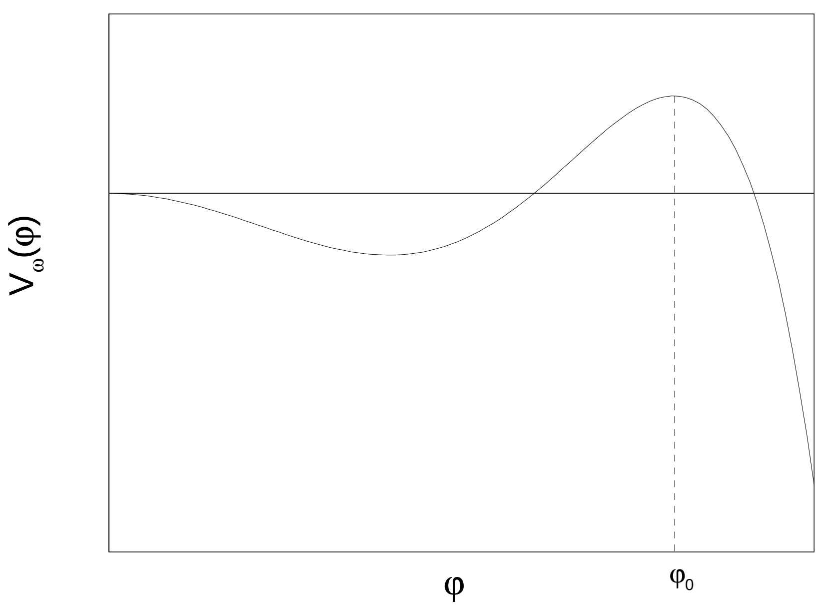

Figure 3.1: A generic potential that allows Q-ball solutions

$\varphi'(0) = 0$ and $\varphi(r \to \infty) = 0$ corresponds in the mechanical analogy to a point particle that is released at rest at such a point that it just reaches the origin. From Fig. 3.1

it is clear that we must release the particle on the hill for it to reach the origin. In the one dimensional case, $D = 1$, there is no dissipation and the point at which the particle must start, φ_0, is clearly at the same level as the origin, $V_\omega(\varphi_0) = V_\omega(0) = 0$. When there is dissipation present in the system, we must then release the particle at a higher point for it to reach $\varphi = 0$. Note that the dissipation term is proportional to r^{-1}, or inversely proportional to "time" in the mechanical analogy, which means that the dissipation term can be made small if the particle starts to slide along the potential very slowly. It is then clear that in the case $D > 1$, there exists a suitable solution for a type of potential shown in Fig. 3.1: If a particle is released too close to the "valley", it will never reach the origin. If, on the other hand, the particle is initially placed very close to the top of the hill, φ_c, it can spend a very long time near its initial position during which the dissipation term becomes negligible and the particle will overshoot. The required solution is hence located somewhere in between these two cases.

We can now understand the requirement for the potential (3.16). It assures that for some range of ω the potential U_ω has a global minimum somewhere away from the origin, or $U_\omega = 0$ has non-zero solutions, which in turn guarantees that solutions exist. As an example, a model polynomial potential, $U_\omega(\varphi) = \varphi^2 - 0.03\varphi^4 + 0.001\varphi^6 - \omega^2\varphi^2$, is shown in Fig. 3.2 for different choices of ω.

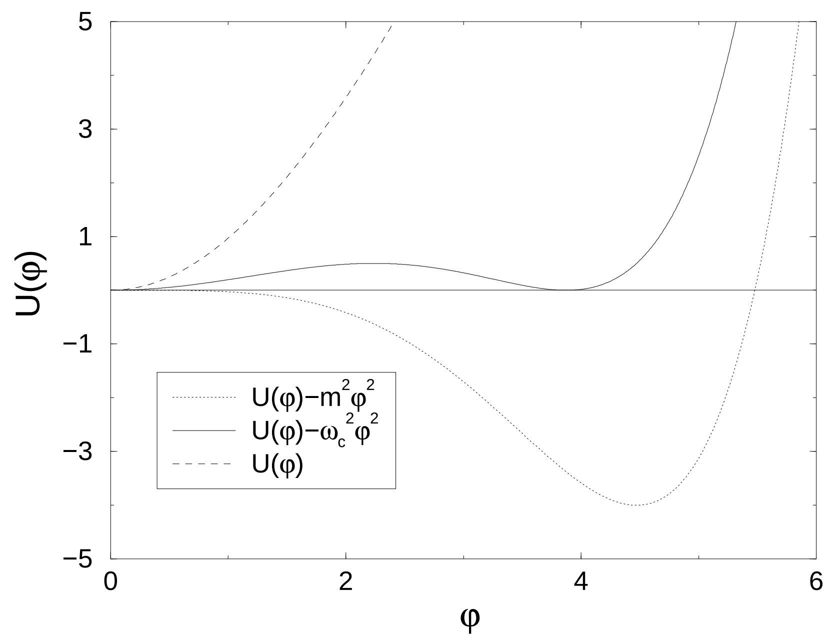

Figure 3.2: A polynomial potential with varying value of ω

The critical value ω_c corresponds to the point at which Q-ball solutions start to exist, *i.e.* the effective potential starts to dip below zero. Q-balls hence exist for ω in the range $\omega_c < \omega < m$. The upper limit, m, is set by the requirement that $dU/d\varphi^2 > 0$ at the origin, or in the context of the mechanical analogy, that the origin is located on top of a hill which enables the particle to stop at the origin.

Consider then the energy of a Q-ball with some charge Q (recall that due to the $U(1)$-symmetry, charge is conserved in the theory). If the energy of the Q-ball configuration, E_Q, is less than the energy, mQ, of a collection of free scalars with an equal total charge, Q, the Q-ball is energetically stable. In other words, to have an energetically stable Q-ball, condition

$$E_Q < mQ, \tag{3.19}$$

must hold. The stable Q-ball solution is hence the ground state of the theory in the sector of fixed charge. In general, the question of finding the energy and charge of a Q-ball configuration is a numerical problem. In some limiting cases, namely in the thin- and thick-wall limits, analytical approximations can be utilized.

3.1 Thin-wall approximation

Consider a spherical configuration with some associated charge Q whose surface is thin compared to its volume so that its profile can be approximated by $\varphi(r) \approx \varphi_0 \theta(r-R)$. The energy and charge of such a configuration are obtained from (3.7) and (3.8) (omitting all contributions coming from the surface terms):

$$Q = 2\omega\varphi_0^2 V \tag{3.20}$$
$$E = \omega^2\varphi_0^2 V + U(\varphi_0^2)V, \tag{3.21}$$

where V is the volume of a D dimensional sphere with radius R,

$$V = \frac{\pi^{\frac{D}{2}} R^D}{\Gamma(D/2+1)}. \tag{3.22}$$

From (3.20) and (3.21) we can eliminate ω, so that the energy of the Q-ball in terms of charge and volume is

$$E(V) = \frac{Q^2}{4V\varphi_0^2} + VU(\varphi_0^2). \tag{3.23}$$

For a fixed charge Q, $E(V)$ obtains its minimum at $V_{\text{min}} = Q/(2\sqrt{U(\varphi_0^2)\varphi_0^2})$. Inserting V_{min} back into (3.23), we find that

$$\frac{E}{Q} = \sqrt{\frac{U(\varphi_0^2)}{\varphi_0^2}}. \tag{3.24}$$

Consider now the potential $U(\varphi)$ as ω approaches ω_c from above. The minimum of U_ω, defined in (3.11), then becomes less and less negative. In this limit φ_0, which in the mechanical analogy corresponds to the starting point of the particle, must approach the minimum point of the potential, φ_c, so that it spends an arbitrarily long time close to it in order to minimize dissipation. The profile of the field configuration is hence such that the value of the field stays approximately constant up to a large radius after which it rapidly decreases (close) to zero, which is exactly what is assumed in the thin-wall approximation. We can hence approximate ω by ω_c and φ_c by φ_0 (*i.e.* , the minimum of $U_\omega(\varphi^2)$ is at φ_0). In this limit the minimum of $U_\omega(\varphi^2)$ is approximately at the same point as the minimum of $U_\omega(\varphi^2)/\varphi^2$, *i.e.* at φ_0. On the other hand, the minimum of $U(\varphi^2)/\varphi^2$ is at the same point as the minimum of $U_\omega(\varphi^2)/\varphi^2$, implying

$$\frac{U(\varphi_0^2)}{\varphi_0^2} \approx \min\left(\frac{U(\varphi^2)}{\varphi^2}\right) < m^2, \tag{3.25}$$

due to (3.16). From (3.24) it then follows that $E/Q < m$ and the Q-ball is stable. The condition (3.16) therefore guarantees that the potential is such that Q-ball solutions exist and are energetically stable in the thin-wall limit.

By the time the particle starts to roll, the dissipation term is negligible and the effective equation of motion during the transition $\varphi_0 \approx \varphi_c \to 0$ is therefore

$$\frac{d^2\varphi}{dr^2} - \varphi\frac{dU_{\omega_c}(\varphi^2)}{d\varphi^2} = 0. \tag{3.26}$$

Integrating this with respect to r gives

$$(\frac{d\varphi}{dr})^2 - U_{\omega_c}(\varphi^2) = 0 \tag{3.27}$$

(the integral vanishes since $\varphi' \to 0$ and $U_\omega \to 0$ as $r \to \infty$). The energy of a thin-walled Q-ball can be approximated by using (3.27):

$$\begin{aligned} E &\approx \int_{\partial V_D}[(\frac{d\varphi}{dr})^2 + U_{\omega_c}]\mathrm{d}^D x + \omega_c Q \\ &\approx \int_{\partial V_D} 2U_{\omega_c}\mathrm{d}^D x + \omega_c Q \\ &\approx \frac{D\pi^{\frac{D}{2}}R^{D-1}}{\Gamma(D/2+1)}\int_0^{\varphi_c} 2\sqrt{U_{\omega_c}}d\varphi + \omega_c Q, \end{aligned} \tag{3.28}$$

where R is the radius of the thin-walled Q-ball. Using (3.22) we see that the energy of the thin-walled Q-ball grows with charge as

$$E(Q) \approx \omega_c Q + D\Big(\frac{\pi^{D/2}}{\Gamma(D/2+1)}\Big)^{1/D}(\frac{1}{2\omega_c\varphi_c^2})^{\frac{D-1}{D}}Q^{\frac{D-1}{D}}\int_0^{\varphi_c} 2\sqrt{U_{\omega_c}}d\varphi. \tag{3.29}$$

It is worth noting that we have actually not showed that the solution is a minimum energy configuration but just a stationary point of E at fixed Q. Coleman [6] has shown that for large Q the Q-ball solution is the minimum energy configuration and that Q-balls exist if condition (3.19) holds. It has also been shown that if stable Q-balls exist at some charge Q, Q-balls in the thin-wall limit will also be stable for any $Q' > Q$ [6].

3.2 Thick-wall approximation

On the basis of the thin-wall approximation one might be lead to believe that only Q-balls with large charges exist. However, Kusenko has showed by studying Q-balls in the thick-wall limit [21], that small stable Q-balls exist in the potential $U(\varphi) = \frac{1}{2}m^2\varphi^2 - A\varphi^3 + \lambda\varphi^4$ This was generalized in Paper I, where a more general potential was considered in D dimensions in the thick-wall approximation.

The basic idea of the thick-wall approximation is simple: Consider a Q-ball configuration corresponding to some value of $\omega > \omega_c$. As ω is increased, U_ω dips deeper and φ_0 moves closer to the origin. In the language of the mechanical analogy, the particle is released farther from the top of the hill as the height of the hill increases. At some point the hill is so high that the particle is released far away from the top and the effect of the top can be ignored, *i.e.* we only need to consider the two lowest order terms in the potential. Clearly this cannot be done for an arbitrary potential since one can easily device a case where the height of the hill cannot be increased enough (recall that ω is bounded from above by m) for the higher order terms to be negligible. Therefore, the condition (3.16) is not sufficient to determine whether thick-walled Q-ball solutions are allowed in an arbitrary potential.

To illustrate these arguments, consider a general potential

$$U(\varphi) = m^2\varphi^2 - \lambda\varphi^A + \lambda'\varphi^B + \mathcal{O}(\varphi^{B+1}), \tag{3.30}$$

where $A > 2$, $B > A$ and λ, λ' are positive. In the thick-wall limit, the higher order terms can be neglected so that

$$U_\omega(\varphi) \approx (m^2 - \omega^2)\varphi^2 - \lambda\varphi^A. \tag{3.31}$$

We hence require that $(m^2-\omega^2)\varphi_0^2 \approx \lambda\varphi_0^A \gg \lambda'\varphi_0^B$ (and that the higher order terms can be neglected), from which it follows that

$$\lambda' \ll \lambda^{\frac{B-2}{A-2}}(m^2-\omega^2)^{\frac{A-B}{A-2}}. \tag{3.32}$$

Introducing dimensionless space-time coordinates, $\xi_i = x_i\sqrt{m^2-\omega^2}$ and re-scaling the field $\psi = (\frac{\lambda}{(m^2-\omega^2)})^{1/(A-2)}$, the energy functional (3.10) can be written as

$$\epsilon_\omega = \lambda^{\frac{2}{2-A}}(m^2-\omega^2)^{\zeta+1}S_\psi^A + \omega Q, \tag{3.33}$$

where S_ψ^A is the action of the bounce in the dimensionless potential $\psi^2 - \psi^A$ and $\zeta \equiv \frac{4+2D-AD}{2(A-2)}$. For Q-balls to exist in the thick-wall limit, (3.33) must have a minimum in the range $0 < \omega < m$, and the minimum energy must less than mQ for the Q-ball to be energetically stable. The derivative of (3.33) with respect to ω is

$$\frac{\partial\epsilon_\omega}{\partial\omega} = Q - 2\lambda^{\frac{2}{2-A}}S_\psi^A(\zeta+1)\omega(m^2-\omega^2)^\zeta. \tag{3.34}$$

Clearly, if $\zeta \leq -1$, (3.33) will be monotone increasing in the considered range and no local energy minimum exists. If $-1 < \zeta \leq 0$, (3.34) can vanish in the required range, but looking at the second derivative of (3.33) we find that the extremum is a local maximum and no stable Q-balls exist in the thick-wall limit. The second derivative of (3.33) with respect to ω has a zero at

$$\omega^2 = \frac{m^2}{1+2\zeta}, \tag{3.35}$$

which for $\zeta > 0$ is in the range $0 < \omega < m$. In order to have a local minimum of ϵ_ω in the appropriate range, the minimum of (3.34) must be negative because then (3.34) will have two zeros and hence ϵ_ω will have a local minimum and a maximum in the appropriate range. Since

$$\left.\frac{\partial\epsilon_\omega}{\partial\omega}\right|_{\omega=m} = \left.\frac{\partial\epsilon_\omega}{\partial\omega}\right|_{\omega=0} = Q, \tag{3.36}$$

for $\zeta > 0$, the local minimum will have energy less than mQ. The requirement for the minimum of $\partial\epsilon/\partial\omega$ to be negative sets an upper limit for the charge,

$$Q < 2\lambda^{\frac{2}{2-A}}S_\psi^A(\zeta+1)m^{2\zeta+1}(2\zeta)^\zeta(1+2\zeta)^{-\zeta-1/2}. \tag{3.37}$$

To summarize, for Q-balls to exist in the thick-wall limit in the potential (3.30) the following conditions must hold:

$$\begin{aligned} \lambda' &\ll \lambda^{\frac{B-2}{A-2}}(m^2-\omega^2)^{\frac{A-B}{A-2}} \\ 0 &< 4+2D-AD \\ Q &< 2\lambda^{\frac{2}{2-A}}S_\psi^A(\zeta+1)m^{2\zeta+1}(2\zeta)^\zeta(1+2\zeta)^{-\zeta-1/2}. \end{aligned} \tag{3.38}$$

For example, in three dimensions ($D=3$), Q-balls in the thick-wall limit can only exist if $A<10/3$. The case $A=3$ was considered in [21] where it was shown that Q-balls exist at the thick-wall limit. If $A=4$, it is easy to see that the only extremum point is an energy maximum for which $E=mQ\sqrt{1+(S_\psi^4/\lambda Q)^2}>mQ$. Note how the requirement for $\zeta>0$ severely restricts the allowed integer values of A so that for $D>3$ Q-balls do not exist in the thick wall limit in a potential of the form (3.30) [1].

The charge in the thick-wall limit was also studied in Paper I, where it was found that

$$Q\sim(m^2-\omega^2)^\zeta, \tag{3.39}$$

as ω tends to m. Hence, since $\zeta>0$, the charge of a thick-walled Q-ball vanishes in the $\omega\to m$ limit.

3.3 Q-ball solutions

Q-balls in the two limits discussed in the previous sections are energetically stable, because they fulfill the energy condition (3.19). In a general case, however, the question of Q-ball stability is a numerical problem. To address it, one first needs to solve the Q-ball profile by solving (3.14) with the initial conditions $\varphi'(0)=0,\ \varphi(r\to\infty)\to 0$. The initial value of the field, $\varphi(0)$, is left as a free parameter and is in practice determined by trial and error.

To illustrate the Q-ball solutions, consider the following two different polynomial potentials:

$$U_1(\varphi) = \frac{1}{2}m_1^2\varphi^2-\alpha_1\varphi^4+\lambda_1\varphi^6 \tag{3.40}$$

$$U_2(\varphi) = \frac{1}{2}m_2^2\varphi^2-\frac{\alpha_2}{3}\varphi^3+\frac{\lambda_2}{4}\varphi^4, \tag{3.41}$$

that were studied in Paper I. Their profiles and energy versus charge curves are shown in Figs. 3.3 and 3.4 ($D=3$) [23]. The parameter values were chosen as $m_1=1,\ m_2=1,\ \lambda_1=0.001,\ \lambda_2=0.01$, and the α values were such that the minimum is degenerate at $\omega=0$.

From the figures it can be seen how the profiles mutate from a thick-walled type to a more thin-walled configuration as ω decreases. Note that the profiles appear quite similar in the two cases. However, when one considers the energy versus charge behaviour, there is a clear difference. In both cases, the charge of the Q-ball starts to decrease as ω increases as is expected from the general behaviour of the Q-ball solution. In the first case, Fig. 3.3, the energy of the Q-ball decreases more slowly so that at

[1]Recently, the thick-wall limit has been further studied in [22]. It was noted that in three dimensions, the thick-wall limit is a bad approximation for $A\geq 6$.

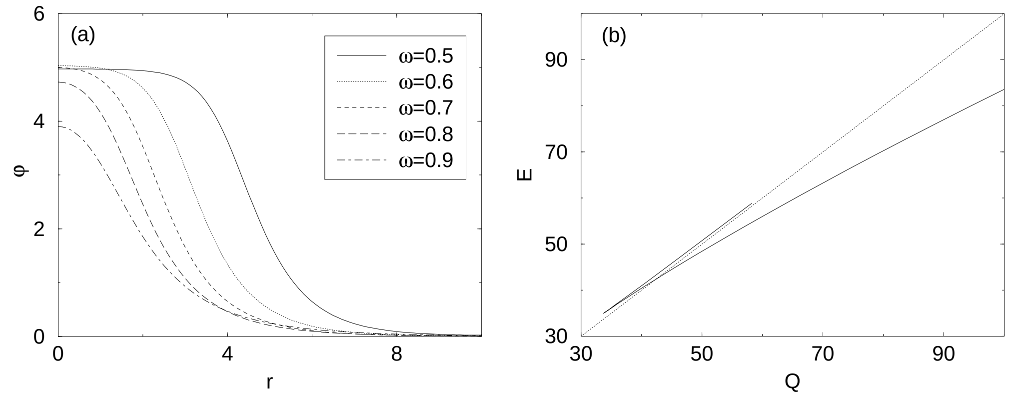

Figure 3.3: Q-ball profiles (a) and energy vs charge (b), $U(\varphi) = U_1(\varphi)$

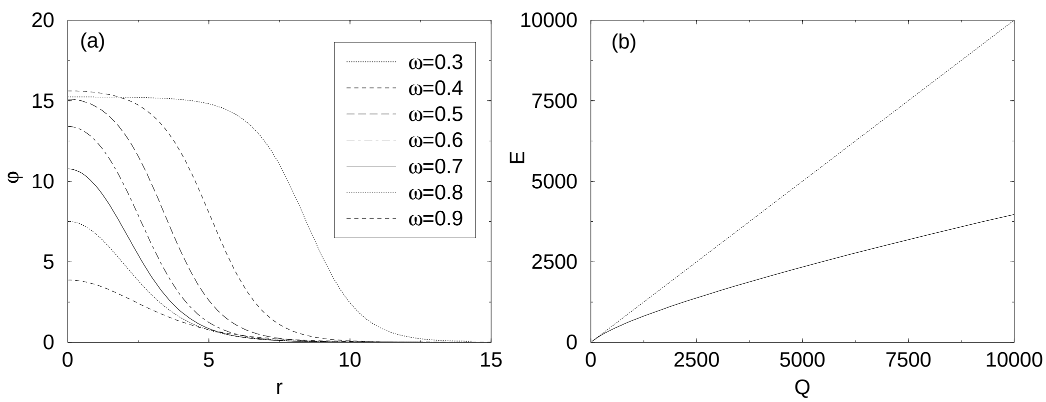

Figure 3.4: Q-ball profiles (a) and energy vs charge (b), $U(\varphi) = U_2(\varphi)$

some point it crosses the stability line $E = mQ$. When ω is increased even further, the energy curve reaches a turning point after which both energy and charge start to increase and asymptotically approach the stability line from above. This is exactly what we expect on the basis of the analytical arguments: Since $A = 4$ and $D = 3$, $\zeta < 0$, and from (3.38) we expect that Q-balls do not satisfy the energy condition (3.19) in the thick-wall limit. Furthermore, looking at (3.39), we can seen that as $\omega \to m$, $Q \to \infty$. In the second case, Fig. 3.4, the energy of the Q-ball decrease with increasing ω so that the energy is below the stability line at all points. Also the energy curve tends to the origin as charge decreases. This is again in accordance with the analytical expectations, since now $A = 3$, $D = 3$, and hence $\zeta > 0$. From (3.38) and (3.39) we expect that Q-balls are energetically stable in the thick-wall limit and that $Q \to 0$ as $\omega \to m$.

Chapter 4

Q-balls in supersymmetric theories

For looking at the consequences of Q-balls for phenomenology and in cosmology, one should obviously consider Q-balls in realistic theories. Based on the discussion in Chapter 3, in order for the theory to have Q-balls in its spectrum, some requirements must be satisfied. First of all, we need to have a suitable $U(1)$-symmetry in order to have a conserved charge to keep the Q-balls stable. In this work we have concentrated on global $U(1)$-symmetry, but Q-balls associated with a local $U(1)$-symmetry have also been considered [24]. In addition, we need to have a scalar field in the theory that carries one of these charges, and a suitably flat scalar potential with a global minimum at the origin (Q-balls associated with scalar fields that carry different charges have been considered in [19]). Clearly, the Standard Model (SM) does not satisfy these conditions so one must consider extensions of the Standard Model.

Supersymmetric (SUSY) extensions of the SM have a large number of scalar fields that commonly have flat directions in their potential. In particular, the Minimally Supersymmetric Standard Model (MSSM) [25] has several flat directions in its scalar potential where the scalar potential nearly vanishes [26, 27]. One can then expect that the MSSM may support Q-balls in its scalar spectrum, at least for some range of parameters.

For any realistic supersymmetric theory, SUSY breaking will play a role in determining the details of the model. Hence the form of the scalar potential in the MSSM is dependent on the exact nature and method of SUSY breaking. To understand the implications of the SUSY breaking mechanism for the scalar potential along a flat direction, different breaking scenarios are briefly discussed in the following.

4.1 SUSY breaking

From experiments we know that supersymmetry is necessarily a broken symmetry of nature at energies accessible to us because of the lack of experimental evidence of supersymmetric particles. The question of understanding how SUSY is broken in the theory is therefore important. While breaking supersymmetry, one obviously wishes to retain the properties of SUSY (namely the cancellation of quadratic divergences) that makes it so desirable. A pragmatic approach to SUSY breaking, that leaves many questions unanswered, is to simply add "soft" SUSY breaking terms to the Lagrangian ("soft" means that the cancellation of quadratic divergences remains) [28]. Such an approach, however, leaves one with the question of the origin of the breaking terms.

Another, maybe a more elegant, approach is to break SUSY spontaneously.

Breaking SUSY dynamically in the context of global supersymmetry, has some difficulties associated with it. A non-supersymmetric vacuum implies that the vacuum has positive energy, which might be problematic for cosmology . Furthermore, the supertrace relations (see *e.g.* [25]) associated with F- and/or D-term breaking lead to troublesome constraints. For example, in pure F-term breaking the following relation holds [29] (at tree level for renormalizable theories):

$$\mathrm{STr}\mathcal{M} = \sum J(-1)^{2J}(2J+1)\mathrm{Tr}\mathcal{M}_J^2 = 0, \tag{4.1}$$

where J is the spin and $\mathcal{M}_J$ the mass matrix of all particles with spin J. Making all of the fermionic superpartners heavy enough not to be seen in present day collider experiments is hence quite problematic due to (4.1).

A way to solve the difficulty in building phenomenologically viable models is to consider SUSY breaking in some sector that is "hidden" to us. The hidden sector then couples to the "visible" sector either via loops or non-renormalizable couplings, hence avoiding the rigorous constraints coming from the supertrace relations. Furthermore, if one considers in more detail the pragmatic approach of simply adding the soft breaking terms to the Lagrangian, one finds that the parameter space of the MSSM is constrained due to phenomenology. Problems arise due to the constraints on flavor changing neutral currents and CP violation [30].

A common way to solve these problems is to consider mechanisms which communicate SUSY-breaking from the hidden sector in such a way, that the problems present in the MSSM with arbitrary parameters are addressed. In addition, such models obviously greatly reduce the large number of free parameters that plague the MSSM.

4.2 Gravity mediated SUSY breaking

A conventional approach to SUSY breaking is the one based on local supersymmetry, which naturally includes gravity and is hence often called supergravity (for an introduction, see *e.g.* [25, 31, 32]). In promoting a globally ($N = 1$) supersymmetric theory to a local one, one finds that now the couplings depend, in addition to the superpotential and the gauge kinetic function, also on the so-called Kähler potential, $G(z^i, z_i^*)$, where z_i are the scalar components of the chiral multiplets. The scalar potential is given in terms of the Kähler potential as

$$V = -e^{-G}[3 + G_k(G^{-1})_l^k G^l] + \frac{1}{2} f_{\alpha\beta}^{-1} D^\alpha D^\beta, \tag{4.2}$$

where $G_i \equiv \partial G/\partial z_i$, $D^\alpha \equiv \tilde{g} G^i T_i^{\alpha j} z_j$, $\tilde{g}$ is the gauge coupling constant, $T_i^{\alpha j}$ the group generators and $f_{\alpha\beta}$ the gauge tensor [25]. Comparing with the case of global SUSY, we see that we can now have $\langle G_k \rangle \neq 0$, $\langle D^\alpha \rangle \neq 0$, *i.e.* , broken SUSY with a vanishing vacuum energy. However, setting the cosmological constant to zero requires fine tuning [25] (and, in light of the recent supernova results [33], may even be disfavored).

Supersymmetry can be broken dynamically by setting a non-vanishing vacuum expectation value (vev) to some of the fields in the hidden sector of the theory. Since the natural scale of the hidden sector is the Planck scale, the vacuum expectation values of the fields are of order $M_{\rm Pl}$. The hidden sector is chosen such that the fields that have large vevs are gauge singlets so that they do not have gauge couplings to the MSSM fields. The other couplings are all suppressed by powers of $M_{\rm Pl}$, hence the sector is hidden from the MSSM, expect that soft breaking terms now appear in the effective low energy theory through Planck scale suppressed operators. Supergravity hence provides a way to couple the visible sector fields to the hidden sector such that the couplings are well suppressed and soft breaking naturally follows from the high energy theory. The soft breaking terms can still be chosen quite freely by appropriate choices of the Kähler potential. However, by imposing a global $U(N)$-symmetry on the Kähler metric, $(G^{-1})^i_j$, where N is the number of superfields on the visible sector, SUSY breaking in the visible sector can be described by only a few parameters [34].

4.3 Gauge mediated SUSY breaking

In addition to the gravity mediated scenario, another commonly considered method is to transit SUSY breaking to the visible sector by gauge interactions [35]. This has the advantage that the communicating mechanism of SUSY breaking respects the flavor symmetry of the SM, whereas in the gravity mediated mechanism this is not necessarily guaranteed by the nature of the breaking mechanism.

The basic principle of the gauge mediated SUSY breaking mechanism is that the supersymmetric partners of the Standard Model fermions receive the dominant part of their mass via gauge interactions with the hidden sector. As an example, consider a generic model [30, 36] with a superpotential that includes a term

$$W = \lambda X \bar{\psi}\psi + \ldots, \tag{4.3}$$

where ψ is a superfield with SM couplings but is not in the spectrum of MSSM and X is a SM singlet superfield in the hidden sector that gets non-zero vacuum expectation values in the D- and F-directions. The scalar vev of X gives masses to the fermionic component of ψ so that $m_{\tilde{\psi}} = \lambda\langle X\rangle$. The masses of the scalar components are $m_\psi^2 = \lambda^2\langle X\rangle^2 \pm \lambda\langle F_X\rangle$, where $\langle F_X\rangle$ is the vev of the auxiliary component of X. The gauginos of the MSSM now get mass corrections at 1-loop level, $m_\lambda \sim \Lambda \equiv \langle F_X\rangle/\langle X\rangle$, while the scalars of the MSSM get soft masses at 2-loop level, $m_\phi^2 \sim \Lambda^2$. The trilinear soft terms arise at 2-loop level as well. These masses then act as effective soft supersymmetry breaking terms in the theory.

4.4 Flat directions

Flat directions in supersymmetric theories appear as combinations of fields for which the F- and D-terms vanish. A flat direction can, however, be lifted by two different mechanisms, by non-renormalizable terms in the superpotential and by supersymmetry breaking which implies that the mechanism of SUSY breaking is also significant. A flat direction can be parametrized by an invariant operator, X, formed from the product of l chiral superfields which constitute the flat direction. The flat direction can then be lifted by a non-renormalizable term X^k, which in terms of the field ϕ then leads to a term in the superpotential of the form [26]

$$W = \frac{\lambda}{dM_{\rm Pl}^{d-3}}\phi^d, \tag{4.4}$$

where λ is a coupling constant, M is a large mass scale and $d = lk$. Another type of a non-renormalizable term is one with a single field (ψ) not in the flat direction and a number of fields (ϕ) in the flat direction [26],

$$W = \frac{\lambda}{M_{\rm Pl}^{d-3}} \psi \phi^{d-1}. \tag{4.5}$$

In the flat space limit, $M_{\rm Pl} \to \infty$ with minimal kinetic terms for the Kähler potential, $G_i^j = -\delta_i^j$, $f_{\alpha\beta} = \delta_{\alpha\beta}$, the lowest order contributions from (4.4) or (4.5) give a contribution to the scalar potential [26]

$$V(\phi) = \frac{\lambda^2}{M_{\rm Pl}^{2d-6}} |\phi|^{2(d-1)}. \tag{4.6}$$

Such a term will always dominate the potential for large enough field values. Note that the contribution to the scalar potential (4.6) is $U(1)$ symmetric, which is exactly what we need for building Q-balls.

In addition to the terms coming from the non-renormalizable terms in the superpotential, the flat directions are lifted by soft terms arising from the supersymmetry breaking mechanism. Since the supersymmetry breaking in the gravity mediated scenario is communicated via gravity, the soft breaking terms stay intact up to $\sim M_{\rm Pl}$. The situation in the gauge mediated case is, however, quite different. Here the soft masses are vanishing at a scale larger than the supersymmetry breaking scale, which for the gauge mediated case can be much less than $M_{\rm Pl}$. This is due to the fact that as ϕ grows larger than $\langle X \rangle$, the loop corrections that lead to the soft mass term become suppressed by $\langle \phi \rangle^{-2}$ so that at $\langle \phi \rangle > \langle X \rangle$ the soft mass of ϕ decreases [36]. The potential can hence be flat at $\langle \phi \rangle > \langle X \rangle$ until the non-renormalizable terms become effective.

In addition to the effect of the non-renormalizable terms and the terms induced by SUSY breaking, the high energy density of the early universe also affects the effective potential along the flat direction. For example, the Kähler potential can have a (D-) term of the form [26, 37]

$$\int d^4\theta M_*^{-2} \chi^\dagger \chi \phi^\dagger \phi, \tag{4.7}$$

where $M_* \equiv M_{\rm Pl}/\sqrt{8\pi}$ is the reduced Planck mass, χ is a field which dominates the energy density of the universe and ϕ again corresponds to the flat direction and the integration is with respect to a superspace coordinate θ. While the energy density of the universe is dominated by the χ field, the interaction (4.7) gives an effective mass of $(\rho/M_*^2)|\phi|^2$, where ρ is the energy density of the χ field. The energy density of the universe in flat space with vanishing cosmological constant is related to the Hubble parameter, H, by Einstein's equations, $\rho = 3H^2 M_*^2$. We can then, for example, identify the χ field with the inflaton which dominates the energy density of the universe during inflation and during the coherent oscillations.

More generally, the flat directions are lifted by a number of different terms coming from the supergravity scalar potential. The general form of these terms differs according to whether they are lifted by the superpotential terms along the flat direction or not. However, to avoid the gravitino problem [38], the lifting terms arising from the supersymmetry breaking are only effective during the inflation and pre-heating [26].

The terms that are independent of the superpotential along a flat direction can be expressed in the following general form during inflaton dominance [26]

$$V(\phi) = H^2 M_{\rm Pl}^2 f(\frac{\phi}{M_{\rm Pl}}), \tag{4.8}$$

where f is a function that depends on the exact form of the Kähler potential. The contributions to the potential that depend on the non-renormalizable terms in the superpotential are of the form of a general A-term [26],

$$H M_{\rm Pl}^3 g(\frac{\phi^d}{M_{\rm Pl}^d}), \tag{4.9}$$

where g is again some function that depends on the details of supersymmetry breaking.

The flat directions of the supersymmetric models are lifted by a number of terms from different sources. To be more concrete and for understanding the relevance of the flat directions for Q-balls and Q-ball formation in the early universe, the MSSM is considered in more detail in the following.

4.5 Flat directions and Q-balls in the MSSM

The MSSM has a number of flat directions along which a scalar field can have a large vacuum expectation value [26]. A condensate of the flat direction can have a net quantum number corresponding to a global $U(1)$ symmetry and therefore, in the MSSM, we can expect to have condensates with non-zero B and L. In view of baryogenesis, the most interesting flat directions are those that carry a non-zero $B - L$ number and unbroken R-parity, since electroweak sphaleron transitions can then convert the non-zero $B - L$ to a baryon asymmetry. Among these flat directions are those which correspond to the $H_u L$ direction ($\langle H_u \rangle = \langle \nu_L \rangle$) and the $u^c d^c d^c$ direction ($\langle u_c \rangle \neq 0,\ \langle d_c \rangle \neq 0,\ \langle d_c' \rangle \neq 0$, the color indices of the squark field are different and d_c and d_c' are orthogonal combinations of the down squark generations) [39]. If R-parity is assumed to be conserved, the superpotential of the MSSM contains terms proportional to $(H_u L)^2$ and $(u^c d^c d^c)^2$. In addition to these terms, another interesting case is the $B - L$ conserving $u^c u^c d^c e^c$ direction, since baryon number carrying Q-balls that decay after the electroweak phase transition can protect the baryons from the sphalerons.

For a large $|\phi|$ the scalar potential along a general flat direction is [39]

$$U(\phi) = m_S^2 |\phi|^2 - cH^2 |\phi|^2 + \frac{\lambda^2}{M_{\rm Pl}^{2(d-3)}} |\phi|^{2(d-1)} + (\frac{(A_\lambda + a_\lambda H)\lambda\phi^d}{M_{\rm Pl}^{d-3}} + h.c.), \tag{4.10}$$

where m_S^2 is the soft SUSY breaking scalar mass, $A_\lambda + a_\lambda H$ the (order H corrected) A-term, d is the dimension of the non-renormalizable term in the superpotential and c, λ constants of order one. The value of a_λ depends on the inflation model: in the case of the F-term inflation, $|a_\lambda|$ is typically of order one [26] whereas for the minimal D-term inflation it vanishes [40].

The non-renormalizable terms are suppressed by the Planck scale. Clearly, if m_S^2 is ϕ independent, (4.10) cannot support Q-ball solutions for $H \ll m_S$. Hence, for Q-balls to exist in potentials of this type, m_S^2 must depend of $|\phi|^2$ so that $U(\phi)/|\phi|^2$

has a global minimum away from the origin. Therefore m_S^2 must decrease over some range of increasing $|\phi|^2$.

The way supersymmetry is broken determines how m_S^2 depends on $|\phi|$. If SUSY is broken by a gauge mediated mechanism, the potential can be completely flat until the non-renormalizable terms begin to dominate [36, 41]. A generic potential in this case is typically modeled by (omitting the A-terms) [41]:

$$U(\phi) = m^4 \log(1 + \frac{|\phi|^2}{m^2}) - cH^2|\phi|^2 + \frac{\lambda^2}{M_{\rm Pl}^{2(d-3)}}|\phi|^{2(d-1)}, \tag{4.11}$$

where H is the Hubble parameter, λ is a dimensionless coupling constant and c is a constant of order one [41]. The mass of the field, m, is typically in the range $10^2 - 10^4$ GeV.

On the other hand, if the soft SUSY breaking terms are generated by a gravity mediated mechanism, the breaking terms are constant at tree level. However, radiative corrections cause the soft parameters, including m_S^2, to be scale dependent. The running of the soft parameters hence depends on the particular direction we are considering.

The H_uL direction was studied in [39, 42] where it was found that even though Q-balls (or L-balls since Q-balls will be carrying lepton number in this case) may exist along this direction, the range of the MSSM parameters that allows for Q-ball solutions, is quite constrained. Furthermore, the L-balls typically have small charges and evaporate too quickly to be cosmologically significant.

The potential in the $d = 4$ ($u^cu^cd^ce^c$) and the $d = 6$ ($(u^cd^cd^c)^2$) directions can be modeled in the general form [39, 43]

$$U(\phi) = m^2(1 + K\log(\frac{|\phi|^2}{M^2}))|\phi|^2 - cH^2|\phi|^2 + \frac{\lambda^2}{M_{\rm Pl}^{2(d-3)}}|\phi|^{2(d-1)}, \tag{4.12}$$

where M is a large mass term, K a constant, and the A-terms have been omitted again. The mass of the scalar is the sum of the constituent fields of the flat direction, *e.g.* in the $(u^cd^cd^c)^2$ direction $m^2 \approx \frac{1}{3}(m^2_{\tilde{u}_i} + m^2_{\tilde{d}_j} + m^2_{\tilde{d}_k})$ $(j \neq k)$ [42]. Radiative corrections to the scalar mass cause it to run, and the value of K can be calculated from the renormalization group (RG) equations for the constituent masses. The sign of the K-term is crucial in determining whether Q-balls exist along the flat direction in question; if $K > 0$, no Q-ball solutions exist, whereas if $K < 0$, Q-ball solutions are possible. The value of K was studied for different flat directions in [42]. There it was found that generally all but the H_uL direction are such that for a large part of the MSSM parameter space the potential is sufficiently flat for Q-balls to exist. The value of K along a suitable direction is typically in the range $K \sim -0.01$ to -0.1.

In addition to the flat directions that allow for large Q-balls to exist in the MSSM, Q-balls can typically also appear in all other realistic supersymmetric extensions of the Standard Model [19]. This is due to the trilinear couplings in the superpotential that arise from the Yukawa couplings of the Higgs fields with quark and leptons. The corresponding scalar potential then has cubic couplings that allow for Q-balls made of sleptons or squarks to exist. The mass of such Q-balls grows linearly with respect to charge, and they are generally unstable with respect to decay into fermions.

Note that the Q-ball scalar fields can also have gauge interactions. The effect of gauge interactions have been studied in [44], where a set of sufficient conditions for the existence of Q-balls in a class of gauge theories have been formulated.

4.6 Q-balls in the gravity mediated SUSY breaking scenario

In the MSSM with SUSY broken by a gravity mediated mechanism, the effective potential of a generic field along a flat direction is given by (4.12). The properties of Q-balls in potentials of this type have been studied in [43], as well as in Papers I and II of this thesis. In [43] the thick- and thin-wall solutions of these Q-balls were studied analytically and numerically. The thin-wall solution behaves as described in Section 3.1. The wall width of the solution is $\delta r \approx \omega_0^{-1}$, where $\omega_0^2 \equiv \omega^2 - m^2(1+K)$. The thick-wall solution, on the other hand, is well described by a Gaussian Ansatz,

$$\phi(r) = \phi_0 \mathrm{e}^{-r^2/R^2}, \tag{4.13}$$

where $R \approx |K|^{-1/2} m^{-1}$. The energy-charge relation was found to be linear in this approximation, $E \propto Q$, and the binding energy per unit charge, $\delta E = (m - E/Q) \sim |K|m$. The evaporation rate of Q-balls in the gravity mediated scenario was studied in Paper I along with their profiles and charge-energy relations. These were again considered in Paper II where the similarity of the energy-charge relations and the profiles in the case of two and three spatial dimensions were noted.

4.7 Q-balls in the gauge mediated SUSY breaking scenario

The behavior of the Q-ball solutions in the gauge mediated scenario can be quite different compared with the gravity mediated case. This is due to the fact that the effective potential at high enough energy can be completely flat due to the switching off of the soft SUSY breaking terms that lift the potential at lower energy scales. In the gravity mediated case the potential was growing more slowly than the mass term $m^2\phi^2$, but was not completely flat due to the nature of the messengers of SUSY breaking. The unusual properties of Q-balls in these completely flat potentials were realized in [36, 45].

The energy of a Q-ball in terms of its charge and ω is given by (3.10). Assuming that the potential is flat, $U(\phi) \approx U_0$, and redefining the field and space variables, $\varphi = \omega\chi$, $\xi = \omega x$, the energy functional can be written as

$$\varepsilon_\omega = \omega \int d^3\xi (|\nabla_\xi \chi|^2 - \chi^2) + \omega^{-3} \int d^3\xi U_0 + \omega Q. \tag{4.14}$$

The integrals are now independent of ω so that solving for the minimum of ε_ω in terms of ω, one finds that $Q \propto \omega^{-4}$. Substituting this into (4.14), it can be seen that $E \propto Q^{3/4}$, *i.e.* , the energy of a Q-ball in a flat potential grows more slowly than Q. It is hence energetically preferable to store charges in large Q-balls.

The profile of a Q-ball in a flat potential can be solved approximately near the origin, $r = 0$, where the potential is effectively $U_0 - \omega^2\varphi^2$ and far away from the Q-ball, where the potential is approximately constant. The Q-ball profile can therefore be modeled by [36]

$$\varphi(r) = \begin{cases} \varphi_0 \frac{\sin(\omega r)}{\omega r}, & r < R, \\ \varphi_1 \mathrm{e}^{-mr}, & r \geq R. \end{cases} \tag{4.15}$$

The constants φ_0, φ_1, ω, and R are chosen in such a way that the energy of the solution is minimized. Using the approximation (4.15), the energy, radius and the value of the field inside the Q-ball can be estimated. The energy minimum of (4.14) corresponds to $\omega_0 \approx (4U_0\pi R^3/Q)^{1/4}$, which with $R_\xi \approx \pi$ and $U_0 \approx m^4$ becomes $\omega_0 \approx \sqrt{2}\pi m Q^{-1/4}$. Substituting this into the expression for energy and dropping the bounce term gives the approximate mass of the Q-ball:

$$E \approx \sqrt{2} m\pi Q^{3/4} \frac{1}{3^{1/4}} \left(\frac{1}{3^{3/4}} + 3^{1/4}\right) = \frac{4\sqrt{2}}{3} m\pi Q^{3/4}. \tag{4.16}$$

The radius is

$$R \approx \pi/\omega_0 = \frac{1}{\sqrt{2}} m^{-1} Q^{1/4}, \tag{4.17}$$

and the field inside the Q-ball can be obtained from the approximate relation $Q \approx \omega_0 (4/3)\pi R^3 \varphi_0^2$, giving

$$\varphi_0 \approx \sqrt{\frac{3}{2\pi^2}} m Q^{1/4} \approx \frac{1}{\sqrt{2\pi}} m Q^{1/4}. \tag{4.18}$$

Note that these expressions are only valid along a completely flat potential and are violated as the non-renormalizable terms begin to dominate at Planck scale.

The profiles and energy-charge relations in the gauge mediated scenario with potential (4.11) in the case of two and three spatial dimensions were calculated in Paper III. It was found that both the profiles and energy-charge curves were similar to each other in the two cases just like in the gravity mediated scenario studied in Papers I and II. The properties of Q-balls along flat directions in supersymmetric potentials hence appear to be quite insensitive to the number of spatial dimensions, in contrast with the polynomial potentials studied in Chapter 3.

In addition to the two scenarios presented above, a hybrid of the two has also been suggested [46]. There the potential is a combination of the gravity and gauge mediated scenarios that allows for stable Q-balls to exist.

Chapter 5

Q-ball formation

For Q-balls to be interesting for cosmology or astrophysics, they must either be produced in the early universe or later in the evolution on the universe by some process. The prospect of actually producing these extended objects in a collider was considered in [36], where it was speculated that large Q-balls, once produced, could be used for high-energy experimentation. Such prospects, however, seem still far off while supersymmetric particles await their discovery, and so we must look for the time being to the universe as a source of Q-balls.

The formation of non-topological solitons has been studied in different models and mechanisms in the literature. In the context of a model of the type (2.2) it was first studied in [47], where solitogenesis was contemplated in a phase transition. (A similar model was studied in [10] for the formation of strange-matter nuggets in the QCD phase transition.) As was already mentioned, the potential of the model (2.2) has two minima; a global minimum at $\sigma = \sigma_-$, where the ϕ field is massless, and a local minimum at $\sigma = \sigma_0$ where $m_\phi^2 = h(\sigma_- - \sigma_0)^2$. The potential is such that there exists a critical temperature $T_c \approx 2\sigma_0$, below which the universe divides into regions of false ($\sigma = \sigma_0$) and true ($\sigma = \sigma_-$) vacua. These vacua are separated by domain walls due to the potential barrier separating the two minima. The energy difference between the two minima, however, leads to a shrinking of the false vacuum domains so that all of the universe is eventually in the true vacuum state (hence avoiding the domain wall problem, see *e.g.* [48]). If the domain walls survive long enough to protect the regions of false vacua from the background of ϕ particles in the true vacua, the ϕ particles in the false vacuum regions can then be trapped into the contracting bag. If the number of ϕ particles, and hence the charge, is large enough, $Q \geq Q_{min} \propto \lambda_1/h^2$, to render the non-topological soliton stable, the collapse will halt leaving a non-topological soliton in the universe. In a sense such a non-topological soliton is hence an artifact of the false vacuum regions present in the early universe. The distribution of the domains of different vacua is determined by the thermal fluctuations, which freeze-out below the Ginzburg temperature, T_G. To efficiently produce non-topological solitons via this generic mechanism, the charge within a false vacuum region must be large enough for the non-topological soliton to be stable. This requirement may be fulfilled by an initial charge asymmetry or, in a zero net charge universe, by statistical fluctuations [49].

The initial papers on solitogenesis did not consider the evolution of non-topological solitons after they have been formed. The distribution of non-topological solitons can obviously be altered by a number of different processes, which in turn can be significant in determining the role of non-topological solitons in cosmology. The non-topological soliton distribution can evolve due to dissociation or evaporation into free ϕ scalars, absorption of ϕ particles, and fusion into larger non-topological solitons. New non-topological solitons can possibly be formed via fusion of free ϕ scalars, called solitosynthesis [17, 18, 41, 50].

In the solitosynthesis, an initial charge asymmetry in the form of free particles at finite temperature start to accrue and form Q-balls. The basis of this mechanism can be understood by considering the free energy of a plasma at temperature T, $F = E - TS$, where S is the entropy of the state and E is its energy. The Q-ball is a state of low entropy compared to the entropy of free scalars carrying an equal amount of charge as the Q-ball. At high temperatures the Q-ball may then well not be the minimum free energy state. As the temperature drops, the increase in free energy coming from the Q-ball state compared to the plasma state becomes smaller and below some critical temperature, the Q-ball becomes the minimum free energy configuration.

The formation of Q-balls by the solitosynthesis is efficiently suppressed by the fact that if there are light fermions in the system carrying the same global charge as the Q-ball scalars, the free energy can be accommodated in the fermionic sector of the theory [50]. This clearly makes solitosynthesis impossible in such a theory. Hence, in the MSSM the solitosynthesis of Q-balls carrying baryon or lepton number is only possible in a B or L breaking minima, where the quarks/leptons are heavy enough [50].

The cosmological evolution of non-topological solitons in the context of the model (2.2) was studied in [17], where it was found that the evolution is strongly dependent on the values of the parameters and that the solitosynthesis appears to require a somewhat special choice of parameters. In [18] it was further noted that in a general class of models, the primordial non-topological solitons tend to disappear quickly unless their typical charge is large.

The formation of Q-balls has been recently considered also via thermal fluctuations [51]. The thermal production rate of Q-balls with charge Q in a generic polynomial potential of finite temperature,

$$V = \frac{1}{2}M(T)^2\phi^2 - A(T)\phi^3 + \lambda\phi^4, \tag{5.1}$$

was approximated by considering the nucleation rate of subcritical bubbles with a charge asymmetry. The obtained result is

$$\Gamma(Q) \approx \frac{2^{13/8}\lambda^{49/8}T^6}{M^2} Q \exp(-\frac{2\lambda^3 M^3 Q^2}{T^3} - c\frac{Q_{\rm min}M^2}{T^2\sqrt{2\lambda}}), \tag{5.2}$$

where $Q_{\rm min}$ is the minimum charge Q-ball and c a constant of order one. Such processes are unlikely to produce a cosmologically significant number of Q-balls, and the production of large Q-balls is virtually impossible unless there exists a mechanism that leads to charge accretion to the initially small Q-balls.

In many of the aforementioned mechanisms the requirement that Q-balls have a minimum charge severely restrict the possibility of producing a large amount of Q-balls. This is not necessarily the case for the Q-balls of supersymmetric theories that allow very small Q-balls to exist [21]. Such small Q-balls can be created by pair production due to thermal fluctuations, after which they can grow by charge accretion [21].

The different mechanisms of producing Q-balls in the early universe are often either restricted or inefficient in generating a cosmologically significant amount of Q-balls. Furthermore, the suggested mechanisms are typically generic and not necessarily in contact with phenomenology. A new mechanism for producing a copious number of Q-balls in the context of the MSSM was realized in [39, 41], where it was noted that the Affleck-Dine (AD) -condensate [52] is generally unstable and breaks up into Q-balls.

The sign of the first term in (4.10) at the origin $\phi = 0$ is crucial for the formation of the AD-condensate and hence Q-balls. In the case of minimal Kähler terms, there is a $H^2|\phi|^2$ contribution with a positive sign in the potential so that typically the average value of the field evolves to the origin during inflation [26]. As inflation ends, the field continues to sit at the origin and no AD-condensate forms. Quantum fluctuations may salvage the situation if the induced mass term $(m_S^2 - cH^2)|\phi|^2$ is very small so that the fluctuations can drive the field up to large values [53].

However, with non-minimal Kähler terms, it is possible that the sign of the induced $H^2|\phi|^2$-term is negative. During inflation the field again evolves to the minimum of the potential but this time it can be located far away from the origin. The relaxation of the field after inflation can in this case lead to the formation of an AD-condensate which, in turn, can then fragment into Q-balls.

5.1 Formation of the AD-condensate

During inflation, the universe expands rapidly as the Hubble parameter acquires a large value. The potential (4.10) during inflation is then effectively given by

$$V(\phi) \approx -cH_I^2|\phi|^2 + (\frac{a_\lambda H_I \lambda \phi^d}{dM_{M_{\rm Pl}}^{d-3}} + \text{h.c.}) + \frac{\lambda^2|\phi|^{2(d-1)}}{M_{M_{\rm Pl}}^{2(d-3)}}, \tag{5.3}$$

where H_I is the value of the Hubble parameter during inflation. The soft terms are omitted here since their contribution during inflation is negligible (except in minimal D-term inflation where $a_\lambda = 0$). The minimum of (5.3) is located at $|\phi_0| = (\beta\lambda^{-1}H_I M_{M_{\rm Pl}}^{d-3})^{1/(d-2)}$, where β is a numerical constant depending on a_λ, c and d. The non-zero A-term violates the $U(1)$-symmetry of the potential and gives discrete minima for the phase of ϕ. During inflation the field quickly settles into one of the minima [26].

Inflation ends with the end of the slow-roll of the inflaton field, after which the field begins to oscillate around the minimum of the inflaton potential. The universe becomes dominated by the coherent oscillations, and the Hubble parameter during this era is $H = 2/3t$. At large values of H the potential $V(\phi)$ is still effectively (5.3). As the Hubble parameter decreases with time, the minimum of the potential starts to approach the origin. The field closely follows the minimum of the potential during this time [26].

As $H \sim m_S$, the coherent production of baryons takes place. The field begins to oscillate about $\phi = 0$ with the initial condition given by the location of the minimum at $t \sim m_S^{-1}$. The presence of the $U(1)$ violating A-terms are now crucial since they lead to the evolution of the phase of the ϕ field with time. Writing $\phi = |\phi|e^{i\theta}$, $\lambda = |\lambda|e^{i\theta_\lambda}$, $a_\lambda = |a_\lambda|e^{i\theta_a}$, $A = |A|e^{i\theta_A}$, the angular potential at large H is effectively proportional to $\cos(\theta_a + \theta_\lambda + d\theta)$. As H decreases the A-term begins to dominate over

$a_\lambda H$ and the angular potential is proportional to $\cos(\theta_A + \theta_\lambda + d\theta)$. A nonzero $\dot{\theta}$ is generated as the ϕ-field begins to oscillate freely, assuming that $\theta_A \neq \theta_a$. The rotation of ϕ generates baryons due to $n_b = i(\dot{\phi}^\dagger \phi - \dot{\phi}\phi^\dagger) = 2|\phi|^2\dot{\theta}$. The CP-violating phase, $\theta_A - \theta_a$, which is the relative phase between the inflaton and the hidden sector, is hence important for determining the amount of baryons produced by this mechanism.

The number density of baryons created in this process is obviously dependent on the parameters of the theory, but for a reasonable choice of these parameters, the ratio of baryon number density to the number density of the ϕ field, $n_\phi \sim m_S^2|\phi|^2$, is $n_b/n_\phi \sim \mathcal{O}(0.1)$ [26]. Note that after the baryon number is created the Hubble parameter decreases to a value that is small compared with the scalar mass, $H \ll m_S^2$, the baryon number violating terms become negligible, and the produced baryon asymmetry is conserved to later times.

In the case of minimal D-term inflation, the mechanism works slightly differently since now the $a_\lambda H_I$-term does not dominate over the A_λ-term at large H_I. Instead, the initial phase of the ϕ-field will be random since the equation of motion for θ is overdamped for $H \gg m_S$ [40]. As $H \sim m_S$, the A_λ-term becomes effective and θ begins to evolve. Again, the rotation of the ϕ field as it oscillates around zero, violates CP-symmetry and baryons are produced. This scenario is described in detail in [40].

The effect of thermal corrections to the AD-potential have been studied in [54], where it has been shown that at least the $d = 4$ directions are generally such that efficient production of baryons does not occur easily. In light of this information, the $d = 6$ directions are favored as far as the Q-ball formation and baryogenesis are concerned.

5.2 Fragmentation of the AD-condensate

The recent interest in Q-balls in the early universe is much due to the realization that the AD-condensate naturally fragments into Q-balls [41, 43]. The seeds of fragmentation are due to the quantum fluctuations during inflation that generate a spectrum of perturbations in the magnitude of the AD-field [39],

$$\delta\phi_0(\lambda_0) \approx \frac{1}{2\pi m_S \sqrt{H_I}\lambda_0^{5/2}}, \tag{5.4}$$

where λ_0 is the perturbation length scale at $H \approx m_S$ and H_I is the value of the Hubble parameter during inflation.

The growth of perturbations has been later on studied both analytically and numerically. To understand the origin of the growing perturbations, it is useful to study the evolution of the homogeneous condensate. Writing the field in the form $\phi = \varphi e^{i\theta}$ and assuming that the baryon number violating terms are negligible at this time so that the potential is $U(1)$-symmetric, the equations of motion in the expanding universe are [41]:

$$\ddot{\theta} + 3H\dot{\theta} - \frac{1}{a^2}\nabla^2\theta + 2\frac{\dot{\varphi}}{\varphi}\dot{\theta} - \frac{2}{a^2\varphi}\nabla\theta\cdot\nabla\varphi = 0 \tag{5.5}$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{1}{a^2}\nabla^2\varphi - \dot{\theta}^2\varphi + \frac{1}{a^2}(\nabla\theta)^2\varphi + \frac{\partial U(\varphi)}{\partial\varphi} = 0, \tag{5.6}$$

where a is the scale factor of the universe. To study the growth of perturbations we need the set of linearized equations obtained by the replacements, $\theta \to \theta + \delta\theta$, $\varphi \to \varphi + \delta\varphi$:

$$\delta\ddot{\theta} + 3H\delta\dot{\theta} - \frac{1}{a^2}\nabla^2\delta\theta + 2\frac{\dot{\varphi}}{\varphi}\delta\dot{\theta} + 2\frac{\dot{\theta}}{\varphi}\delta\dot{\varphi} - 2\frac{\dot{\varphi}\dot{\theta}}{\varphi^2}\delta\varphi = 0 \quad (5.7)$$

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} - \frac{1}{a^2}\nabla^2\delta\varphi - 2\varphi\dot{\theta}\delta\dot{\theta} + U''(\varphi)\delta\varphi - \dot{\theta}^2\delta\varphi = 0. \quad (5.8)$$

Since we are interested in the time development of the field, the terms proportional to the spatial derivatives of φ or θ have been omitted for simplicity.

Consider now a perturbation of φ and θ, such that both $\delta\varphi$ and $\delta\theta$ are proportional to $\exp(S(t) - i\mathbf{k}\cdot\mathbf{x})$, where k is the spectral index. An exponentially growing mode is present if $\mathrm{Re}\,(\frac{dS(t)}{dt}) > 0$ (which obviously means that the linear approximation becomes rapidly inappropriate as the non-linear mode begins to grow). Substituting the *Ansatz* for the perturbation into (5.7) and (5.8), collecting terms proportional to $\delta\varphi$ and $\delta\theta$ and requiring that the solution is non-trivial, we get the dispersion relation [41]

$$[\alpha^2 + 3H\alpha + \frac{k^2}{a^2} + 2\frac{\dot{\varphi}}{\varphi}\alpha][\alpha^2 + 3H\alpha + \frac{k^2}{a^2} - \dot{\theta}^2 + U''] + 4\dot{\theta}^2[\alpha - \frac{\dot{\varphi}}{\varphi}]\alpha = 0, \quad (5.9)$$

where $\alpha \equiv dS/dt$ and the adiabatic approximation $\dot{\alpha} \ll \alpha$ has been used. Since we are interested in the possibility of rapidly growing instability modes, we can assume that the amplitude of the homogeneous field evolves slowly. From (5.9) we can then see that for growing modes to exist, $U''(\varphi) - \dot{\theta}^2$ must be negative since then α has solutions that have a non-zero real component. The band of growing modes is roughly

$$0 < k < k_{\rm max} \equiv a(t)\sqrt{\dot{\theta}^2 - U''(\varphi)}. \quad (5.10)$$

The time evolution of the band depends on the potential; if $k_{\rm max}$ is constant or grows with time, the instabilities can develop indefinitely (or more precisely, at least until the linear approximation breaks down). On the other hand, if $k_{\rm max}$ decreases rapidly, an instability may not have enough time to grow significantly before it is red-shifted away from the resonance.

In the case of gravity mediated SUSY breaking, with a potential of the form (4.12), the perturbations grow as [43]

$$\delta\phi = (\frac{a_0}{a})^{3/2}\delta\phi_0 \exp(\int\sqrt{\frac{k^2|K|m^2}{2a^2\dot{\theta}^2}}\mathrm{dt})\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}, \quad (5.11)$$

$$\delta\theta = \delta\theta_0 \exp(\int\sqrt{\frac{k^2|K|m^2}{2a^2\dot{\theta}^2}}\mathrm{dt})\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}, \quad (5.12)$$

where ϕ_0, θ_0, a_0 are the initial values at t_0, and it has been assumed that conditions $k^2/a^2 \lesssim 2|K|m^2$, $H^2 < m^2$ and $|K| \ll 1$ are satisfied. The negativity of K is crucial for the formation of condensate lumps since the pressure in the potential (4.12) is $P_\phi \approx (K/2)\rho_\phi$ [39]. According to the stability analysis of [43, 55], the band of growing modes lies between

$$0 < \frac{k^2}{a^2} < 3m^2|K|. \quad (5.13)$$

Hence the band of growing modes, in this scenario, becomes larger with time due to the expansion of the universe.

The behavior of the instability band in the gauge mediated case corresponding to the potential (4.11) is somewhat similar. At large ϕ the instability band is now [41, 56]

$$\frac{k}{a} \lesssim \frac{m^2}{|\phi|}, \tag{5.14}$$

where it has been assumed that the potential is dominated by the logarithmic part. Since the homogeneous field ϕ decreases as $a(t)^{-3}$, the instability band grows with time, which again allows fluctuations to grow.

The growth of fluctuations into Q-balls in the two scenarios, were first considered in [39, 41, 43]. These analytical and numerical studies, which were based on the linear approximation, were supplemented by a number of numerical works [55]-[57]. In these studies the formation of Q-balls from an initial homogeneous condensate with small random perturbations has been demonstrated in three dimensional lattices in both the gravity and gauge mediated SUSY breaking cases. Nearly all of the charge in the condensate was found to be contained in Q-balls (and anti-Q-balls) in all cases.

Three dimensional simulations require a large amount of CPU time and that is why only a few simulations have been done [55, 56]. In particular, the initial charge-energy density ratio of the condensate,

$$x \equiv \frac{\rho(\phi)}{mq(\phi)}, \tag{5.15}$$

was assumed to be approximately one in these studies. The condensate was seen to fragment into Q-balls with no anti-Q-balls visible.

To further explore the fragmentation of the AD-condensate, the Q-ball formation process in the gravity mediated case was studied in Paper IV by two dimensional lattice calculations. Simulations were done only in two spatial dimensions simply to reduce the required CPU time so that a larger parameter space could be explored. Furthermore, analytical considerations done in Paper IV show that, at least for $x \gg 1$, the two and three dimensional cases should give similar results.

If one assumes that most of the baryon asymmetry of the universe was contained in Q-balls formed from the AD-condensate, it is likely that the energy-charge density ratio is naturally much larger than one, $x \gg 1$. Obviously the exact value of x will depend on the details of the theory but if we wish to have a small baryon asymmetry, $\Delta Q = (Q_+ - Q_-)/(Q_+ + Q_-)$, contained in Q-balls at early times, a large value of x is necessary [48].

A large x implies that there is a significant amount of extra energy in the system compared with the energy that is stored in Q-balls with an equal amount of charge. It is then expected that the fragmentation of the AD-condensate produces also anti-Q-balls and that small Q- and anti-Q-balls are relativistic. In Paper IV it was argued that the Q-ball distribution will reach an equilibrium state with a one-particle partition function

$$Z_1 = 2gV_D\beta\left(\frac{m}{2\pi\beta}\right)^{\frac{D+1}{2}} \int_{-\infty}^{\infty} dQ|Q|^{\frac{D+1}{2}} \mathrm{e}^{\mu Q} K_{\frac{D+1}{2}}(\beta m Q), \tag{5.16}$$

where D is the number of spatial dimensions, μ is the chemical potential related to the charge of the Q-balls, $K_n(z)$ is the modified Bessel function, and it has been assumed that the energy-charge relation of a Q-ball in this potential is $E \approx mQ$. Fixing the total energy and charge of the distribution, the chemical potential can be found in terms of x. In Paper IV it was found that the chemical potential nearly vanishes at large x and that the average velocity of Q-balls is relativistic but that large Q-balls move slowly. The reaction rate is hence found to be larger than the Hubble rate and one can expect that the small quickly moving Q-balls thermalize the distribution.

The numerical simulations performed in Paper IV confirm what one expects on the basis of analytical arguments. As x was varied, it was found that at $x \approx 1$ only Q-balls are present at the end of the simulation, whereas at larger x a large number of Q-balls as well as anti-Q-balls appear. This is quite natural simply from the conservation of energy and charge; in the $x \approx 1$ case the amount of energy in the condensate is just right to accommodate the charge in Q-balls, no anti-Q-balls are needed. However, if there is a large amount of extra energy in the system, it must be accounted for in some form. Some of the energy invariably goes into the kinetic energy of small Q-balls, but it is also possible that Q-ball - anti-Q-ball pairs with equal and opposite charge appear.

The simulations also confirms the analytical expectations of an equilibrium distribution of Q-balls. The condensate relaxes into the state of maximum entropy and the Q-ball charge distribution is well represented by a Maxwell-Boltzmann distribution.

A copious production of anti-Q-balls is also expected to occur in the gauge mediated scenario for large x, because again the energy cannot all be accommodated in Q-balls. Due to the non-linear energy-charge relation, the analytical study is less straightforward than in the gravity mediated case. Future studies are needed to shed light on this question.

Chapter 6

Properties of Q-balls

Q-balls possess a number of properties that are interesting in themselves and need to be studied for better understanding of the significance of Q-balls for cosmology.

6.1 Evaporation of Q-Balls

In realistic theories the Q-ball field will obviously couple to other fields as well as to itself. The interactions with other fields may cause charge loss from the Q-ball through an evaporation process. Evaporation was first studied in [58], where a semi-infinite Q-ball with a step-function boundary was considered.

Consider a field theory with the Lagrangian density

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* - U(\phi^* \phi) + \psi^\dagger (i\partial_0 + i\sigma \cdot \nabla)\psi - ig\phi\psi^\dagger \sigma_2 \psi^* + ig\phi^* \psi^T \sigma_2 \psi, \tag{6.1}$$

where ψ is a two component Weyl spinor, ϕ is the Q-ball scalar field, and σ are the Pauli matrices. From (6.1) we can see that, in addition to the Q-ball field, the theory also includes a massless fermion field which couples to the Q-ball with a coupling g. In other words, we are considering massless fermions in the presence of a classical background field, the Q-ball. The Fermi sea inside the Q-ball is filled, and hence evaporation will only occur at the surface of the Q-ball [58]. From (6.1) we can also read that evaporation proceeds via pair production.

The equations of motion for ψ and $\chi \equiv i\sigma_2 \psi^*$ can be calculated from (6.1), and by choosing the normal modes such that $\psi \sim \exp(-(\omega/2+\omega')it)$ and $\chi \sim \exp(-(-\omega/2+\omega')it)$, the field equations can be written as

$$\left(\frac{\omega}{2} + \omega' + i\sigma \cdot \nabla\right)\psi - g\phi\chi = 0, \tag{6.2}$$

$$\left(-\frac{\omega}{2} + \omega' + i\sigma \cdot \nabla\right)\chi - g\phi\psi = 0. \tag{6.3}$$

Writing the fields by using spherical spinors [59], $\psi = g_1(r)\Omega_{jlm} + if_1(r)\Omega_{jl'm}$, $\chi = g_2(r)\Omega_{jlm} + if_2(r)\Omega_{jl'm}$, $(l = j - 1/2,\ l' = j + 1/2)$, we get the set of radial equations that can be solved analytically in terms of spherical Bessel and Hankel functions at the center of the Q-ball and far away from the Q-ball, respectively.

From the solutions of the radial equations we can identify the in- and out-moving ψ and χ waves outside the Q-ball. The evaporation rate can then be calculated by assuming that there is no incoming χ wave: all outgoing χ flux must have been transmutated by the Q-ball from the ψ wave. The transmutation coefficient T then leads us to the expression for the evaporation rate [58]:

$$\frac{dQ}{dt} = \sum_j \int_0^{\omega/2} \frac{dk}{2\pi}(2j+1)|T(k,j)|^2, \tag{6.4}$$

where $k = \omega/2 + \omega'$. The upper bound for the evaporation rate per unit area of the Q-ball surface was calculated in [58] to be

$$\frac{dQ}{dtdA} \leq \frac{\omega^3}{192\pi^2}. \tag{6.5}$$

For the step-function boundary, the in- and out- solutions can be easily matched and the transmutation coefficient expressed analytically. In the general case, where the Q-ball profile is not necessarily well represented by a step-function, the calculation of the transmutation rate is a numerical problem that was studied in Paper I. The radial equations can be solved numerically with the initial conditions coming from the known solution near the origin. The problem hence reduces to finding the right initial conditions in such a way that far away from the Q-ball there is no incoming χ-wave. The transmutation coefficient and the evaporation rate can then be calculated in a straightforward fashion.

In addition to checking that the numerical calculation reproduces the results of [58], realistic Q-ball profiles were also studied in Paper I. The evaporation rates of Q-balls were calculated in three different potentials; in two polynomial potentials (3.40) and along a flat direction in the MSSM with supersymmetry broken by a gravity mediated mechanism.

In all of the three cases it was found that Q-balls lose their charge the faster the smaller they are. An evaporating Q-ball will then lose its charge at an accelerating rate as time passes. The fate of the Q-ball is therefore governed by the characteristics of the energy-charge relation, which determines whether scalar decays begin below some critical charge or not. Note however, that if a Q-ball is energetically stable for all values of Q the semi-classical description applied here is not valid all the way to zero charge and a quantum mechanical approach is required to understand how such a small Q-ball loses its charge[1].

The evaporation of Q-balls in supersymmetric theories breaking was considered recently in [61]. There a thin-walled Q-ball was studied, with a mass-to-charge ratio greater than 100 MeV. It was found that the evaporation rate of large Q-ball in the gauge mediated case increases with increasing charge, but that the evaporation rate of a Q-ball with charge $Q \lesssim 10^{34}$ in the gravity mediated scenario, decreases with increasing charge.

6.2 Q-ball collisions

After Q-balls have been formed, *e.g.* from the AD-condensate, they may collide with each other. Collisions can affect their charge distribution and thereby their evolution in the early universe. For example, fusion of Q-balls can increase the average charge

[1] Recently, quantum corrections to Q-balls in a polynomial potential have been studied in [60].

of the distribution leading to longer decay times. In addition, the scattering of non-topological solitons is by itself an interesting problem that has been studied in the literature [55],[62]-[68] for various potentials. These studies were mostly done in one or two dimensions, except for [68] where Q-ball collisions were studied in three dimensions. Realistic potentials from the MSSM have been considered in [55, 68]. There collisions between two similar Q-balls and between a Q-ball and an anti-Q-ball were studied, but with only a very limited range of phase differences and only for a zero impact parameter.

In Papers II and III, Q-ball collisions were studied more extensively in two spatial dimensions for the two realistic potentials corresponding to the different SUSY breaking scenarios. Due to the similarity of both the Q-ball profiles and energy-charge relations in the cases of two and three spatial dimensions noted already in Chapter 4, the two dimensional simulations are expected to give a good picture of the dynamics in three dimensions as well. To understand the dynamics of the collision processes in more detail, the velocity and the phase difference between the Q-balls were varied, as well as the impact parameter. The potentials studied in Papers II and III were

$$U_1(\phi) = m_1^2\phi^2(1 + K\log(\frac{\phi^2}{M^2}) + \lambda_1\phi^{10}, \tag{6.6}$$

$$U_2(\phi) = m_2^4 \log(1 + \frac{\phi^2}{m^2}) + \frac{\lambda_2}{M_{\rm Pl}^2}\phi^6, \tag{6.7}$$

which correspond to the potentials in the gravity and gauge mediated SUSY breaking scenarios (4.12) and (4.11), respectively (note that there is a typographical error in Eq. (7) in Paper III, the potential should read as Eq. (6.7)). The potential in the gravity mediated scenario corresponds to the $d = 6$ flat direction and the parameters were chosen to have values $m_1 = 100$ GeV, $K = -0.1$, $\lambda_1 = M_{\rm Pl}^{-6}$ and $M \sim 10^{11}$ GeV. The parameters in the gauge mediated case were chosen as $m_2 = 10^4$ GeV, $\lambda_2 = 0.5$. A different choice of parameters is not expected to give significantly different results.

Collisions were studied for different values of charge, relative phase, and impact parameter. The relative phase between Q-balls simply refers to a non-zero phase difference in the Q-ball frequencies, $0 < \Delta\omega < 2\pi$ (note that in Papers II and III, the relative phase difference $\Delta\omega$ was referred to as $\Delta\phi$). The colliding Q-balls had for the most part of the simulations equal initial charges, but a limited study was done for Q-balls of unequal charges in the gravity mediated case.

The studied collision processes can be roughly divided into three types: fusion, charge exchange (also referred to as charge transfer), and elastic scattering. Fusion is defined as a process where most of the initial charge is after the collision in a single Q-ball and the rest of the charge is lost as radiation or small lumps of charge. In the charge exchange process some of the charge is transferred from one Q-ball to the other while the total charge of the two Q-ball system is conserved. Elastic scattering was defined to be a process where less than 1% of the initial charge was transferred from one Q-ball to another.

The relative phase difference is a critical parameter in determining the type of a Q-ball collision process, assuming that the speed of the Q-balls is not too large ($v \lesssim 10^{-2}$). If the Q-balls are in phase, *i.e.* their complex phases rotate in uniform, colliding Q-balls fuse. As the phase difference is increased, at some value of $\Delta\omega$ the Q-balls begin to scatter while charge is transferred between them. The amount of charge transfer in the collision process decreases with increasing $\Delta\omega$, so that after a critical value Q-balls start to scatter elastically. Varying the impact parameter, the scattering cross-section for each process can be calculated. The cross-sections for fusion, σ_F, charge transfer, σ_Q, as well as the geometric cross-section, σ_G, have been calculated for a range of charges in Papers II and III.

In both cases the cross-sections decrease with increasing velocity, which is as expected because the Q-balls have less time to interact with one another as their velocity increases. In the gauge mediated case the charge transfer cross-section is negligible and the total and geometric cross-sections appear to converge with increasing charge. From the simulations a lower limit for the fusion cross-section in terms of geometric cross-section can be estimated for large, $Q \gtrsim 10^8$, Q-balls:

$$\sigma_F \gtrsim 0.6\sigma_G. \tag{6.8}$$

By using the approximation $R \approx 2^{-1/2}m^{-1}Q^{1/4}$ for the radius of the Q-ball, we get an estimate for the fusion cross-section in three dimensions of large Q-balls in the gauge mediated scenario:

$$\sigma_F \gtrsim 3.8\frac{Q^{1/2}}{m^2}. \tag{6.9}$$

The gravity mediated scenario behaves slightly differently from the gauge mediated case. Now the charge transfer cross-section is significant and is found to be weakly dependent on ω, so that the relative average charge increase of the larger Q-ball after the collision is 10% for $v = 10^{-3}$ and 14% for $v = 10^{-2}$. The total cross-section that includes all the aforementioned processes is also weakly charge dependent. The values of $\sigma_{\rm tot}$ averaged over the different charges are in this scenario

$$\begin{aligned} \sigma_{\rm tot} &= 0.27 \pm 0.01\ \mathrm{GeV}^{-2}\ (v = 10^{-3}), \\ \sigma_{\rm tot} &= 0.19 \pm 0.01\ \mathrm{GeV}^{-2}\ (v = 10^{-2}). \end{aligned}$$

Colliding Q-balls can create an excited intermediate state that has excess energy compared to the energy of a Q-ball with equal charge. Such configurations can also emerge from other processes, which raises the question of the dynamics of an excited Q-ball.

6.3 Excited Q-balls

Lumps of charge that are not Q-balls, have excess energy compared with the energy of a Q-ball with equal charge. Such charge lumps can be produced, for example, during the formation of Q-balls as the AD-condensate fragments, in Q-ball collisions where two Q-balls fuse, and in thermal processes where thermal particles can transfer excess energy to the Q-ball.

The evolution of a single spherical condensate lump in the gravity mediated scenario was studied in [69]. It was found that a condensate lump that is not a Q-ball, *i.e.* it has excess energy compared with the energy of the Q-ball configuration, generally forms a pulsating configuration that loses charge until it reaches a quasi-equilibrium pseudo-breather solution, the Q-axiton. Such a configuration can have a much larger energy per charge ratio than a Q-ball and it can exist even if $Q = 0$ [69]. The Q-axiton is expected to relax to a Q-ball, albeit very slowly compared to the natural time scale, m^{-1}, of the field. However, the numerical studies of Q-axitons were done assuming a spherically symmetric configuration which may constrain the process too much to be accurate. Furthermore, no Q-axitons have been reported in numerical simulations [55, 56], which may be explained by the fact that they can be difficult to observe as intermediate states in the highly non-linear fragmentation process.

Excited Q-ball states have been identified as intermediate states in the studies of Q-ball collisions [62, 68, 70, 71]. In Papers II and III it was found that the fusion of two Q-balls typically created an excited state that had excess energy compared with its charge. This excess energy was then found to emit slowly as the lump relaxed into a Q-ball. It was found in [68] that in high-velocity collisions Q-rings were produced as intermediate states that subsequently fragmented into Q-balls.

A more systematic study of excited Q-balls was conducted in Paper V, where Q-balls with varying excess energies were studied using $2+1$ dimensional numerical simulations in the framework of the gravity mediated SUSY breaking scenario. It was found that a Q-ball with an added random perturbation relaxes slowly compared with the dynamical scale of the field so that there is a suppression factor $\mathcal{O}(10^{-2})$ in the rate at which excess energy is emitted compared with m. This suppression factor increases slightly with increasing charge in the studied range of Q-ball charges, $Q \sim 10^{16} - 10^{18}$, therefore larger Q-balls are expected to emit their excess charge even more slowly. Another observation made from the simulations was that Q-balls can withstand a large amount of excess energy without losing a significant amount of their charge. To significantly reduce the charge of a Q-ball in the gravity mediated scenario, it seems that an excess energy comparable to the energy of the unperturbed Q-ball is needed.

The slow emission rate and the robustness of Q-balls obviously has an effect on how efficient thermal effects are in corrupting a Q-ball. This was discussed in Paper V where it was estimated that *e.g.* a Q-ball with charge 10^{20} should survive reheat temperatures of 10^5 GeV for $|K| = 0.1$, assuming that all of the energy of an incoming thermal particle is absorbed and that the charge replenishment rate of the soft edge is m. However, if also the Q-ball reconfiguration rate is suppressed, which may well be the case on the basis of the slow relaxation rate seen in Paper V, the absorption and charge replenishment will be less effective and Q-balls can survive higher reheat temperatures.

6.4 Q-ball variants

In addition to the Q-balls associated with supersymmetry mostly discussed in this introductory part, a number of other types of Q-balls arising from different mechanisms have been studied. Some of these models are very briefly reviewed here for completeness.

The type of Q-ball mostly discussed in the literature is an abelian Q-ball associated with an abelian group. However, also non-abelian Q-balls associated with a polynomial potential have been studied [8]. The non-abelian group in these models is either $SO(3)$ or $SU(3)$.

Q-balls can also exist in models where the scalar sector of the Standard Model is extended by an additional $U(1)$-singlet, under which none of the Standard Model particles is charged [72]. The breaking of the electroweak symmetry allows for stable, electrically neutral Q-balls to exist. Such a so-called electroweak Q-ball couples to the SM spectrum via Higgs bosons and can serve as a dark matter candidate.

The recent interest in large extra dimensions has lead to a suggestion that Q-balls may arise in that framework [73]. The compactification of an extra dimension leads to a $U(1)$-invariance for a bulk scalar field generated by the translations along the compact dimensions. The conserved charge is associated with the number of the Kaluza-Klein excitations in the compact direction.

Non-commutative field theories, inspired by string theories, have also been considered as a possible framework for Q-balls [74]. A non-commutative scalar field theory with a global $U(1)$-symmetry can have Q-ball solutions that are classically and quantum mechanically stable.

As was mentioned earlier, Q-balls associated with a local $U(1)$-symmetry have also been studied [24]. In this case there exists a maximum charge above which Q-balls become unstable.

Chapter 7

Cosmological implications of Q-balls

The copious formation of Q-balls from the fragmentation of the AD-condensate can leave the universe filled with Q-balls. Such a phase in the history of the universe may obviously be significant for the evolution of the universe and possibly leave a detectable trace. As the effective and natural production of Q-balls from the AD-condensate was realized, a number of different cosmological scenarios involving Q-balls were suggested. For the most part, the discussion of these scenarios presented in this chapter is a review of literature and is included for completeness and to discuss the interesting cosmological aspects of Q-balls.

7.1 SUSY Q-balls as dark matter

As discussed in Sect. 4.7, the flatness of the potential in the gauge mediated scenario, leads to a non-linear energy-charge relation, $E \approx \frac{4\pi\sqrt{2}}{3} m Q^{3/4}$. A B-ball is therefore absolutely stable if $m_b > dE/dQ = \pi\sqrt{2} m Q^{-1/4}$, where m_b is the mass of the lightest baryon that can be emitted from the B-ball. The stability limit for a B-ball of charge Q is then

$$Q \gtrsim 4\pi^4 (\frac{m}{m_b})^4. \tag{7.1}$$

Assuming that the lightest baryon is a nucleon, $m_b \sim 1$ GeV, this translates into a lower limit of $Q \gtrsim 10^{10}$ for $m = 10^2$ GeV and $Q \gtrsim 10^{18}$ for $m = 10^4$ GeV. Smaller Q-balls, as well as lepton number carrying Q-balls that can emit (nearly) massless neutrinos, can evaporate their charge as described in Sect. 6.1. Depending on the details of the potential, Q-balls formed from the fragmentation of the AD-condensate can satisfy the stability bound [41, 56]. Such absolutely stable Q-balls can then be responsible for at least some of the dark matter content of the universe [41]. Recently it was also pointed out that Q-balls possess a number of interesting features that make them a candidate for self-interacting dark matter [77]. The detection of dark matter Q-balls is discussed in Sect. 7.6.

7.2 B-ball baryogenesis

Q-balls in the gravity mediated SUSY breaking case have a linear charge-energy relation and are therefore susceptible to decay. Their role in cosmology is then somewhat different from the role of the absolutely stable Q-balls in the gauge mediated scenario.

Depending on the flat direction of the potential, a Q-ball formed from the AD-condensate can carry either baryon or lepton number. Its lifetime, due to evaporation is [39]

$$\tau \approx \frac{48\pi}{R^2\omega^3}Q, \tag{7.2}$$

where R is the radius of a thick-walled Q-ball and it has been assumed that the upper bound for the evaporation rate given in Eq. (6.5) has been saturated. The temperature of the universe at which Q-balls decay, is given by [43]

$$T_d \approx 0.06(\frac{f_s}{|K|})^{1/2}(\frac{m}{100 \text{ GeV}})^{1/2}(\frac{10^{20}}{Q})^{1/2} \text{ GeV}, \tag{7.3}$$

where f_s is a possible enhancement factor of the decay rate due to a decay mode into pairs of light scalars. For the purely baryonic $d = 6$ $(u^c d^c d^c)^2$ direction, $f_s \approx 1$, whereas for a direction containing sleptons, $f_s \approx 170(1 + 2.1g^4)|K|^{-1/2}$ (g is the coupling between the condensate scalars and the light decay products) [43]. The charge of a typical Q-ball coming from the fragmentation of the AD-condensate in this scenario has been estimated to be [43]

$$Q \approx 2 \times 10^{15}\sqrt{|K|}(\frac{100 \text{ GeV}}{m})(\frac{10^9 \text{ GeV}}{T_R})(\frac{40}{\alpha})(\frac{\eta_B}{10^{-10}}), \tag{7.4}$$

where T_R is the reheat temperature, $\alpha = -\log(\delta\phi_0/\phi_0)$, ϕ_0 is the value of the field when the first perturbation goes non-linear, and η_B is the value of the baryon asymmetry during the period dominated by the oscillations of the inflaton. The reheat temperature enters (7.4) due to the fact that the baryon asymmetry of the universe during inflation oscillation domination [39, 43] is

$$n_B = \frac{\eta_B H^2 M_{\rm Pl}^2}{2\pi T_R}. \tag{7.5}$$

Independently of these analytical arguments, the numerical simulations, at least in the $x \approx 1$ case [55], show that depending on the dimension of the non-renormalizable operator, the largest Q-balls produced are in the range $Q \sim 10^{16} - 10^{23}$. Q-balls can then, depending on the reheat temperature and details of the model, decay after the electroweak (EW) transition.

The fact that possibly a large fraction of the baryon number of the universe is stored in decaying Q-balls offers a way to protect the baryons from the sphaleron transitions that are effective at high temperature [75]. The weak sphaleron transitions violate $B + L$ at temperatures above the EW transition and can wash out the baryon number created by the conventional AD-mechanism. If the baryon number is stored in Q-balls, however, baryons can be protected from the electroweak wash out, assuming that the Q-balls decay after the EW phase transition [39, 43].

7.3 Baryon to dark matter ratio

In addition to the B-ball baryogenesis, where the baryon asymmetry is created from decaying B-balls, the fact that Q-balls once might have stored a large fraction of the total baryon number may provide us with an a natural explanation of the baryon to dark matter ratio of the universe [43, 76]. The observed baryon number will be a combination of the baryon number from the decays of Q-balls and what was left from the original AD-condensate (if it has not been destroyed by anomalous $B+L$ violation). The decay of squarks in the Q-ball leads to the production of supersymmetric particles, that decay into the lightest supersymmetric particle (LSP), typically the neutralino. After Q-ball decay there will hence be a background of LSPs from the Q-balls, in addition to the thermal relic background of LSPs. The fractions of LSPs coming from the two sources depend on the reheat temperature and the LSP freeze-out temperature. If the reheating temperature is low compared with the LSP freeze-out temperature, which is favorable for B-ball formation and baryogenesis, the thermal relic density is negligible compared with the density from the Q-balls decays. The LSP density is then determined by the decays of Q-balls [43, 76], *i.e.*

$$n_{BB} \approx 3 f_B n_B, \tag{7.6}$$

where n_{BB} is the LSP density, f_B is the fraction of baryons in Q-balls, and n_B is the total baryon number density. In order for the decays of Q-balls to explain naturally the baryon to dark matter ratio of the universe, the annihilations of LSPs must also be insignificant. This then requires that for a light neutralino, the B-ball decay temperature must be $T_d \lesssim 1$ GeV, which requires a reheat temperature in the range $T_R \lesssim (10^3 - 10^5) f_s^{-1}$ [43]. Assuming that the LSP annihilations are negligible and the Q-ball decay temperature is low enough, the baryon to dark matter ratio can be accounted for if [43]

$$3.7 \text{ GeV} \lesssim (\frac{N_\chi}{3}) f_B m_\chi \lesssim 67 \text{ GeV}, \tag{7.7}$$

where N_χ is the number of LSPs produced per baryon number and m_χ is the mass of the neutralino.

The baryon to dark matter ratio has been studied in the gauge mediated scenario in [78]. It was suggested that thermal effects transport charge from Q-balls to the surrounding plasma. The thermal effects are most effective at temperatures $T \sim m_\phi$, where m_ϕ represents the mass of squarks (suppressed by some couplings to the Q-ball field). To survive thermal erosion, the charge of a Q-ball should obey

$$Q \gtrsim (\delta \frac{M_0}{m} \ln(\frac{m}{m_\phi}))^{4/3}, \tag{7.8}$$

where $\delta \lesssim 1$ parametrizes the deviation from the maximum evaporation rate and $M_0 \sim 3 \times 10^{17}$ GeV [78]. A Q-ball loses some of its baryon number to the surrounding plasma so that the amount of baryons and Q-balls, which later act as dark matter, are related. In [78] it was estimated that the correct baryon photon ratio $\eta = \eta_B/\eta_\gamma \sim 10^{-10}$ can be obtained for $m \sim 10^3 - 10^4$ GeV if the initial charge of Q-balls is in the range $Q \sim 10^{23} - 10^{28}$. This calculation was, however, critically reviewed in [80], where it was argued that due to a suppression of the rate at which diffused charge can be transported away from the Q-ball, the number density of baryons outside the Q-balls is smaller that suggested in [78]. This then implies that to obtain the correct baryon to photon ratio at the present epoch, the Q-ball charge must satisfy $Q \gtrsim 10^{21} - 10^{23}$.

The fraction of the total baryon number of the universe contained in Q-balls, f_B, is obviously important in explaining the baryon to dark matter ratio. In numerical simulations it has been found that in the both SUSY breaking scenarios nearly all of the baryon number in the AD-condensate is in Q-balls after the condensate fragments [55, 56, 57]. Therefore, if the AD-condensate is the only source of baryons, $f_B \approx 1$. However, $f_B = 1$ may be inappropriate for the both cases, unless thermal effects are strong enough to change f_B, which in turn requires a large enough reheat temperature. In the gauge mediated scenario $f_B = 1$ implies that all of the baryon number is contained in Q-balls, which obviously is unacceptable, assuming that a large majority of the Q-balls are stable. Thermal effects, as discussed previously, can remedy the situation by decreasing f_B. In the numerical simulations a range of Q-ball charges have been observed to emerge of which only a fraction is actually stable but still contain most of the charge [55, 56].

Also in the gravity mediated scenario $f_B \approx 1$ may be inappropriate [79, 81]. Assuming that the energy-charge ratio of the AD-condensate is $x \approx 1$ and that there are no subsequent annihilation of neutralinos, the experimental bound for the mass of the neutralino from ALEPH [82] implies $f_B \lesssim 0.64$, assuming a conservative nucleosynthesis limit on primordial element abundances [79]. Increasing x would not in general remedy the situation since then there will simply be more neutralinos and anti-neutralinos produced in the decay of Q- and anti-Q-balls, unless neutralino annihilations take place. A possible solution is that a $x > 1$ condensate may fragment into charged lumps with excess energy that then evolve into Q-balls before the neutralino freeze-out [69, 79]. In this case f_B can be small enough to allow MSSM neutralinos compatible with experimental constraints to arise from Q-ball decays [79].

7.4 Q-balls and phase transitions

The charge accretion mechanism that leads to the formation of Q-balls at a phase transition can also enhance the phase transition itself that would otherwise be effectively suppressed by a negligible tunneling rate of a critical bubble [50, 51]. A Q-ball may, via charge accretion, grow large enough to become unstable and fill the space with the true vacuum, *i.e.* it can precipitate a phase transition [50, 51]. Again, a critical property of Q-balls that other non-topological solitons often do not possess, is that Q-balls associated with supersymmetric models may exist for very small charges. Therefore, an initial Q-ball is more likely to appear from thermal fluctuations than a large non-topological soliton of some other type.

7.5 Thermal effects

As was already discussed, thermal effects can be significant in the early universe. A Q-ball can lose some of its charge to the surrounding plasma or possibly be completely erased by thermal effects. Thermal effects on Q-balls have been discussed in [41, 43, 78, 80, 83].

Q-balls in a flat potential at a non-zero temperature were studied by thermodynamical considerations in [78], where it was found that for a system with non-zero conserved charge density, the ground state at any temperature is a single Q-ball in the infinitely large volume limit. If on the other hand, the total charge in the considered volume is fixed, *i.e.* charge density decreases as $V \to \infty$ like in the early universe, a state that includes a Q-ball is more favorable than a homogeneous plasma if [78]

$$V \lesssim \frac{Q_{\rm tot}^{5/4}}{M(T)T^2}, \tag{7.9}$$

where $M(T)$ is the effective mass of the Q-ball scalar and $Q_{\rm tot}$ is the total charge. From (7.9) one can then estimate at which temperature a Q-ball will evaporate due to thermal effects. All of these conclusions are due to the non-linear energy-charge relation of the Q-ball in a flat potential, $E \sim Q^{3/4}$, which makes it energetically favorable to store charge in a large Q-ball.

A different approach to study the thermal effects on Q-balls has been adopted in [41, 43, 83], where the collisions of thermal particles with a Q-ball were considered. These effects can be divided into two classes [43]: dissociation, where thermal particles transfer energy to the Q-ball that can overcome the binding energy, and dissolution, where charge is removed from the soft edge of a thick-walled Q-ball due to thermalization.

In a dissociation process [41, 43], a thermal particle with energy T, interacting with the Q-ball field, φ, with coupling g gains an effective mass of $g\varphi$ inside the Q-ball and can hence penetrate it up to a distance where $T \sim g\varphi$. If the time scale over which the particle stops is short compared with the absorption time scale of the Q-ball, some of the kinetic energy of the particle will be transferred to the Q-ball. If the rate at which energy is transferred to the Q-ball is larger than the energy emission rate, excess energy can build up in the Q-ball and overcome its binding energy. Hence, to avoid dissociation, condition

$$\Delta E(\delta t_r) < \delta m(Q)Q, \tag{7.10}$$

must be satisfied. Here $\Delta E(\delta t_r)$ is the energy delivered to the Q-ball in a time over which a Q-ball can radiate its excess energy without losing charge, and $\delta m(Q)$ its binding energy per unit charge.

Dissolution [43] progresses via charge transfer from the edge of the Q-ball to the surrounding plasma. A thick-walled Q-ball has a soft edge over which the the φ field is approximately constant, $\delta\varphi/\varphi \lesssim 1$. Thermal background quarks have a much shorter mean free path, $\lambda \approx k_q/T$ ($k_q \approx 6$) [84], than the width of the soft edge of a Gaussian Q-ball in the gravity mediated SUSY breaking scenario [43]. A thermal equilibrium should hence exist within the soft edge and charge can leave the Q-ball by diffusion. The diffusion, rate as well as the rate at which the Q-ball can replenish charge in the soft edge, are therefore crucial in evaluating the significance of thermal effects. Assuming that the charge diffusion is a random walk effect and that the Q-ball can replenish charge in the soft edge approximately at rate m, the charge loss can be estimated to happen at rate [43],

$$\frac{1}{B}\frac{\partial B}{\partial t} \approx \frac{4\pi\beta^3 k_q T}{g^2\varphi_0^2 R^2}, \tag{7.11}$$

where $\beta = \sqrt{\log(g\varphi_0/3T)}$ and φ_0 is the magnitude of the field inside the Q-ball. Note that (7.11) is effective only at high temperatures, *i.e.* when $T \gtrsim 4\beta^2 k_q |K| m$, where the Q-ball has time to replenish charge in the soft edge (charge is diffused more slowly at higher temperatures).

Dissolution and dissociation clearly limit the allowed reheat temperature of the universe if Q-balls are to survive, since any previously formed Q-balls can be erased by the thermal bath if the universe is reheated too much. In the gravity mediated SUSY breaking case it was estimated that, using (7.4) for the charge of Q-balls coming from the fragmentation of the AD-condensate, reheat temperature should be less than $10^3 - 10^5$ GeV for $|K| = 0.01 - 0.1$ to avoid significant thermal erosion of Q-balls [43]. In the gauge mediated case, the survival of a Q-ball from dissolution leads to the following bound for the charge of a Q-ball [41]:

$$Q \gtrsim (\frac{T_{\rm R}}{m})^2 (\frac{M_{\rm Pl}}{1.7\sqrt{n_{\rm eff}} m}), \tag{7.12}$$

where $n_{\rm eff} \sim 100$ is the number of effective degrees of freedom. For $m = 10^4$ GeV, this give an upper limit of $T_{\rm R} \lesssim 10^{-3} Q^{1/2}$ for the reheat temperature.

The dissociation and dissolution processes were also studied in Paper V. These were discussed in more detail in association with excited Q-balls in Sect. 6.3.

7.6 Detection of Q-balls

Q-balls produced in the early universe can, at least in the gauge mediated scenario, survive till the present time. It is then an interesting question to study the possibility of detecting Q-balls either directly with present experiments or indirectly via some astrophysical processes.

The detection and properties of a Q-ball depend on the details of the fields that the Q-ball is made of. The flat direction along which the Q-ball has a large vev obviously dictates which fields acquire these large expectation values. Typically some combination of squark, slepton or Higgs fields have a non-zero vev inside a Q-ball. If a Q-ball carries baryon number ($B \neq 0$), the $SU(3)$ symmetry inside it will be broken. On the other hand, the electroweak $SU(2)$ symmetry may be restored if all of the fields inside the Q-ball are $SU(2)$ singlets, *e.g.* a squark Q-ball associated with the $u^c d^c d^c$ direction. The exact properties of a Q-ball with $B \neq 0$ hence depend on whether the squark fields are accompanied by slepton or Higgs fields. A nucleon entering a baryonic Q-ball will experience QCD deconfinement and dissociate into quarks. The energy released in the process is typically emitted as pions [45]. An electrically neutral Q-ball passing through matter will therefore absorb nuclei by transforming the deconfined quarks into squarks via a gluino exchange [45]. The squarks are then absorbed into the condensate within the Q-ball.

The fate of the electrons in matter are determined by the electric properties of the passing Q-ball. The absorption of quarks into the Q-ball leads to a positively charged condensate unless electrons can be trapped at an equal rate. Depending on whether electroweak symmetry is restored inside the Q-ball or not, electrons can either enter the core of the Q-ball or form a bound state with the positively charged core [45]. The ability to retain charge divides the Q-balls into two classes: supersymmetric electrically neutral solitons (SENS) and supersymmetric electrically charged solitons (SECS) [45].

SECS passing through matter at a velocity $v \sim 10^{-4} - 10^{-2}$ lose their energy mainly via the interaction of the SECS core with nuclei and electrons in the transversed medium [45, 85]. The rate of energy loss is dominated by the electronic losses for $v > 10^{-4}$ [85, 86]. SECS can also catalyze proton decay, but only if they have large masses and velocities [87]. The energy release from SENS, is mainly via their interactions with nucleons that dissociate and get absorbed into the core of the Q-ball. The energy is released typically as pions, so that only a small amount of the kinetic energy of SENS is lost. For example, a large, $Q \sim 10^{24}$, Q-ball passing through matter will release about 100 GeV of energy per centimeter in both cases [45].

Experimental limits on the flux of large Q-balls have been obtained from different sources. The Baikal deep underwater array, "Gyrlyanda", constrains the flux of SENS [88]

$$F_{\rm SENS} \lesssim 10^{-16}{\rm cm}^{-2}{\rm s}^{-1}{\rm sr}^{-1}, \tag{7.13}$$

for a cross-section $\sigma > 1.9 \times 10^{-22}{\rm cm}^2$. The MACRO search [89] constrains the flux of SECS [45]

$$F_{\rm SECS} \lesssim 10^{-14}{\rm cm}^{-2}{\rm s}^{-1}{\rm sr}^{-1}. \tag{7.14}$$

Furthermore, the Kamiokande Cherenkov detector results can be reinterpreted for SENS [90], giving

$$F_{\rm SENS} \lesssim 10^{-15}{\rm cm}^{-2}s^{-1}{\rm sr}^{-1} \tag{7.15}$$

for $\sigma = 10^{-26}{\rm cm}^2$.

If one assumes that stable Q-balls are the main constituents of galactic dark matter, *i.e.* that their mass density in the galactic halo is $\rho_{DM} \approx 0.3$ GeV/cm^3, these limits translate into lower limits for the baryon number of the dark matter Q-balls: for SECS, $Q \gtrsim 10^{21}$ and for SENS, $Q \gtrsim 10^{22}$, for $m = 1$ TeV [45]. The smaller SECS dissipate their energy so quickly, however, that they may never reach the underground detectors, which leaves a window for SECS in the range $Q \sim 10^{12} - 10^{13}$ [45]. No such window exists for SENS since they pass through matter quite freely. On the other hand, the upper limit on the flux of SECS could probably be improved to $10^{-16}{\rm cm}^{-2}{\rm s}^{-1}{\rm sr}^{-1}$ by the MACRO detector [85]. Furthermore, a number of present and forecoming experiments can significantly constrain the flux of non-topological solitons in the near future [90].

In addition to the direct detection, stable (or long living) Q-balls can play an important role in various astrophysical processes. The role of large, baryonic Q-balls that are not associated with supersymmetry, in nucleosynthesis has been studied in [91]. It was found that the presence of such electrically charged Q-balls can reduce the production of ^4He. Furthermore, the possibility of Q-stars [92] as neutron stars further constrains the range of allowed non-supersymmetric Q-balls. However, the nucleosynthesis constraint is not relevant for the Q-balls associated with supersymmetry due to the compactness of these objects.

On the other hand, SUSY Q-balls in the gauge mediated scenario can play a major role in the evolution of a neutron star. In [93] it was suggested that SUSY Q-balls trapped inside neutron stars can absorb the neutrons into the scalar condensate. The Q-ball, due to the non-linear energy-charge -relation, is at a lower energy state than the neutron star and hence energy is released as thermal neutrinos and photons in the absorption process. The Q-ball grows until the star becomes unstable and explodes with a large burst of energy. This has been offered as a possible explanation for gamma-ray bursts [93]. A similar process was also suggested to occur for white dwarfs. These suggestions have since been critically studied in [94], where it was noticed that the capture of SUSY Q-balls in both neutron stars and white dwarfs is probably less likely than what was envisaged in [93]. However, in contrast with [93], the most important trapping mechanism was suggested to be the trapping of Q-balls in supernova progenitors before the neutron star is formed. These considerations may improve the direct detection constraints by several magnitudes for SECS and are hence potentially sensitive detectors of Q-balls. However, further study is still needed to understand the details of the conversion process and the amount of appropriate Q-balls in the cloud forming the neutron star progenitor.

An indirect detection method of primordial Q-balls in the gravity mediated SUSY breaking case arises from the isocurvature fluctuations that should be detectable by the forthcoming cosmic microwave background experiments [95]. In D-term inflation models with low reheat temperature, the fluctuations of the phase of the AD-field are unsuppressed after inflation [40]. Fluctuations of the phase correspond to fluctuations in the local baryon number density, isocurvature fluctuations, while the fluctuations of the amplitude correspond to adiabatic density fluctuations. If the AD-condensate fragments into Q-balls that later decay into the dark matter particles and baryons, the dark matter can have both isocurvature and adiabatic density fluctuations. These result in an enhanced isocurvature contribution relative to the purely baryonic isocurvature fluctuations in the case of conventional AD baryogenesis. The enhancement is so large that it should be observable by MAP and PLANCK [95].

Chapter 8

Conclusions

Any scalar field theory, which is $U(1)$-symmetric and hence possesses a conserved charge, may carry non-topological solitons, Q-balls, in its spectrum. The only stringent requirements are that the potential of the theory has a global minimum at the origin, and it grows slowly enough over some range of field values. The properties of the Q-balls then depend on the details of the theory and the spectrum of Q-balls can range from large thin-walled ones to very small thick-walled Q-balls.

Supersymmetric theories offer a natural framework for Q-balls. The large number of scalars and the existence of flat directions in their scalar potentials provide all the necessary ingredients needed for Q-balls to exist in a theory. Such Q-balls are typically composed of supersymmetric scalars, *e.g.* in the Minimal Supersymmetric Standard Model Q-balls are made of supersymmetric partners of the Standard Model fermions, *i.e.* squarks and sleptons, and possibly from Higgs fields.

In addition to its composition, the details of the flat direction along which the Q-ball fields gain their non-zero value affects the properties of the Q-ball. The supersymmetry breaking mechanism is therefore important for understanding the Q-ball properties. In this work two mechanisms of supersymmetry breaking have been considered where the breaking occurs in a hidden sector and is then transmitted to the visible sector by messenger fields. The nature of the messenger fields determines the details of the potential. If supersymmetry is broken by a gravity mediated mechanism, the energy of a Q-ball grows linearly with charge. The binding energy of such a Q-ball is relatively small and the Q-ball is susceptible to decay. If, on the other hand, the supersymmetry breaking mechanism is transmitted to the visible sector via gauge interactions, the energy of a Q-ball grows typically as $E \sim Q^{3/4}$ over some range of charges. Q-balls in this scenario have a greater binding energy and, if they are large enough, Q-balls can be absolutely stable.

The same framework that naturally leads us to expect Q-balls to appear in supersymmetric theories also provides means to copiously create them in the early universe. The Affleck-Dine -condensate is unstable with respect to fragmentation into Q-balls as the universe expands and cools. The distribution of the formed Q-balls is strongly dependent on the energy and charge present in the condensate before it breaks into fragments, as was presented in Chapter 5.

The studies of the Q-ball properties provide us with an insight to the interesting aspects of these extended objects. The energy-charge -relation, which guides us in determining whether a Q-ball is energetically stable or not, depends critically on the details of the potential, as was emphasized in Chapter 3. A Q-ball, that interacts with other fields and is hence detectable, can evaporate its charge. Such an evaporation process, at least in the three cases studied in Paper I, proceeds at an increasing rate as the Q-ball becomes smaller, much like black holes that lose their mass by Hawking radiation [96]. This behaviour was noticed in studying the evaporation rates of Q-balls with realistic properties as discussed in Sect. 6.1. In Sect. 6.2 it was seen that the relative phase difference in the rotating complex fields of the colliding Q-balls determines the type of the scattering process. Scattering processes shed light on the dynamics of Q-balls. They can also be of importance in altering the distribution of primordial Q-balls coming from the fragmentation of the Affleck-Dine -condensate. The relaxation of Q-balls, which was studied in Sect. 6.3, provides us with an understanding how quickly excess energy is emitted from an excited Q-ball state. Furthermore, the conducted work shows how robust objects Q-balls are in withstanding excess energy.

If Q-balls are produced in the early moments of the evolution of the universe they are evidently of great interest from a cosmological perspective. Again the properties of Q-balls, arising ultimately from the details of supersymmetry breaking, lie in the heart of matters. Stable Q-balls could be responsible for a considerable fraction of the dark matter content of the universe, whereas decaying Q-balls can protect the baryon asymmetry that otherwise could be erased by electroweak processes. In both cases one can offer an explanation for the baryon to dark matter ratio of the universe as coming from a single source. The thermal bath of the early universe plays a role in the Q-ball cosmology as it can remove charge from Q-balls or erase them completely. On the basis of the information gained in studying the relaxation of Q-balls, the effectiveness of the thermal erosion processes can be estimated, as was discussed in Sect. 6.3.

The property of Q-balls which ultimately determines their fate in cosmology is, of course, whether they actually exist or not. If Q-balls are to be considered as a serious dark matter candidate, their mass density must be large and experimental searches are hence important in looking for them in the halo of our galaxy. Such searches, as described in Sect. 7.6, already limit the possible charges and hence masses of the dark matter Q-balls. Future experiments will constrain the limits even more, unless, of course, a positive Q-ball signature is seen in a detector. Even though a significant Q-ball dark matter fraction may be effectively ruled out in the future, unstable, decaying Q-balls would still be of interest in the early universe. A possible signature left by such Q-balls may be imprinted into the cosmic microwave background as enhanced isocurvature fluctuations arising from late decaying Q-balls.

In summary, Q-balls have proved to be interesting objects that can have far-reaching consequences for cosmology. In addition to being typically a part of the spectrum of realistic supersymmetric extensions of the Standard Model, they can also be quite naturally produced copiously in the early universe. However, before supersymmetric particles are actually seen in the detector and a further understanding of what lies beyond the Standard Model is acquired, some healthy skepticism toward the cosmological significance of Q-balls is still in order. Whatever the final verdict on Q-balls might be, Q-balls as field theoretical objects are interesting by themselves. They have been shown to posses a number of interesting qualities, the understanding of which might prove one day to be valuable in different branches of physics. Furthermore, the study of Q-balls has reminded us again how field theories can exhibit a rich phenomenology that may influence and broaden our perspectives on nature.

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